The Nuclear Equation of State: What Do Neutron Stars Tell Us?

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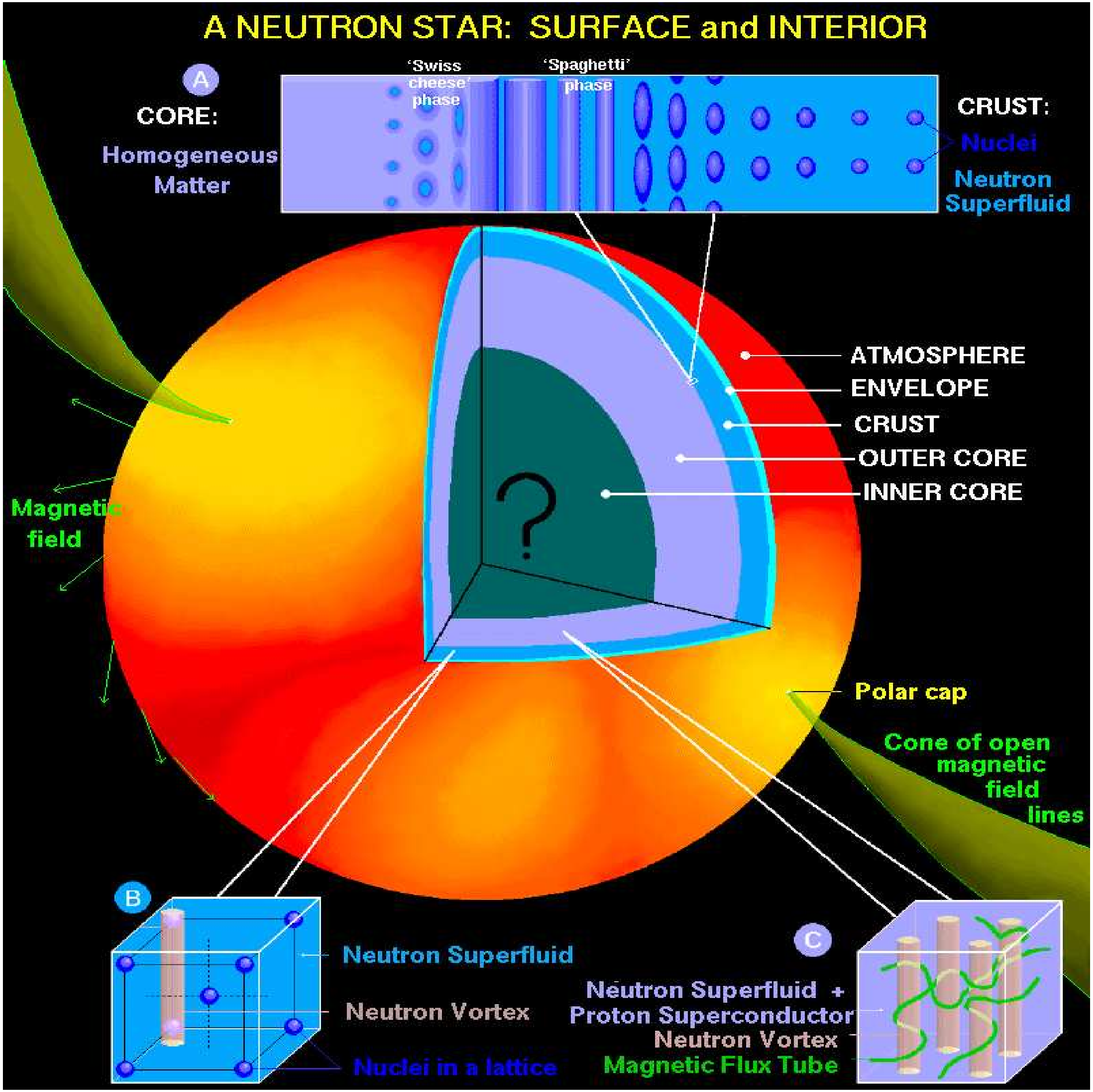


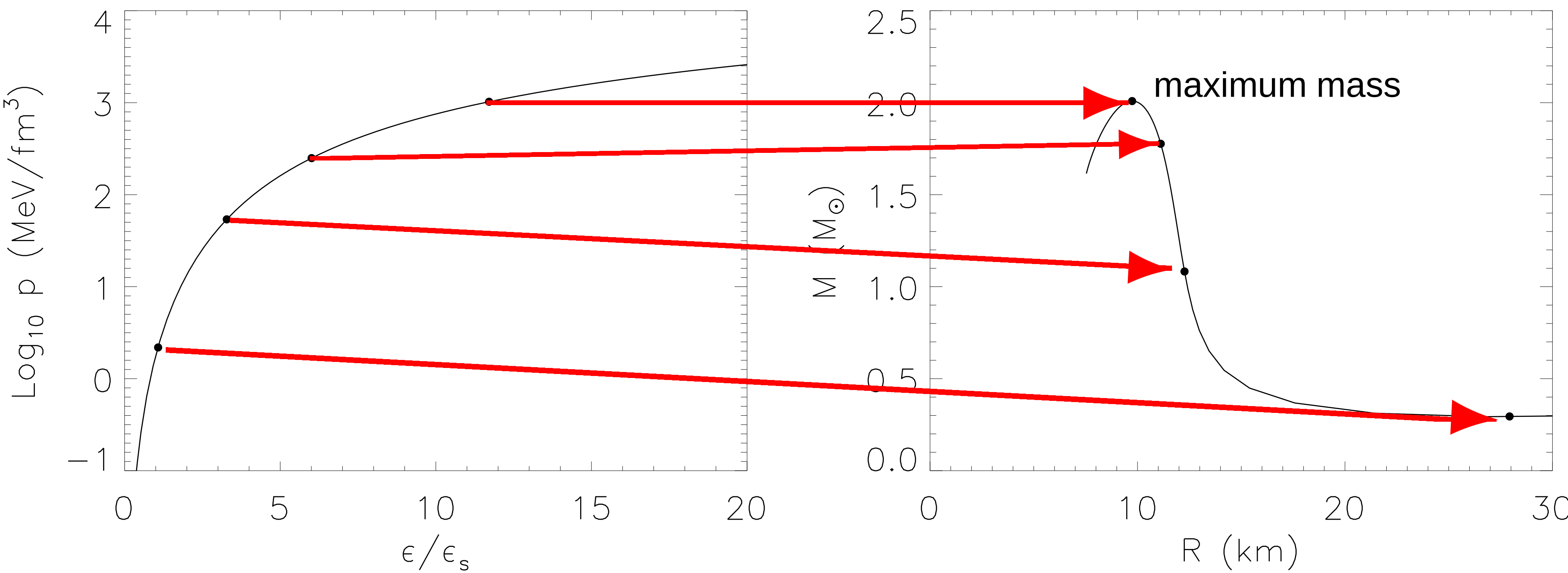
Credit: Dany Page, UNAM

Neutron Star Structure

Tolman-Oppenheimer-Volkov equations

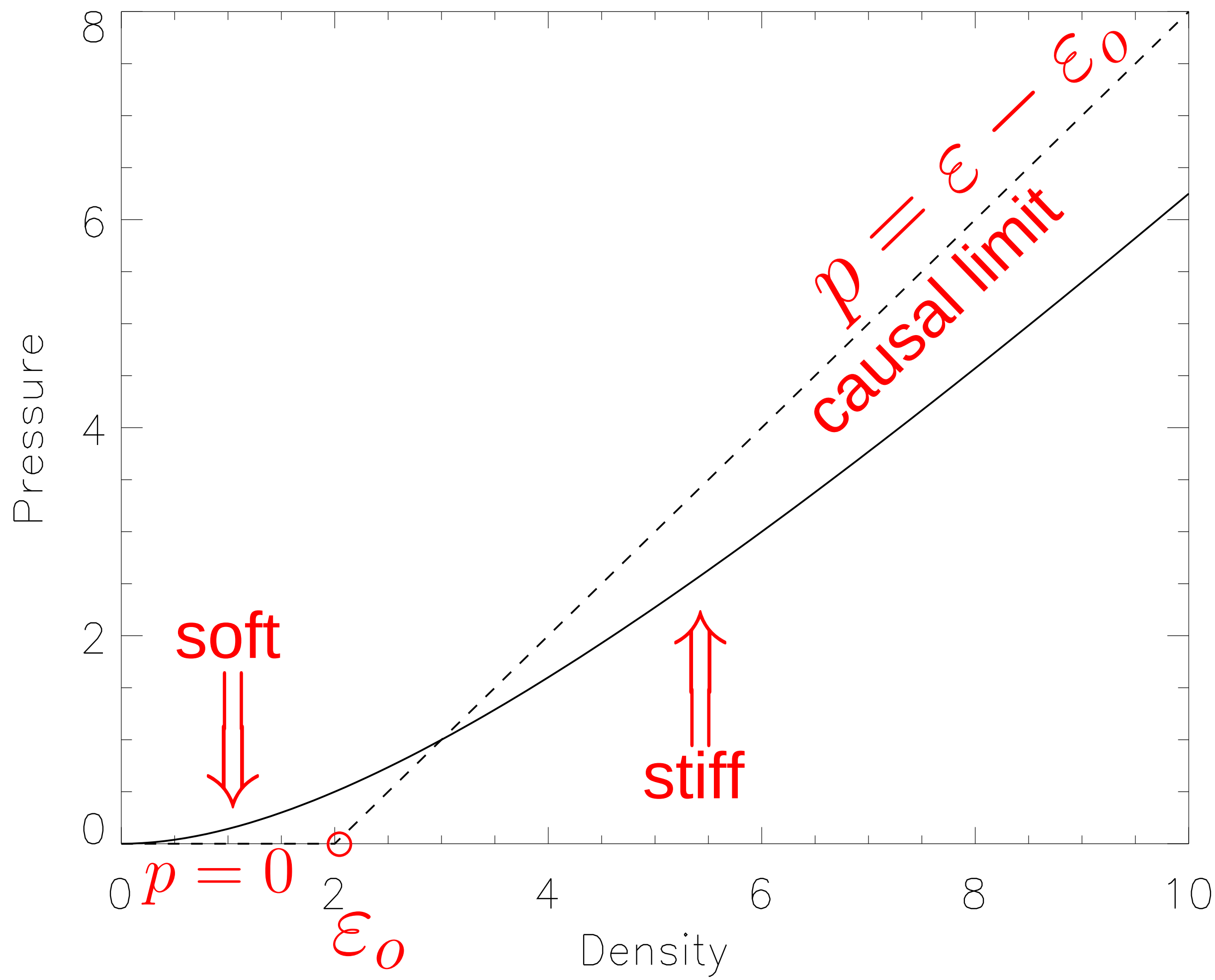
$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m + 4\pi p r^3)(\varepsilon + p)}{r(r - 2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi\frac{\varepsilon}{c^2}r^2$$



Extreme Properties of Neutron Stars

The most compact configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff".



ε_0 is the only EOS parameter

TOV solutions scale with ε_0 even in 2-D

Koranda, Stergioulas & Friedmann (1997)

Maximum Mass, Minimum Period

Theoretical limits from GR and causality

- $M_{max} = 4.2(\epsilon_s/\epsilon_f)^{1/2}\ \mathrm{M}_\odot$ **Rhoades & Ruffini (1974), Hartle (1978)**
- $R_{min} = 2.9GM/c^2 = 4.3(M/\mathrm{M}_\odot)$ km

  Lindblom (1984), Glendenning (1992), Koranda, Stergioulas & Friedman (1997)
- $\epsilon_{central} < 4.5 \times 10^{15}(\mathrm{M}_\odot/M_{largest})^2$ g cm^{-3}

  Lattimer & Prakash (2005)
- $P_{min} \simeq 0.74(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\text{ km})^{3/2}$ ms

  Koranda, Stergioulas & Friedman (1997)
- $P_{min} \simeq 0.96 \pm 0.03(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\text{ km})^{3/2}$ ms

  (empirical) **Lattimer & Prakash (2004)**
- $\epsilon_{central} > 0.91 \times 10^{15}(1\text{ ms}/P_{min})^2$ g cm^{-3} (empirical)
- $cJ/GM^2 \lesssim 0.5$ (empirical, neutron star)

Mass-Radius Diagram and Theoretical Constraints

GR:

$R > 2GM/c^2$

$P < \infty$:

$R > (9/4)GM/c^2$

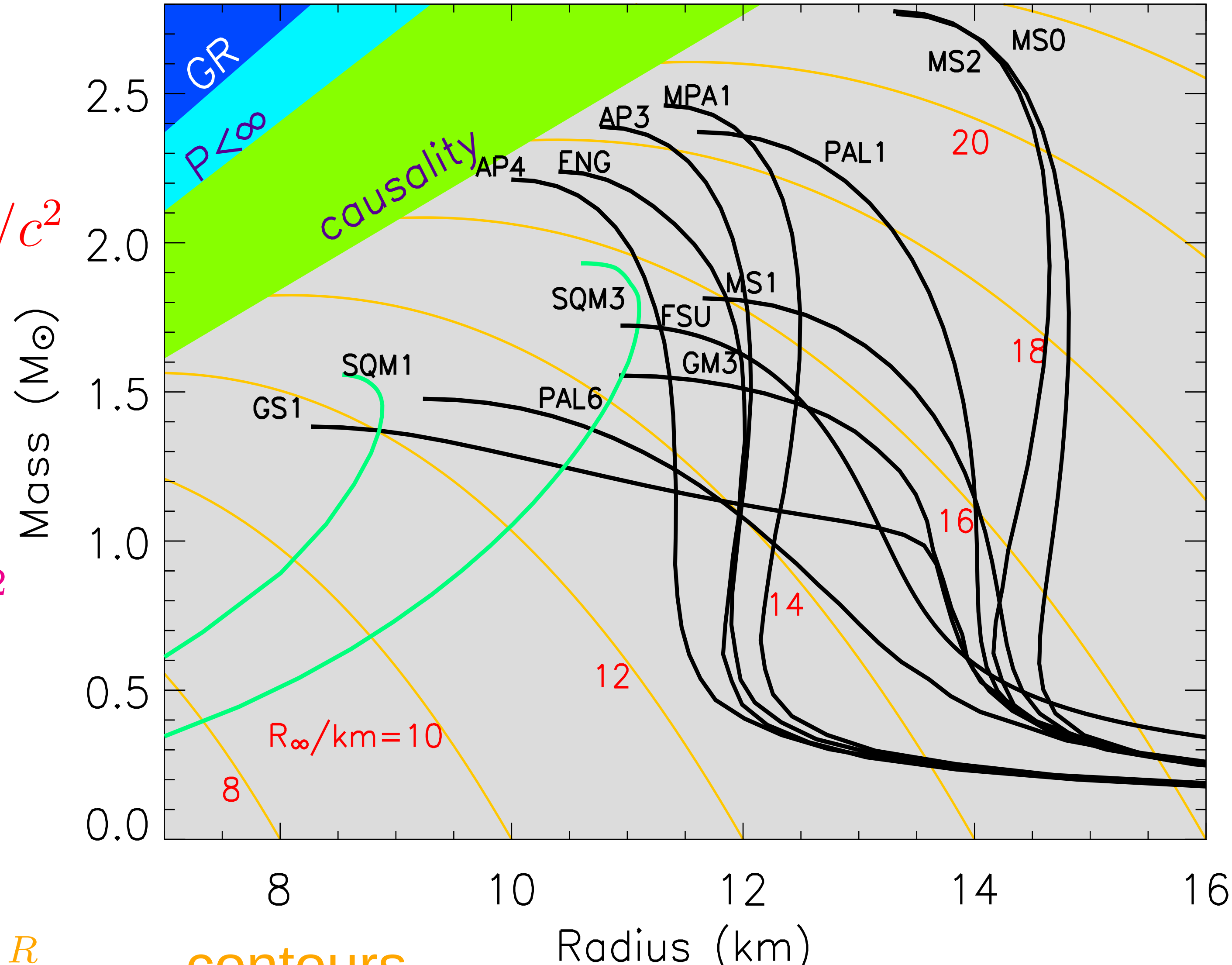
causality:

$R \gtrsim 2.9GM/c^2$

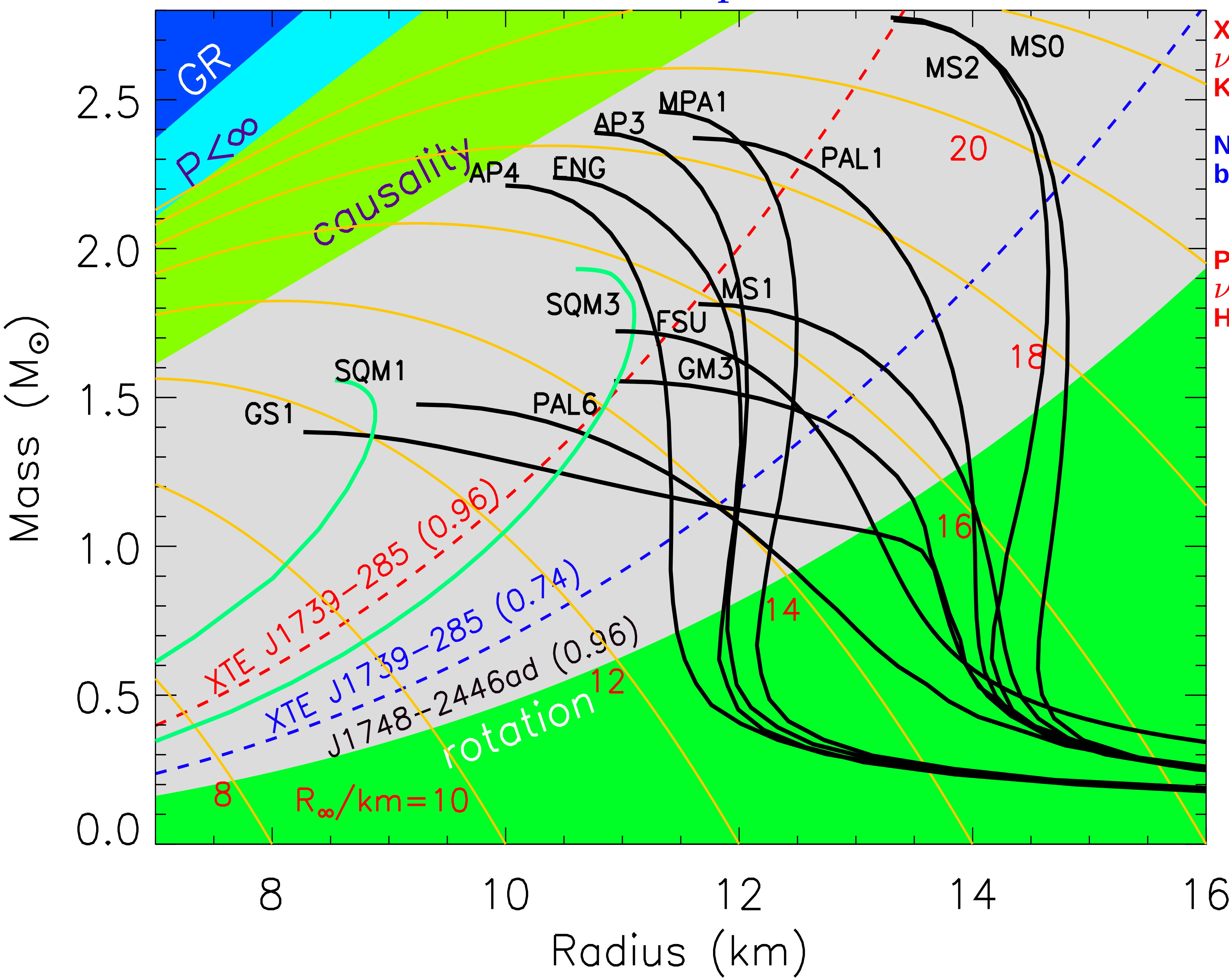
— normal NS

— SQS

— $R_\infty = \frac{R}{\sqrt{1-2GM/Rc^2}}$ contours



Constraints from Pulsar Spins

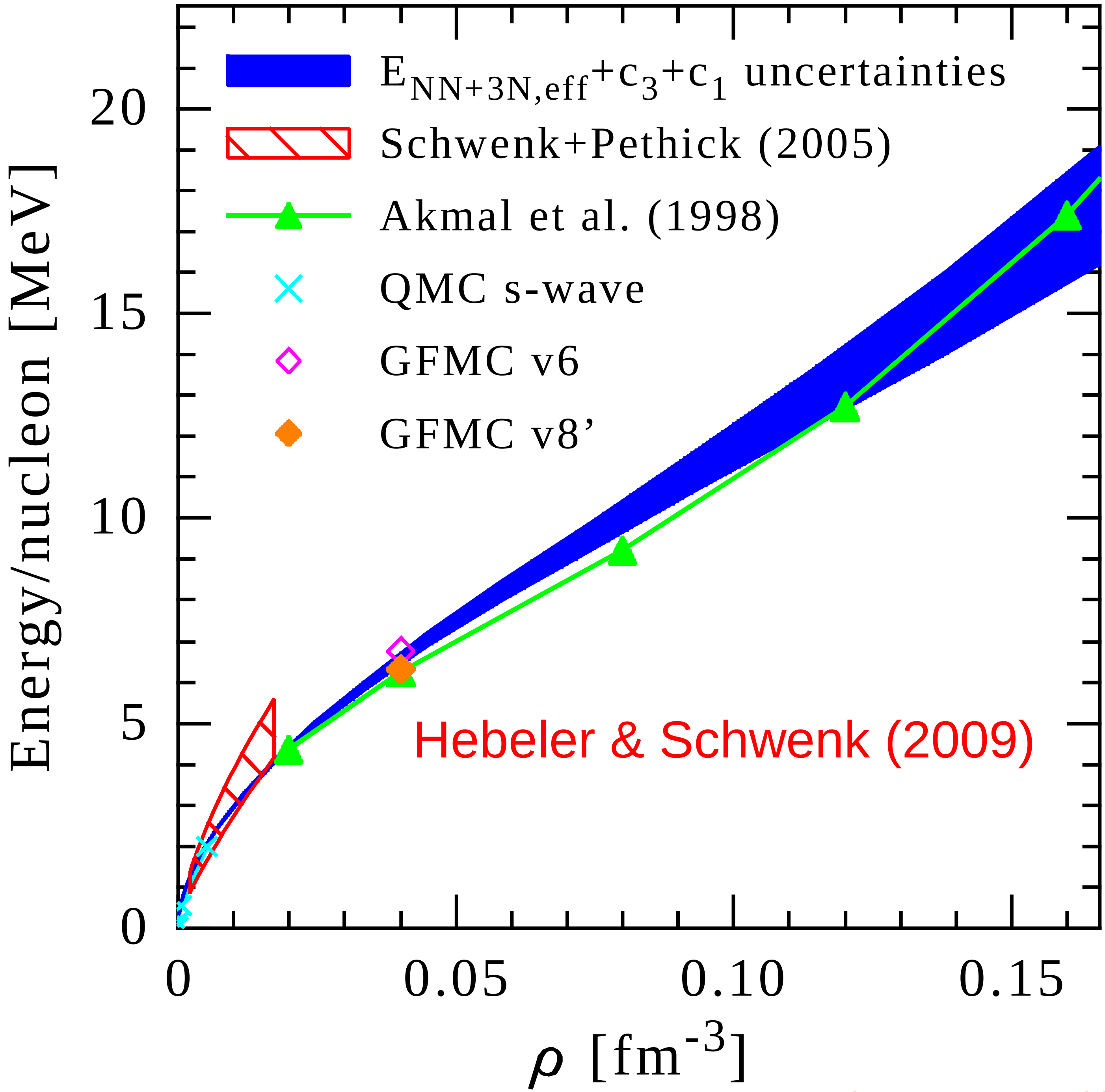


XTE J1739-285
$\nu = 1122$ **Hz**
Kaaret et al. (2006)

Not confirmed to be a rotation rate

PSR J1748-2446ad
$\nu = 716$ **Hz**
Hessels et al. (2006

Pure Neutron Matter



Estimating Symmetry Parameters From Neutron Matter Results

$$E_n = E(n_s, x=0) \simeq 16.3 \pm 2.1 \text{ MeV}, \quad P_n \simeq 2.5 \pm 0.7 \text{ MeV fm}^{-3}$$

$$S_v' \equiv \left.\frac{\partial \ln E_{sym}(n)}{\partial \ln n}\right|_{n_s} = P_n/n_s = 15.6 \pm 4.4 \text{ MeV}$$

- Simple Model

$$E_{sym}(n) = S_v (n/n_s)^p$$

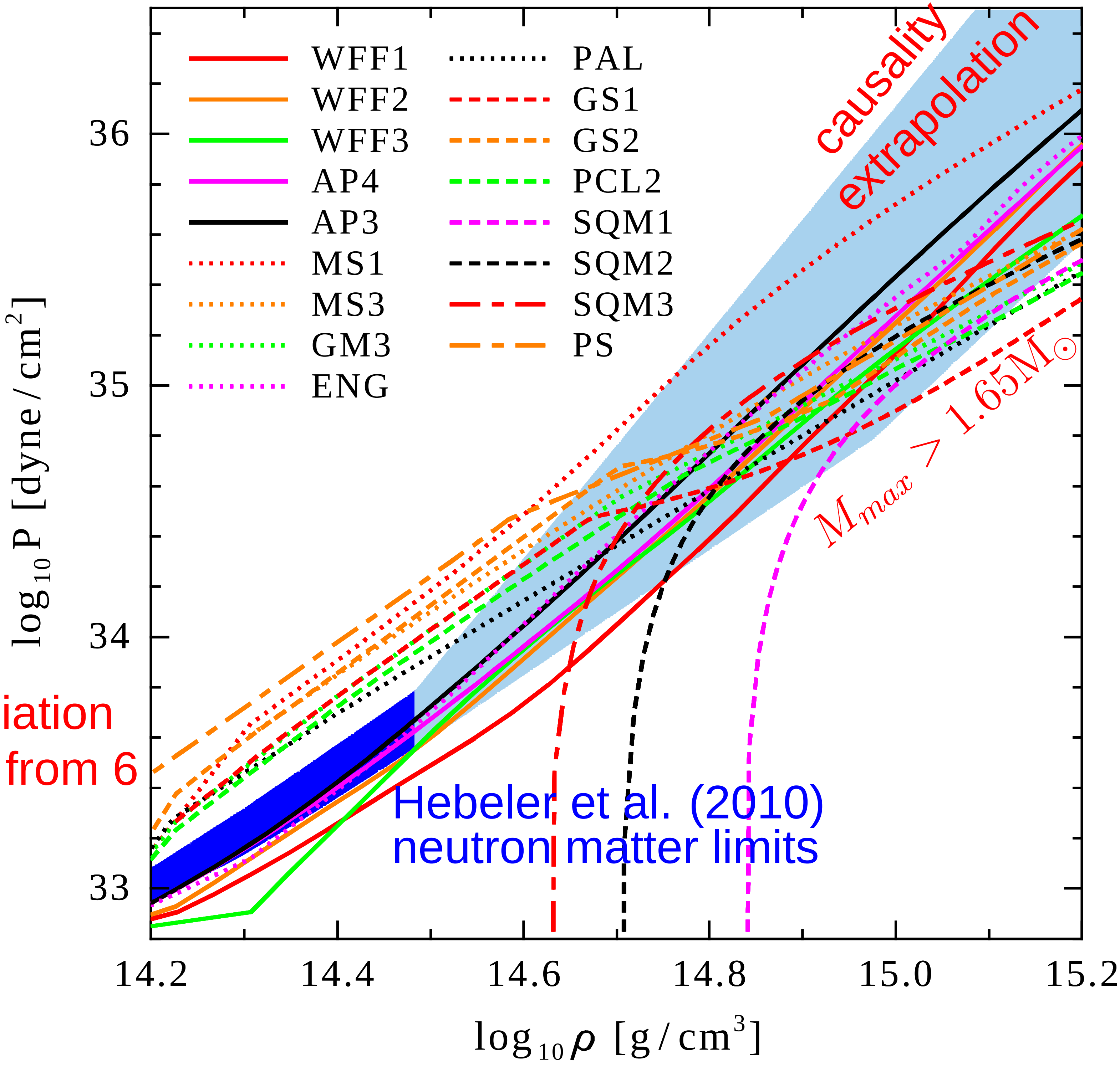
$$S_v = E_n + B \simeq 32.3 \pm 2.1 \text{ MeV}, \qquad p = S_v'/S_v \simeq 0.48 \pm 0.14$$

- More Accurate Model

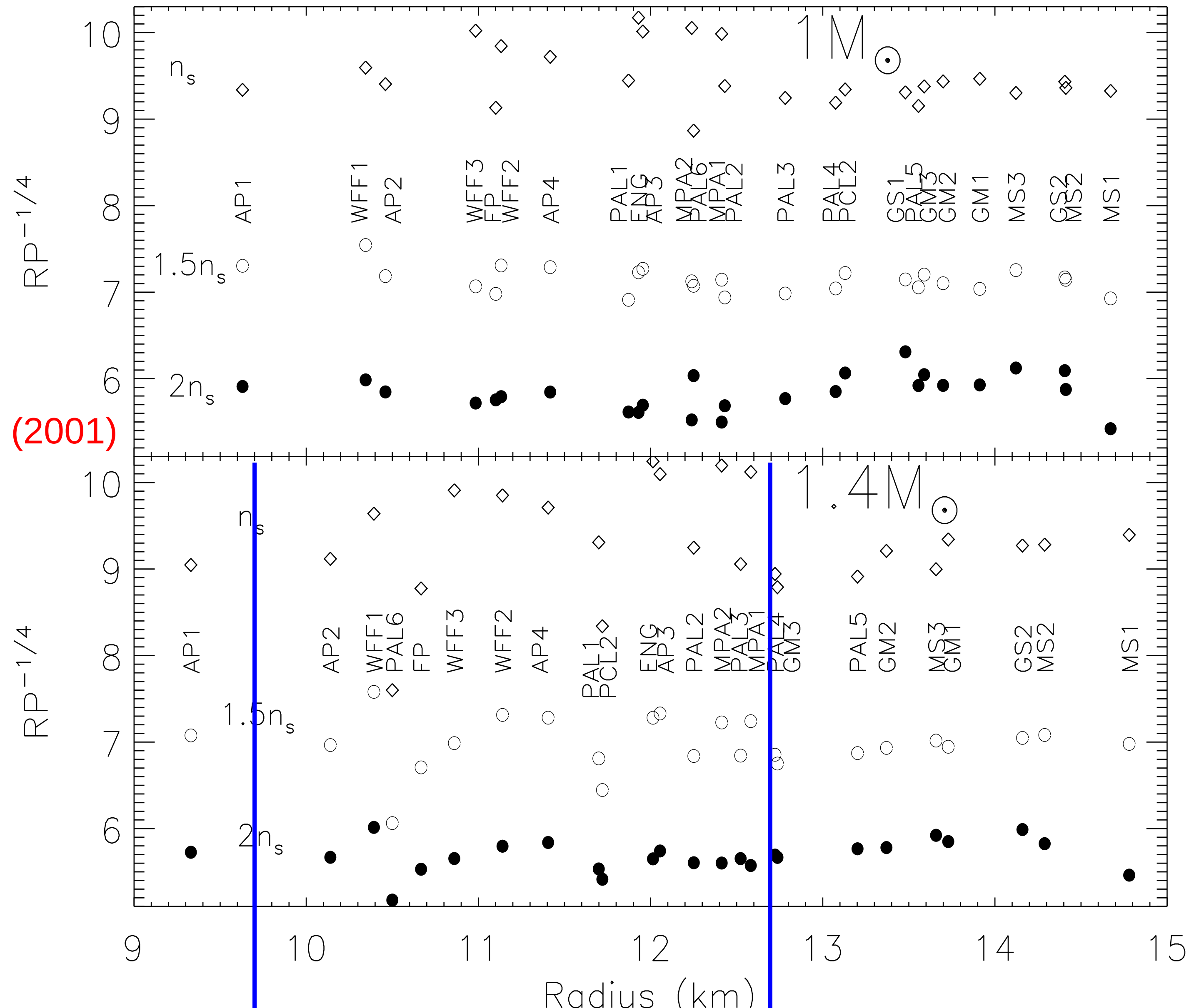
$$E_{sym}(n) = S_k (n/n_s)^{2/3} + (S_v - S_k)(n/n_s)^\gamma$$

$$S_k \simeq 17 \text{ MeV}, \qquad \gamma = \frac{S_v' - 2S_v/3}{S_v - S_k} \simeq 0.28 \pm 0.29$$

$p(\rho_0)$ variation reduced from 6 to 1.5



The Radius – Pressure Correlation

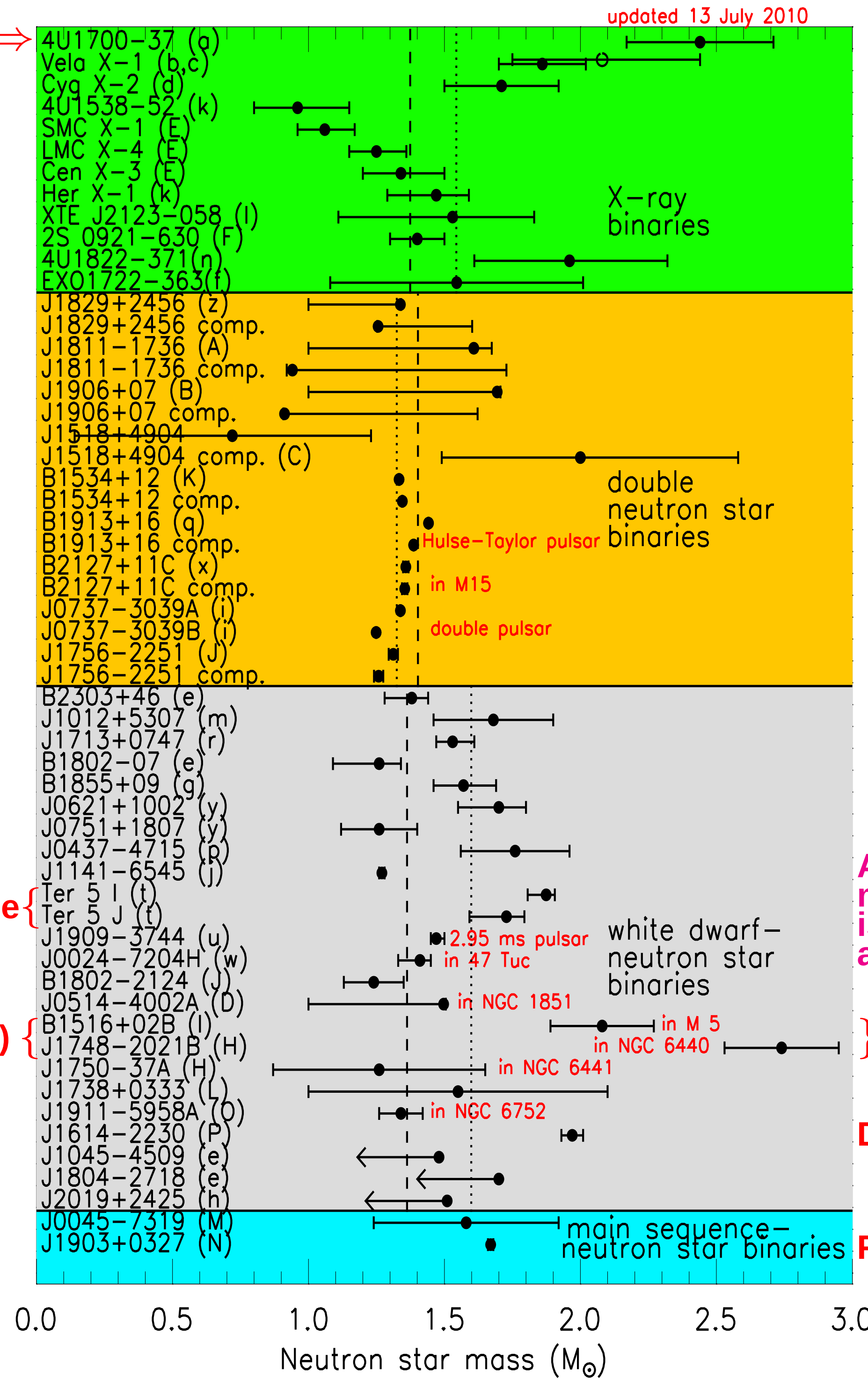


$R \propto p^{1/4}$

Lattimer & Prakash (2001)

⇐ neutron matter ⇒
prediction

Black hole? ⇒



$M > 1.68$ **M⊙, 95% confidence** {

Freire et al. (2007) {

Although simple average mass of w.d. companions is 0.27 M⊙ larger, weighted average is 0.04 M⊙ smaller

} **w.d. companion? statistics?**

Demorest et al. (2010)

Freire et al. (in prep.)

Possible Kinds of Observations

- Maximum and Minimum Mass (binary pulsars)
- Minimum Rotational Period*
- Radiation Radii or Redshifts from X-ray Thermal Emission*
- Crustal Cooling Timescale from X-ray Transients*
- X-ray Bursts from Accreting Neutron Stars*
- Seismology from Giant Flares in SGR's*
- Neutron Star Thermal Evolution (URCA or not)*
- Moments of Inertia from Spin-Orbit Coupling*
- Neutrinos from Proto-Neutron Stars (Binding Energies, Neutrino Opacities, Radii)*
- Pulse Shape Modulations*
- Gravitational Radiation from Neutron Star Mergers* (Masses, Radii from tidal Love numbers)

* Significant dependence on symmetry energy

Potentially Observable Quantities

- Apparent angular diameter from flux and temperature measurements
  $\beta \equiv GM/Rc^2$
  $$\frac{R_\infty}{D} = \frac{R}{D}\frac{1}{\sqrt{1-2\beta}} = \sqrt{\frac{F_\infty}{\sigma}}\frac{1}{f_c^2 T_\infty^2}$$
- Redshift
  $$z = (1-2\beta)^{-1/2} - 1$$
- Eddington flux
  $$F_{Edd} = \frac{GMc}{\kappa c^2 D^2}(1-2\beta)^{1/2}$$
- Crust thickness
  $$\frac{m_b c^2}{2}\ln\mathcal{H} \equiv \int_0^{p_t}\frac{dp}{n} = \mu_{n,t} - \mu_{n,t}(p=0)$$
  $$\frac{\Delta}{R} \equiv \frac{R-R_t}{R} \simeq \frac{(\mathcal{H}-1)(1-2\beta)}{\mathcal{H}+2\beta-1} \simeq (\mathcal{H}-1)\left(\frac{1}{2\beta}-1\right).$$
- Moment of Inertia
  $$I \simeq (0.237 \pm 0.008)MR^2(1+2.84\beta+18.9\beta^4)\ \mathrm{M}_\odot\ \mathrm{km}^2$$
- Crustal Moment of Inertia
  $$\frac{\Delta I}{I} \simeq \frac{8\pi}{3}\frac{R^6 p_t}{IMc^2}$$
- Binding Energy
  $$\frac{\mathrm{B.E.}}{Mc^2} \simeq (0.60 \pm 0.05)\frac{\beta}{1-\beta/2}$$

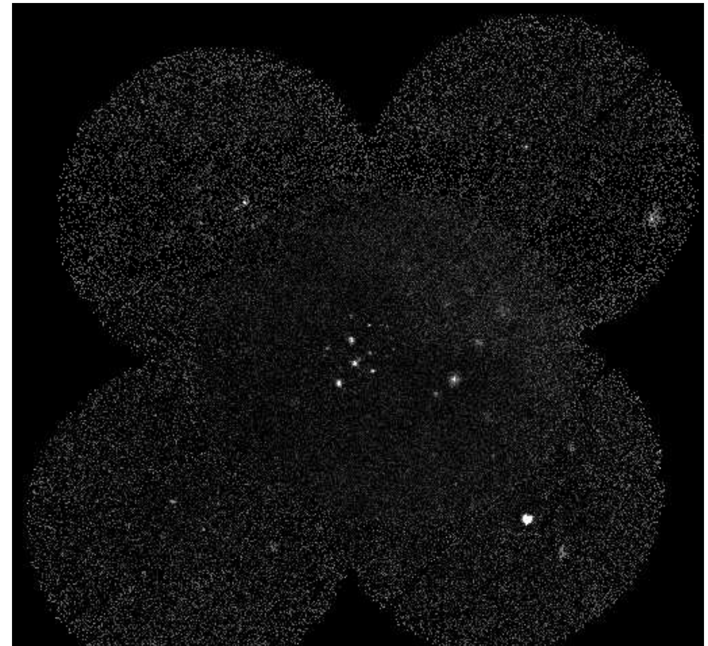
Radiation Radius

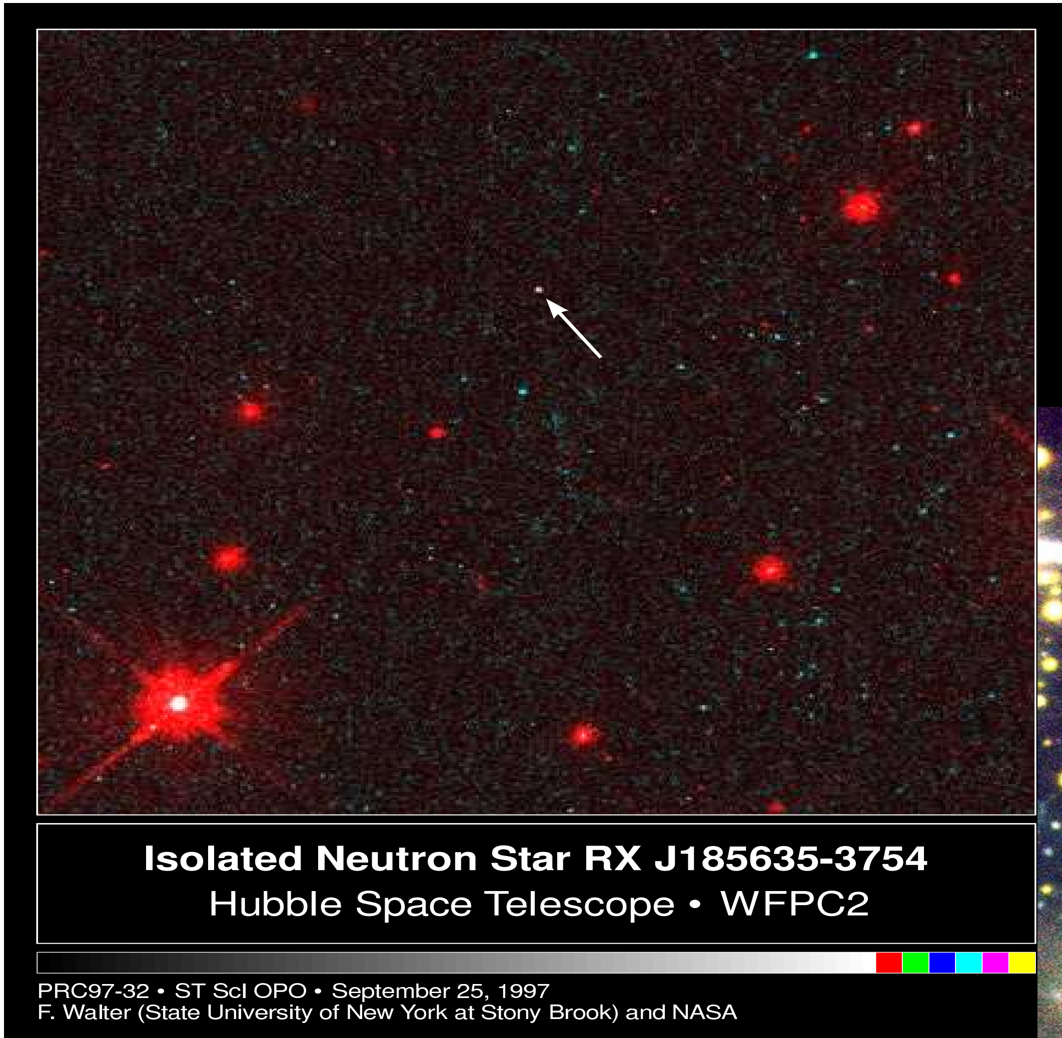
- Combination of flux and temperature measurements yields apparent angular diameter (pseudo-BB):

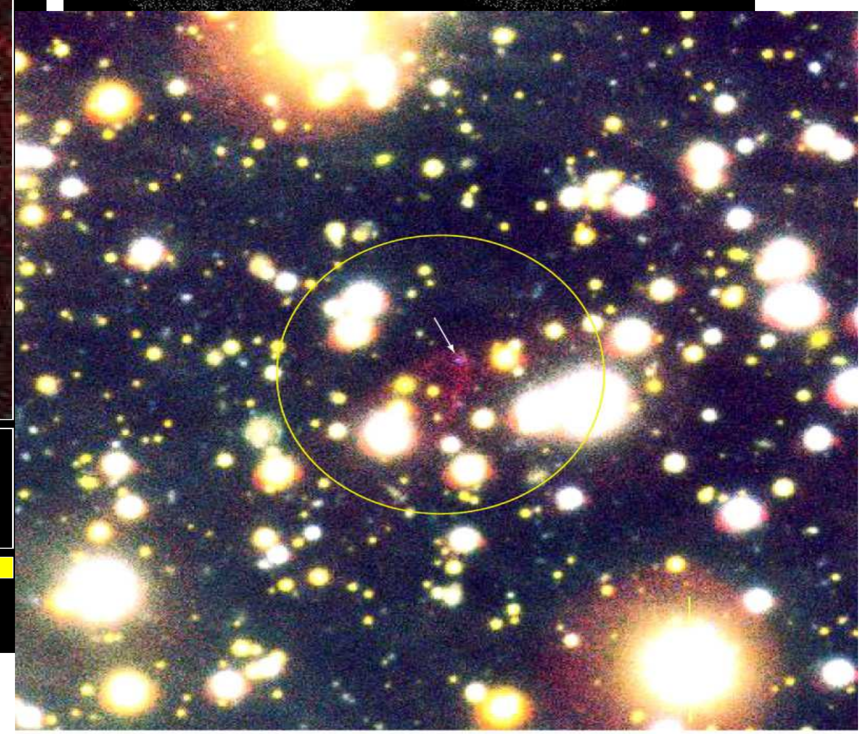
$$\frac{R_\infty}{d} = \frac{R}{d} \frac{1}{\sqrt{1 - 2GM/Rc^2}}$$

- Observational uncertainties include distance, interstellar H absorption (hard UV and X-rays), atmospheric composition
- Best chances for accurate radii are from
    - Nearby isolated neutron stars (parallax measurable)
    - Quiescent X-ray binaries in globular clusters (reliable distances, low B H-atmosperes)

RX J1856-3754





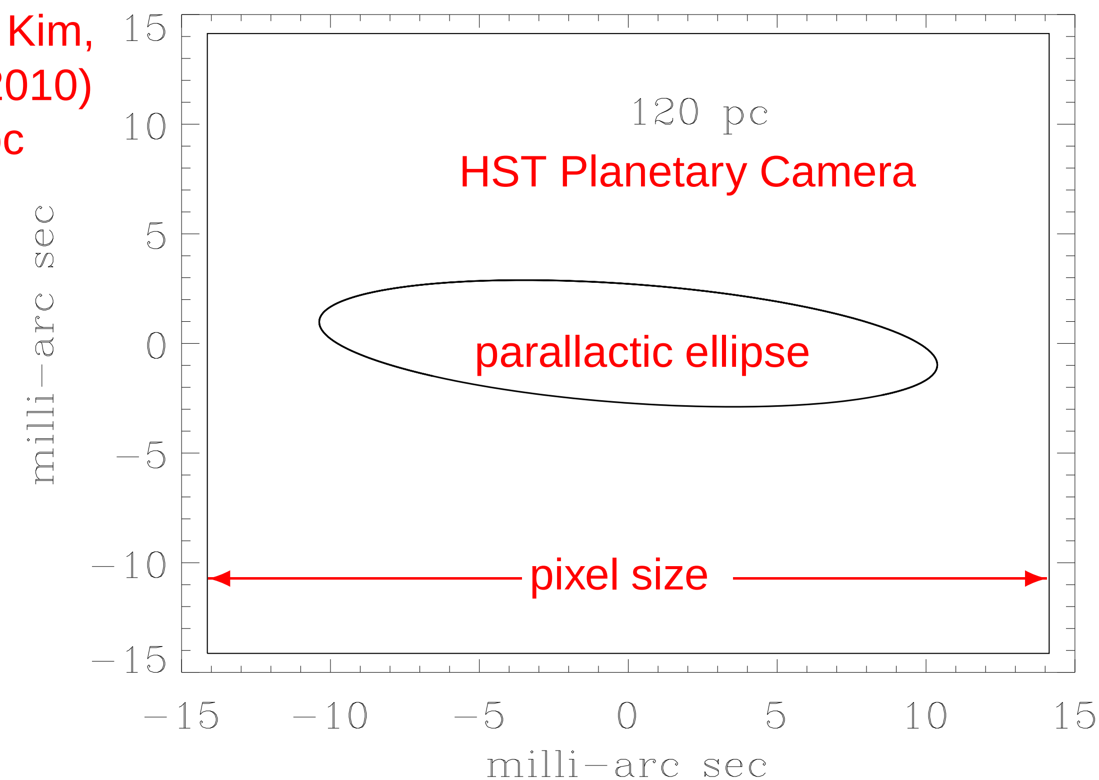


A Bowshock Nebula Near the Neutron Star RX J1856.5-3754 (Detail)
(VLT KUEYEN + FORS2)

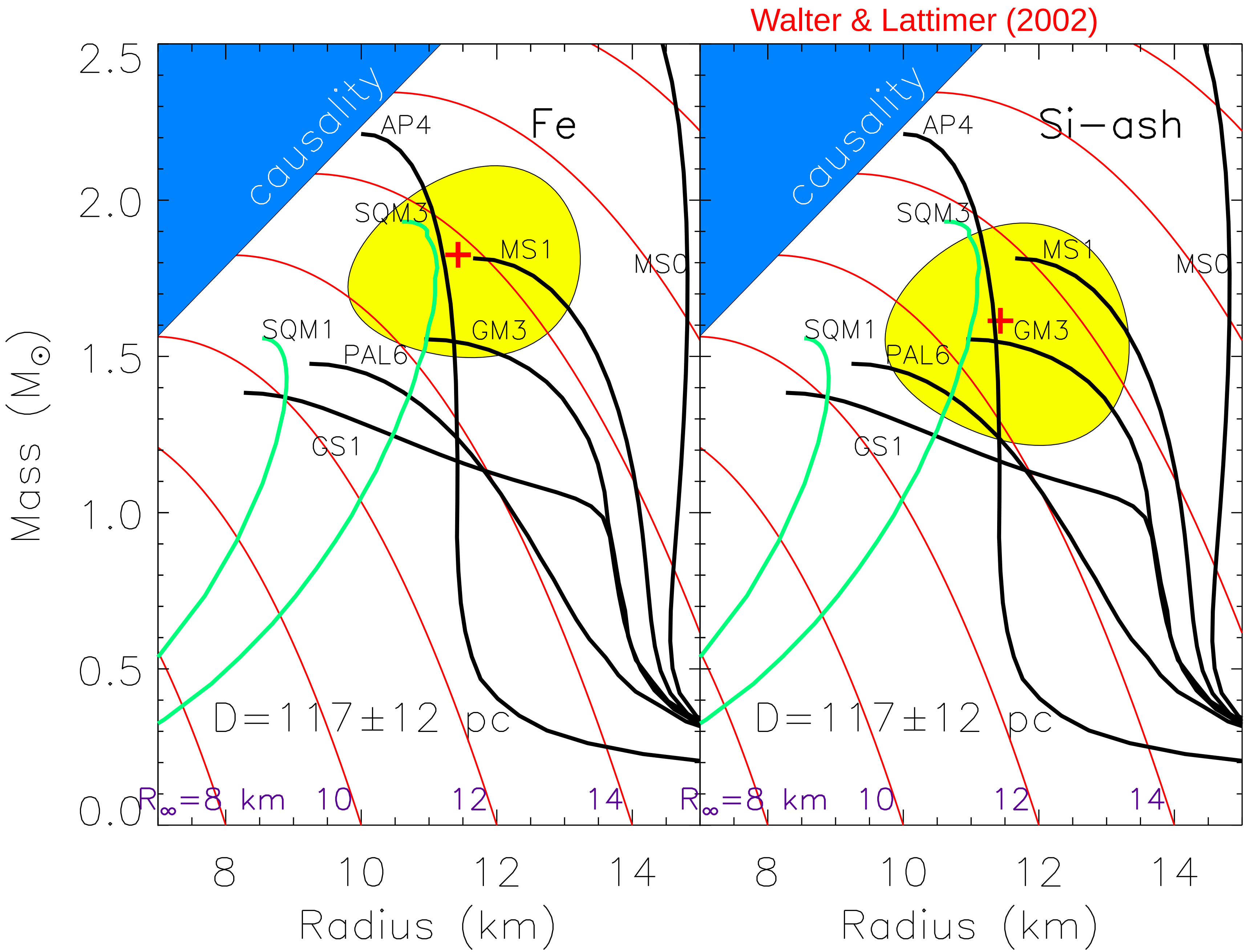
ESO PR Photo 23b/00 (11 September 2000) ©European Southern Observatory

Astrometry of RXJ 1856-3754

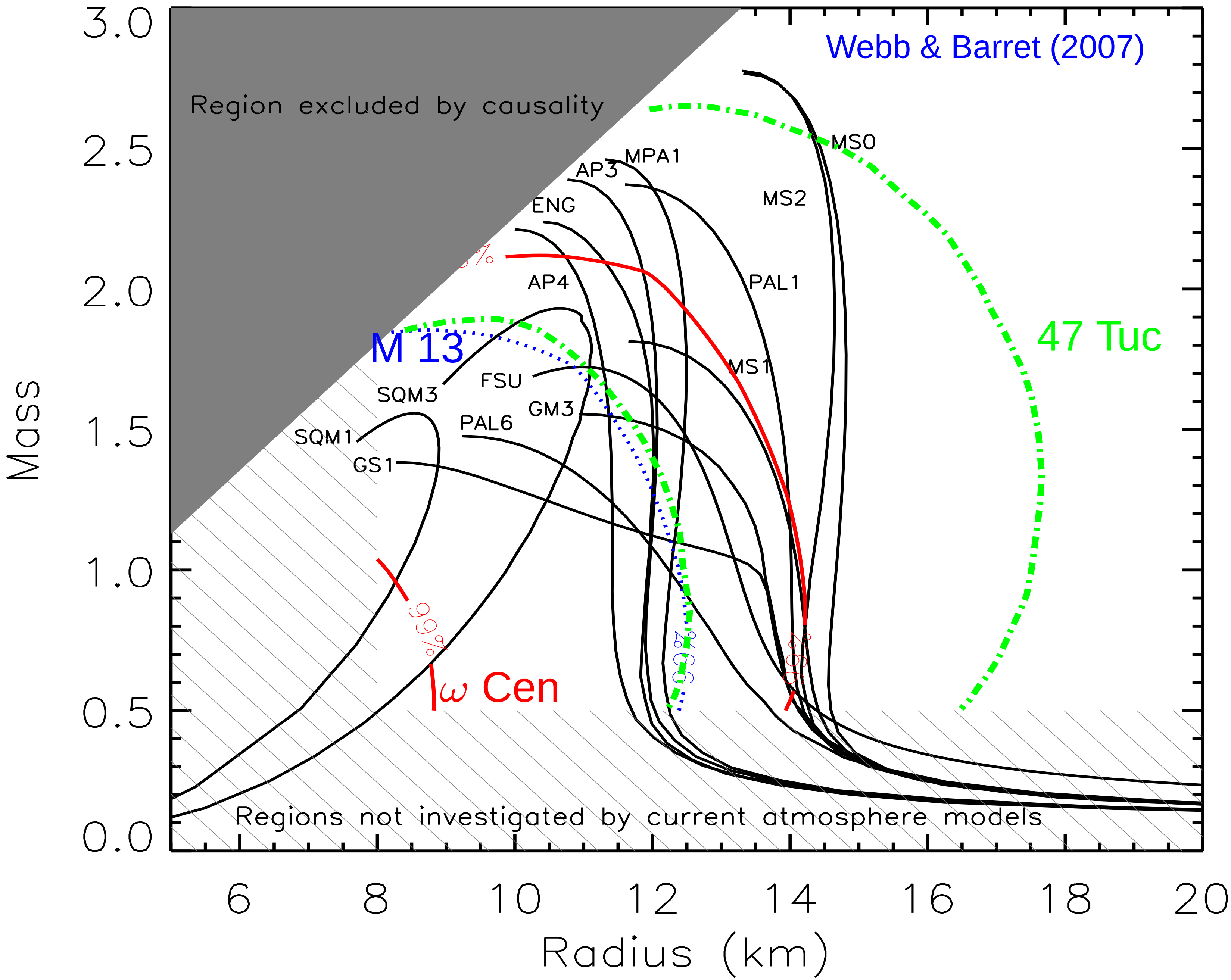
- Walter & Lattimer (2002) determined $D = 117 \pm 12$ pc and $v \simeq 190$ km/s from 1996-1999 HST Planetary Camera observations
- Star's age is probably 0.5 million years, origin Cor Aus
- Kaplan, van Kerkwijk & Anderson (2002): $D = 140 \pm 40$ pc using same data
- van Kerkwijk & Kaplan (2007, conference proceeding) revised this to $D = 161 \pm 16$ pc based on 2002-2004 High-Resolution Camera of the Advanced Camera for Surveys HST observations (double the resolution)
- Walter, Eisenbeiβ, Lattimer, Kim, Hambaryan & Neuhäuser (2010) determined $D \simeq 122 \pm 13$ pc with 2002-2004 HST data



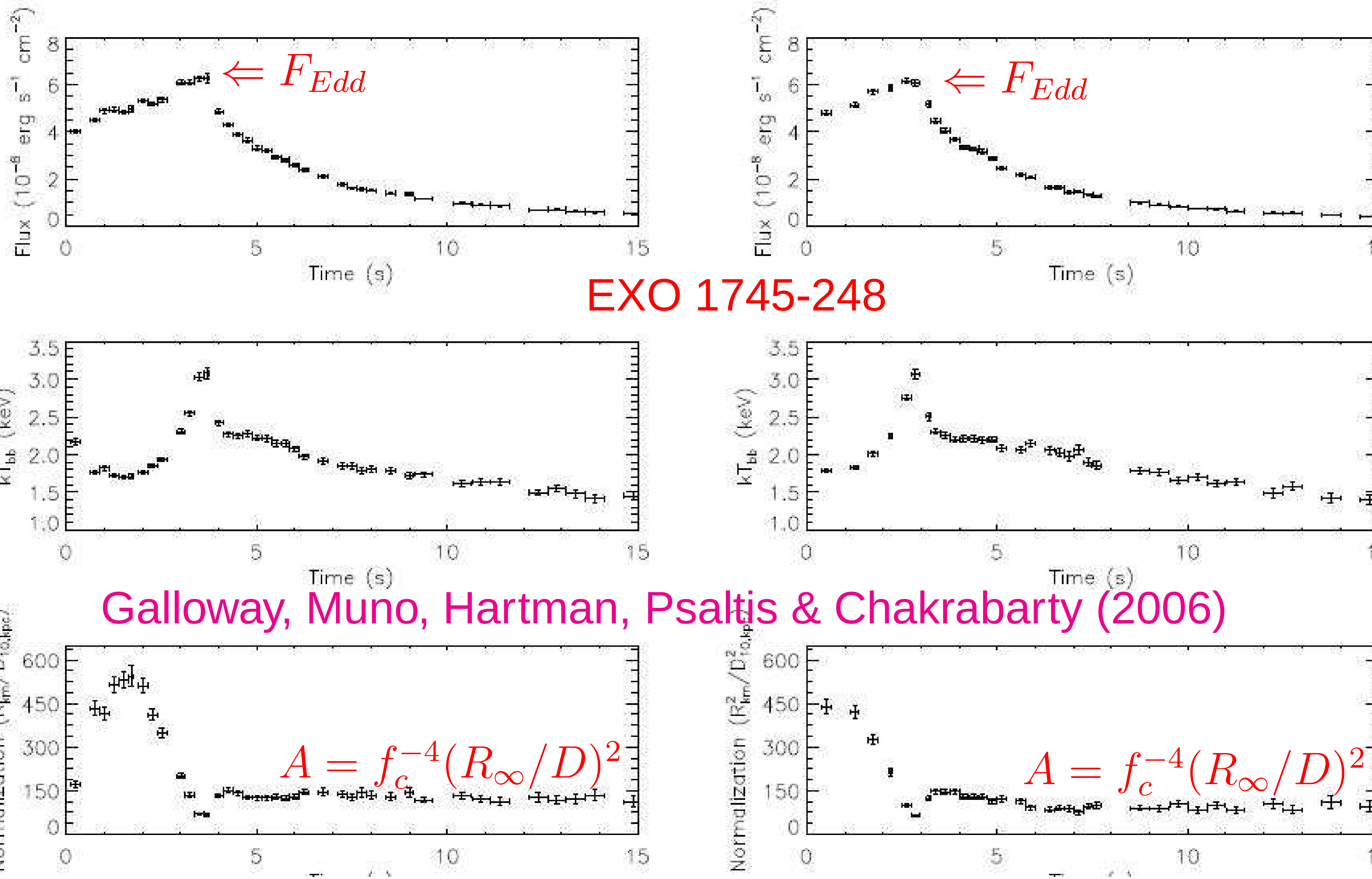
RX J1856-3754



Radiation Radius: Globular Cluster Sources



Photospheric Radius Expansion X-Ray Bursts



EXO 1745-248

Galloway, Muno, Hartman, Psaltis & Chakrabarty (2006)

Systematics assuming $R_{ph} = R$

$$F_{Edd} = \frac{GMc}{\kappa D^2}\sqrt{1-2\frac{GM}{R_{ph}c^2}} = \frac{GMc}{\kappa D^2}\sqrt{1-2\beta}$$

$$\kappa \simeq 0.2(1+X)\ \mathrm{cm^2 g^{-1}}$$

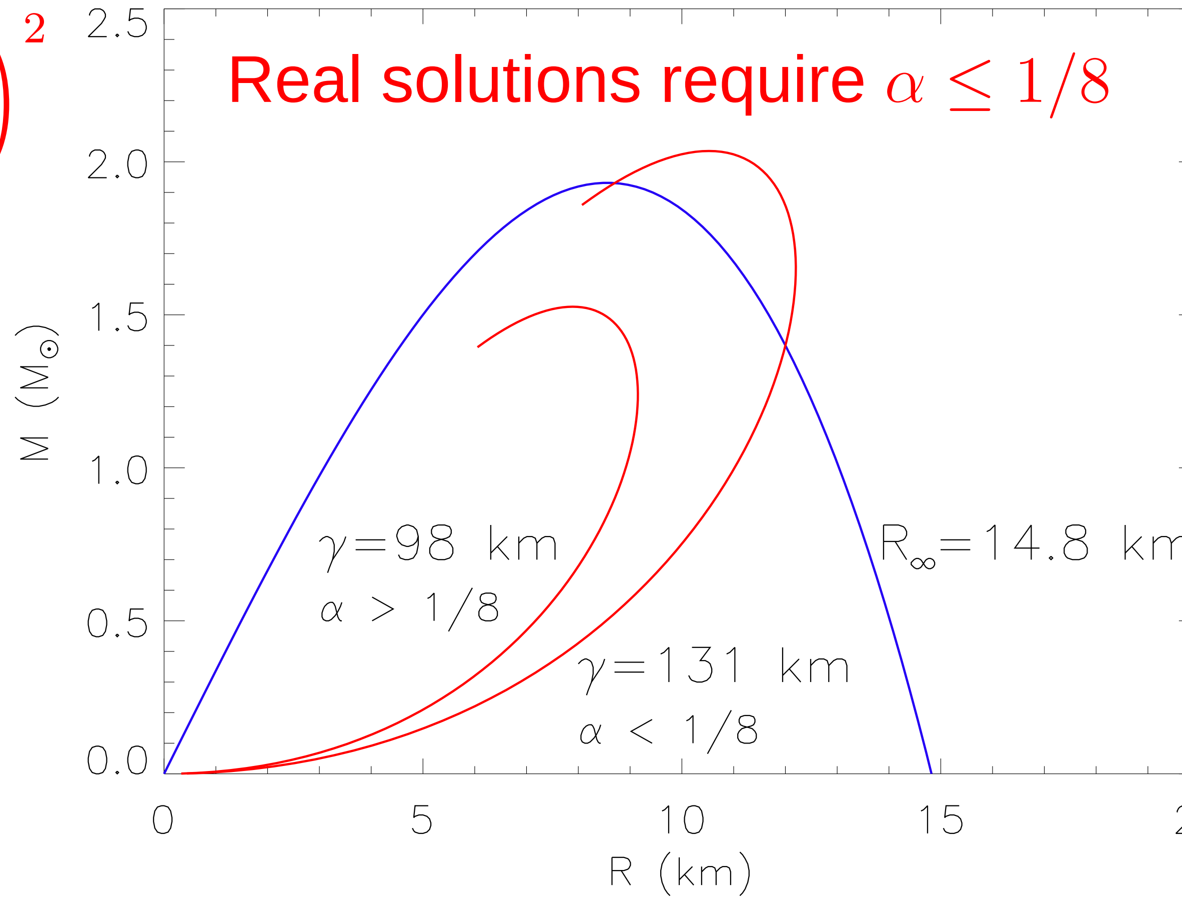
$$A = \frac{F_\infty}{\sigma T_\infty^4} = f_c^{-4}\left(\frac{R_\infty}{D}\right)^2$$

$$\alpha = \frac{F_{Edd}}{\sqrt{A}}\frac{\kappa D}{c^3 f_c^2} = \beta(1-2\beta)$$

$$\gamma = \frac{Ac^3 f_c^4}{F_{Edd}\kappa} = \frac{R}{\beta(1-2\beta)^{3/2}}$$

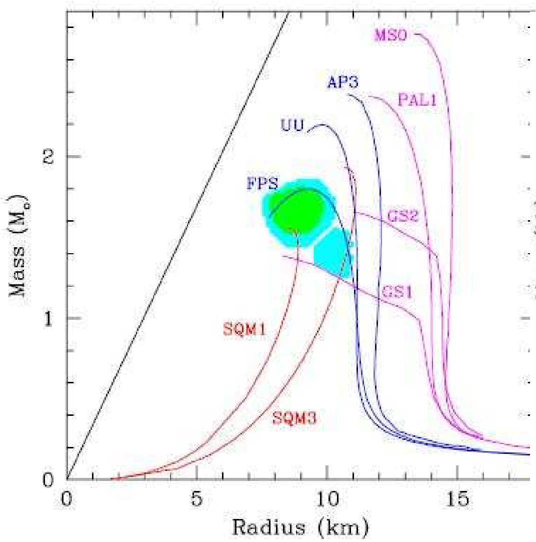
$$\beta = \frac{1}{4} \pm \frac{1}{4}\sqrt{1-8\alpha}$$

$$R_\infty = \alpha\gamma$$

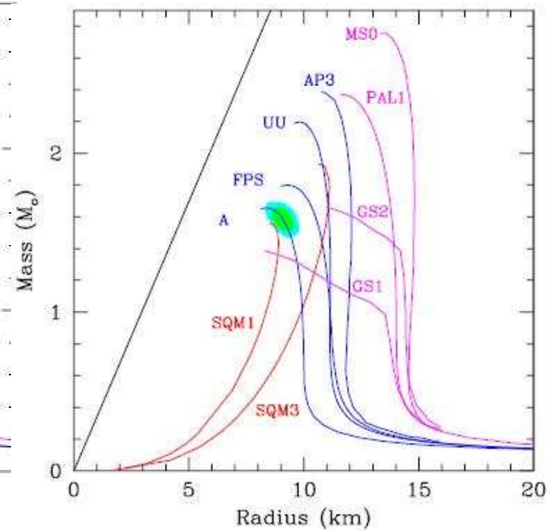


$R_{ph} = R$

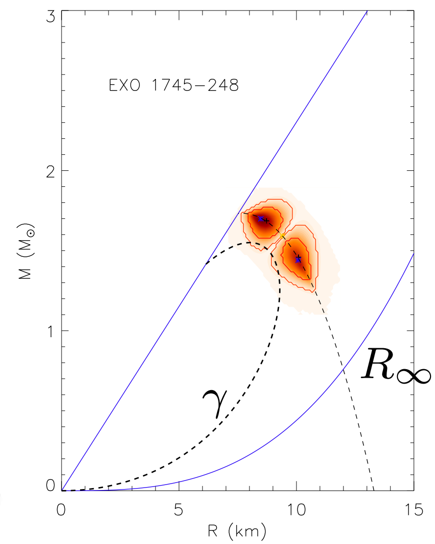
EXO 1745-248
Özel et al. (2009)



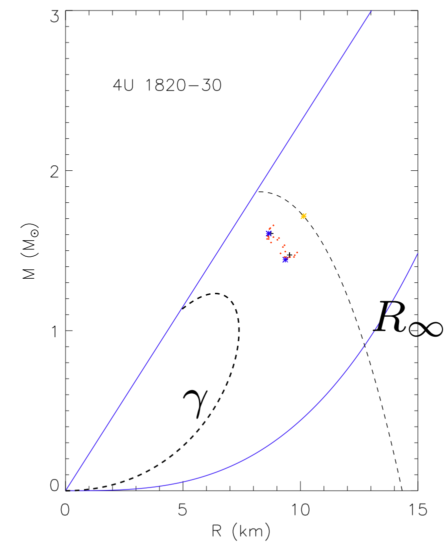
4U 1820-30
Güver et al. (2010)



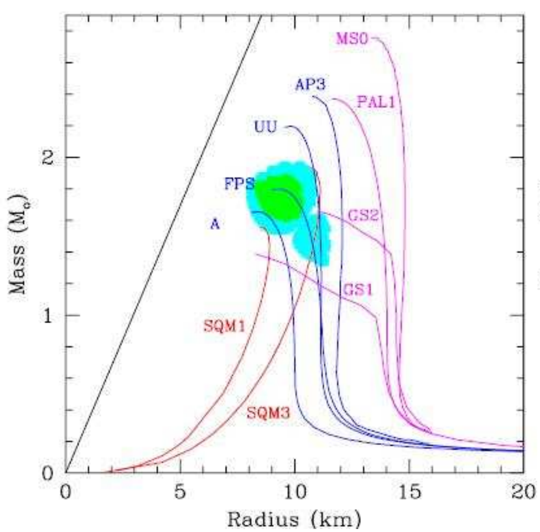
EXO 1745-248
$\alpha = 0.14 \pm 0.02$



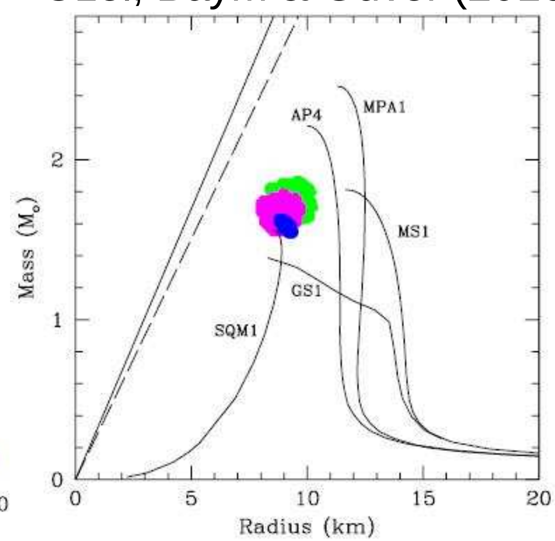
4U 1820-30
$\alpha = 0.18 \pm 0.02$



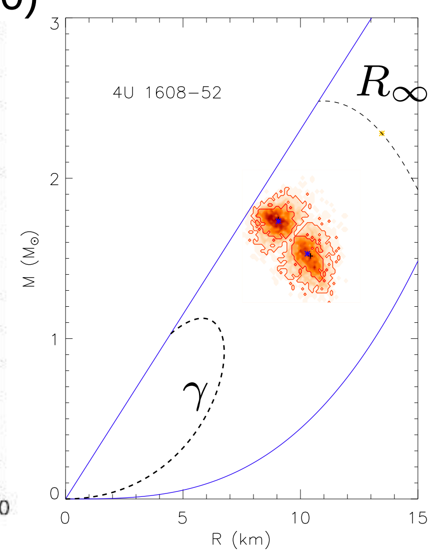
4U 1608-52
Güver et al. (2010)



Özel, Baym & Güver (2010)



4U 1608-52
$\alpha = 0.26 \pm 0.11$



Systematics assuming $R_{ph} >> R$

$$F_{Edd} = \frac{GMc}{\kappa D^2}\sqrt{1 - 2\frac{GM}{R_{ph}c^2}} \simeq \frac{GMc}{\kappa D^2}$$

$$\kappa \simeq 0.2(1+X)\ \mathrm{cm}^2\mathrm{g}^{-1}$$

$$A = \frac{F_\infty}{\sigma T_\infty^4} = f_c^{-4}\left(\frac{R_\infty}{D}\right)^2$$

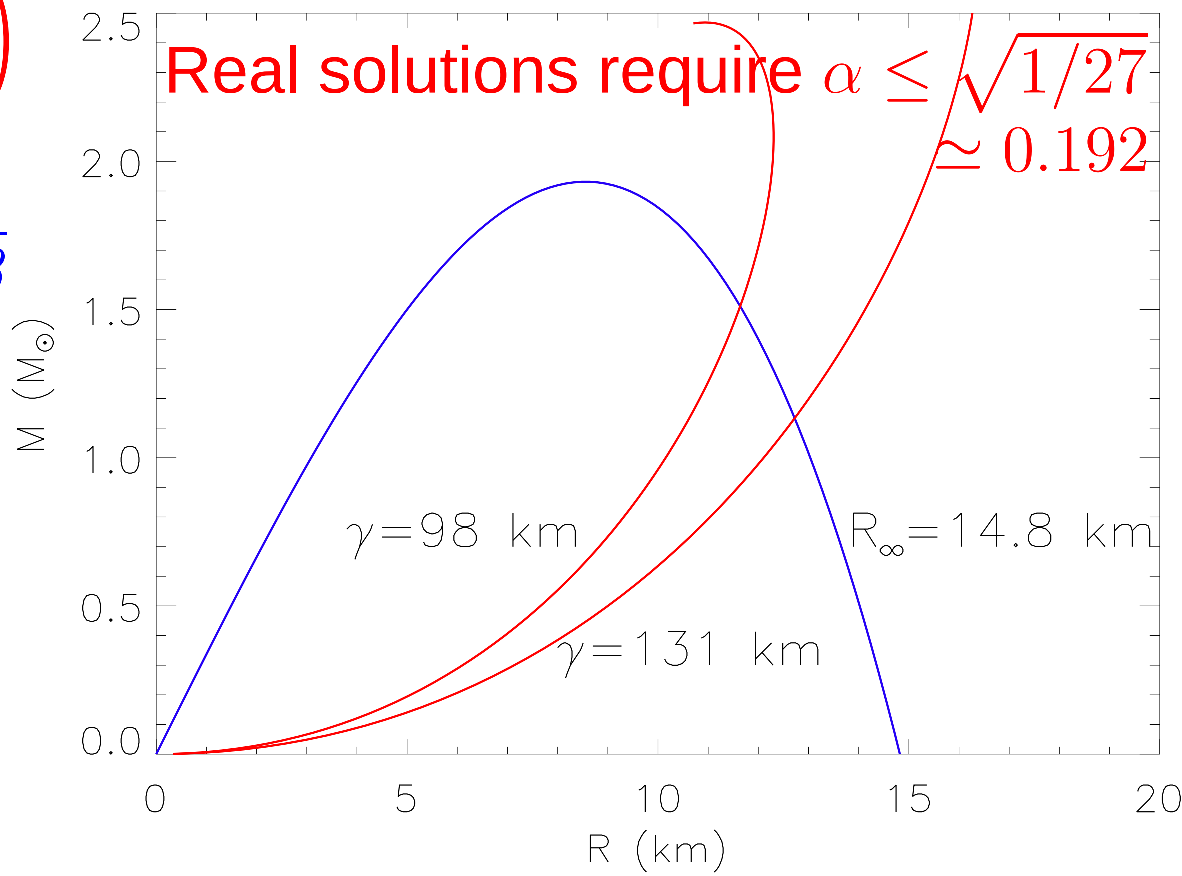
$$\alpha = \frac{F_{Edd}}{\sqrt{A}}\frac{\kappa D}{c^3 f_c^2} = \beta\sqrt{1-2\beta}$$

$$\gamma = \frac{Ac^3 f_c^4}{F_{Edd}\kappa} = \frac{R}{\beta(1-2\beta)}$$

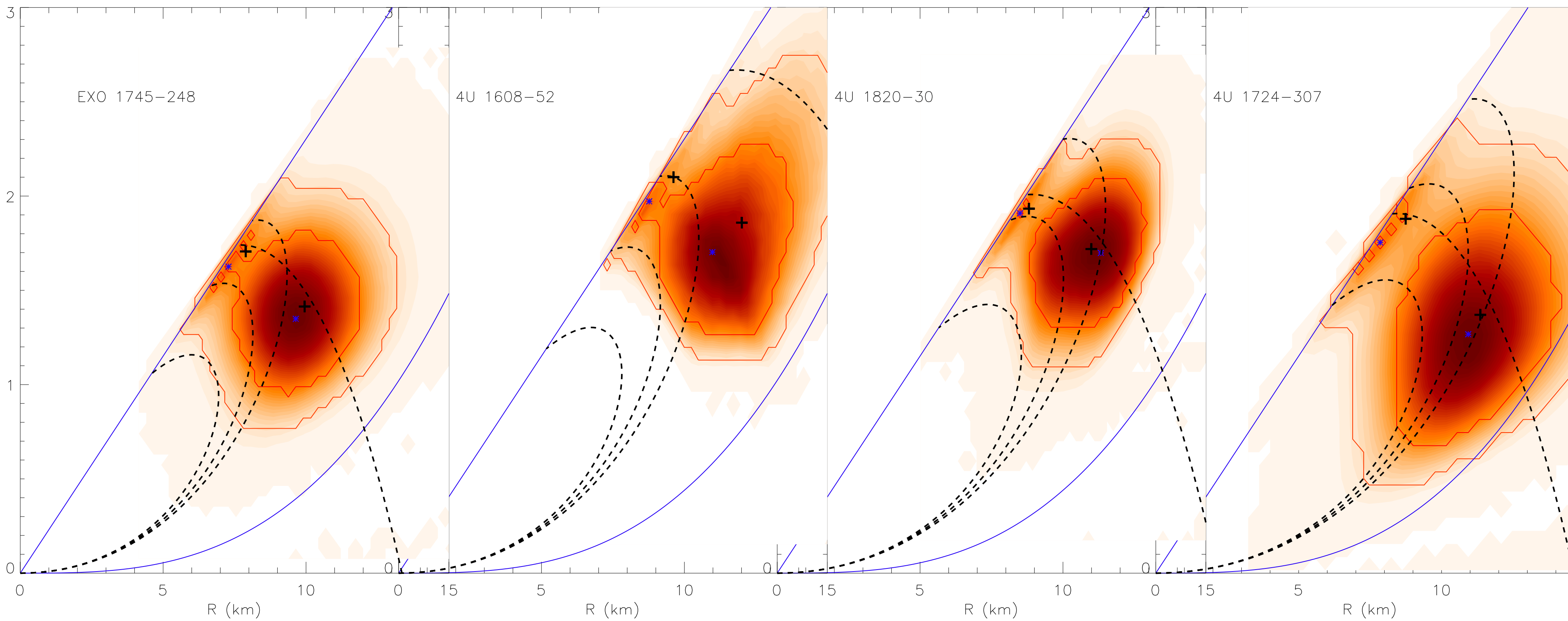
$$\beta = \frac{1}{6}\left[1 + \sqrt{3}\sin\theta - \cos\theta\right]$$

$$\theta = \frac{1}{3}\cos^{-1}(1 - 54\alpha^2)$$

$$R_\infty = \alpha\gamma$$



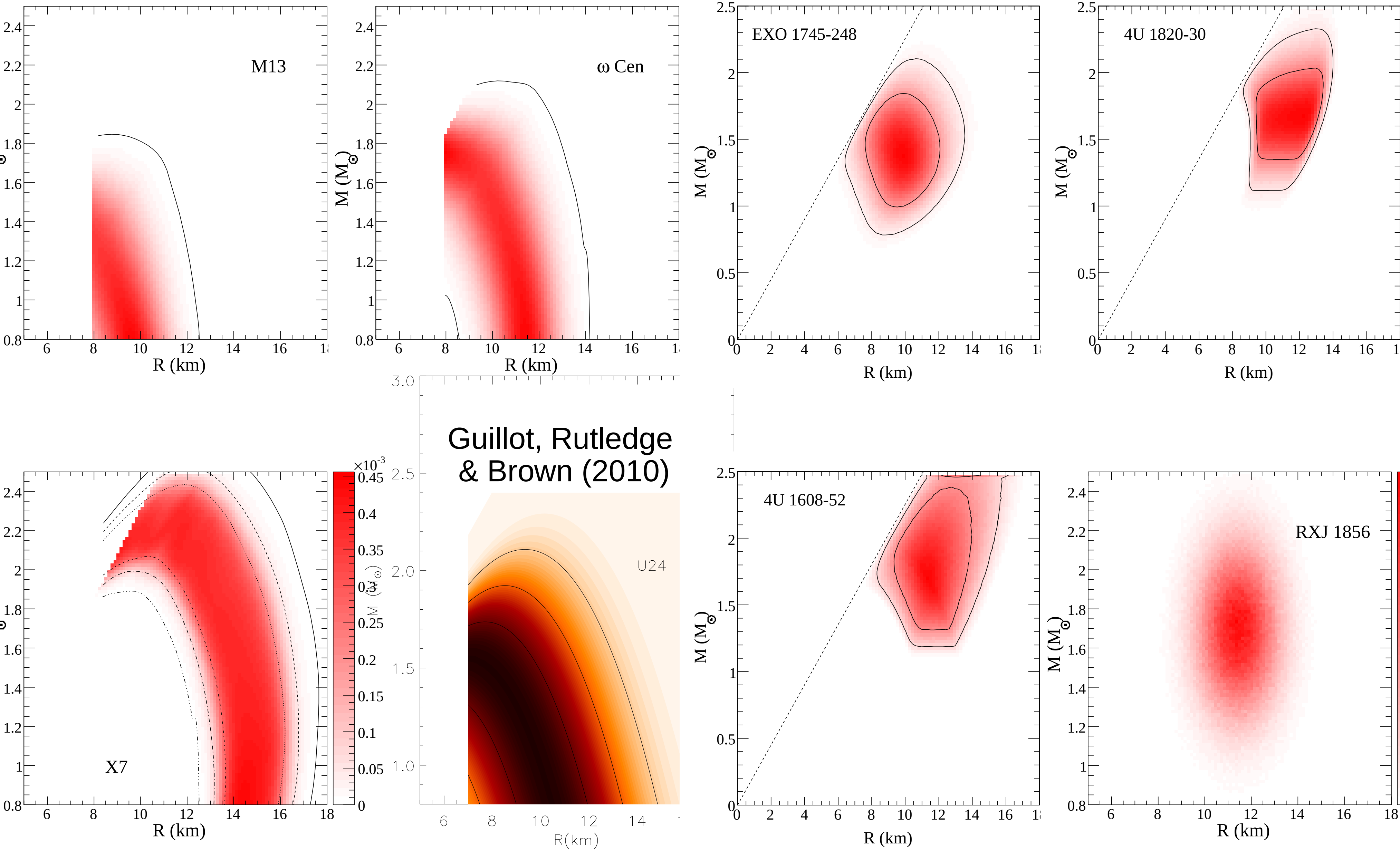
Arbitrary r_{ph}/R



Sources of observational data:

4U 1745-248: Özel et al. (2009)
4U 1608-522: Güver et al. (2010)
4U 1820-30: Güver et al. (2010)
4U 1724-307: Suleimanov et al. (2010)

M-R Probability Distributions



Bayesian Analysis

$P(\mathcal{M}_i|D)$: conditional probability of the model $\mathcal{M}_i$ given the data D (what we want).

$$P(\mathcal{M}_i|D) = \frac{P(D|\mathcal{M}_i)P(\mathcal{M}_i)}{\sum_k P(D|\mathcal{M}_k)P(\mathcal{M}_k)} \propto P(D|\mathcal{M}_i)\,.$$

$P(\mathcal{M}_i)$: prior probability of the model $\mathcal{M}_i$ without any information from the data D (assumed uniform).

$P(D|\mathcal{M}_i)$: conditional probability of the data D given the model $\mathcal{M}_i$.

i is the EOS parameter set and the set of specific masses j.

$$P(D|\mathcal{M}_i) \propto \prod_{k,j} \mathcal{D}_{k,j}|_{M=M_j,R=R_k(M_j)}$$

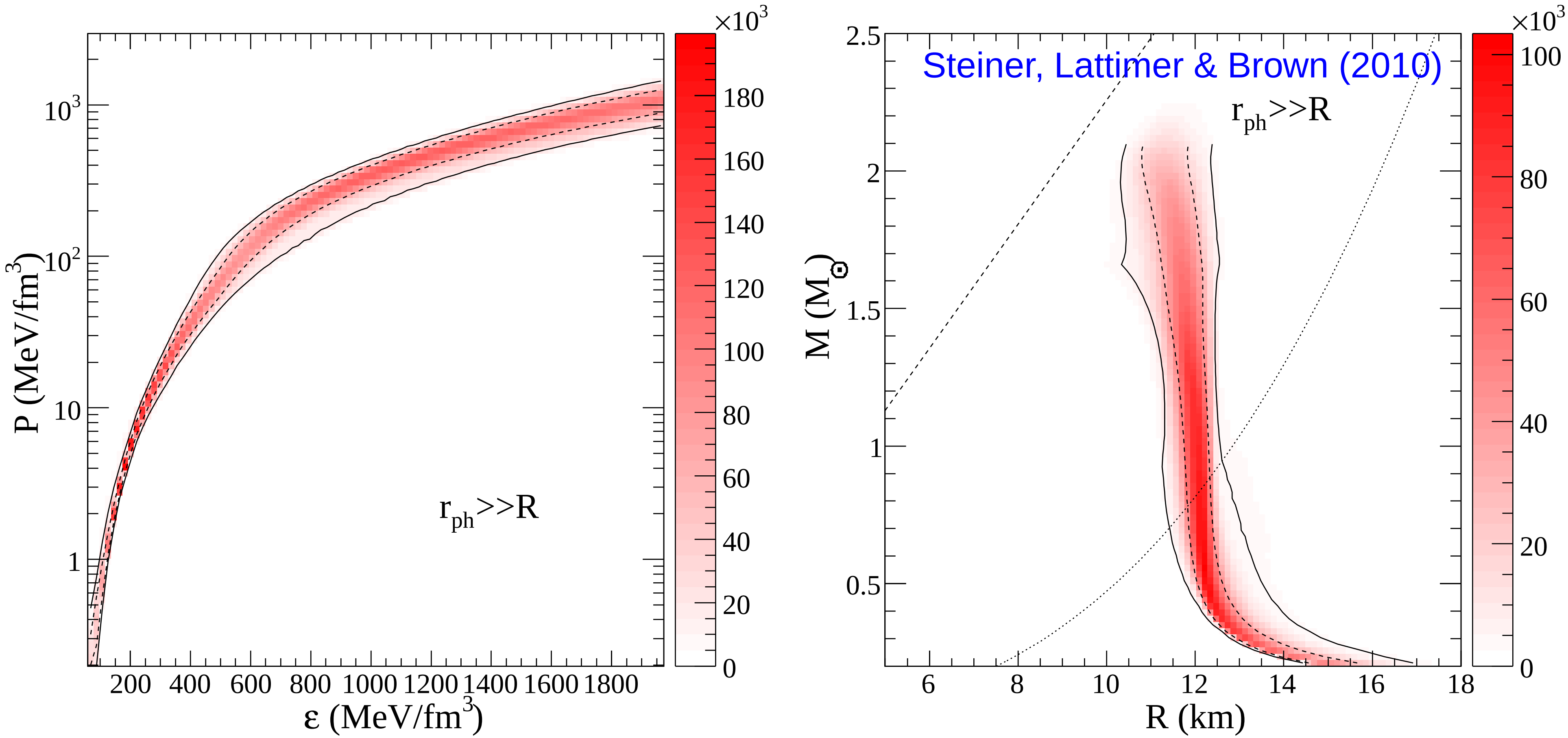
$\mathcal{D}_{k,j}$: probability of observing the radius R_j given a mass M_j in observation k ($M-R$ probabilities).

Posterior probability for a model parameter p_i (marginal estimation):

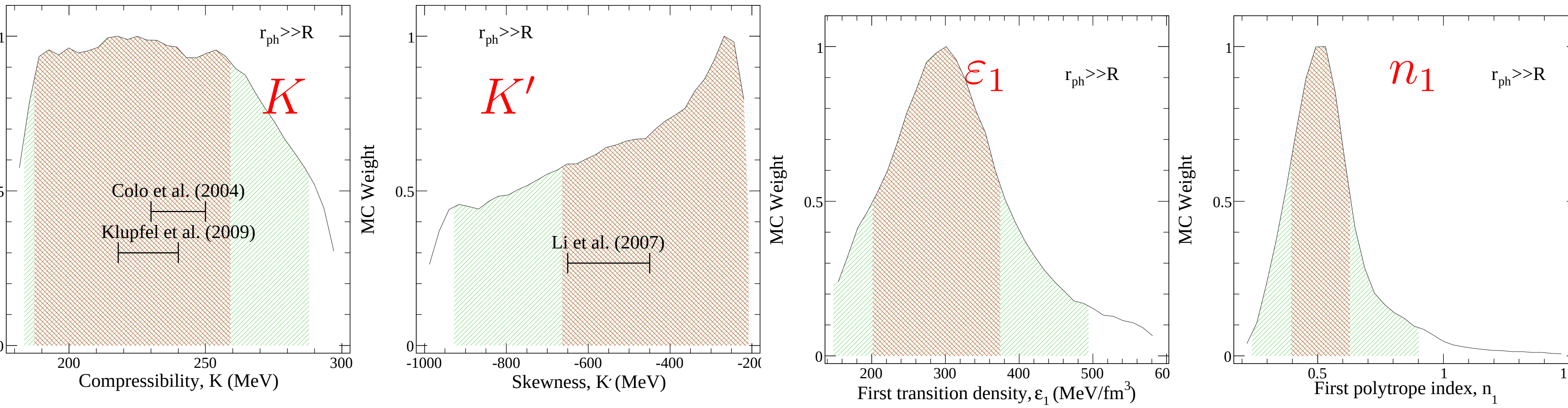
$$P(p_i|D) \propto \int P(D|\mathcal{M}_i)\,dp_1\,dp_2\,\ldots\,dp_{i-1}\,dp_{i+1}\,\ldots\,dp_{N_p}\,dM_1\,dM_2\,\ldots\,dM_{N_M}$$

Bayesian TOV Inversion

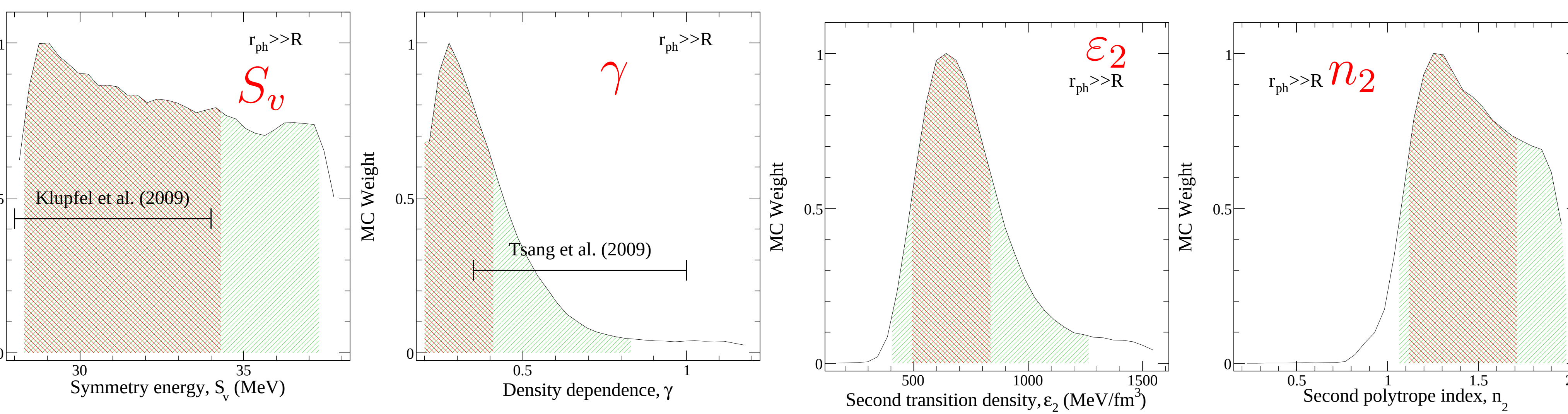
- $\varepsilon < 0.5\varepsilon_0$: EOS from BBP and NV
- $0.5\varepsilon_0 < \varepsilon < \varepsilon_1$: EOS parametrized by K, K', S_v, γ
- $\varepsilon_1 < \varepsilon < \varepsilon_2$: EOS is polytrope with n_1; $\varepsilon > \varepsilon_2$: EOS is polytrope with n_2
- A-priori EOS parameters ($K, K', S_v, \gamma, \varepsilon_1, n_1, \varepsilon_2, n_2$) uniformly distributed
- M and R probability distributions for 7 neutron stars treated equally ($0.8\ \mathrm{M}_\odot < M < 2.5\ \mathrm{M}_\odot$; $5\ \mathrm{km} < R < 18\ \mathrm{km}$)



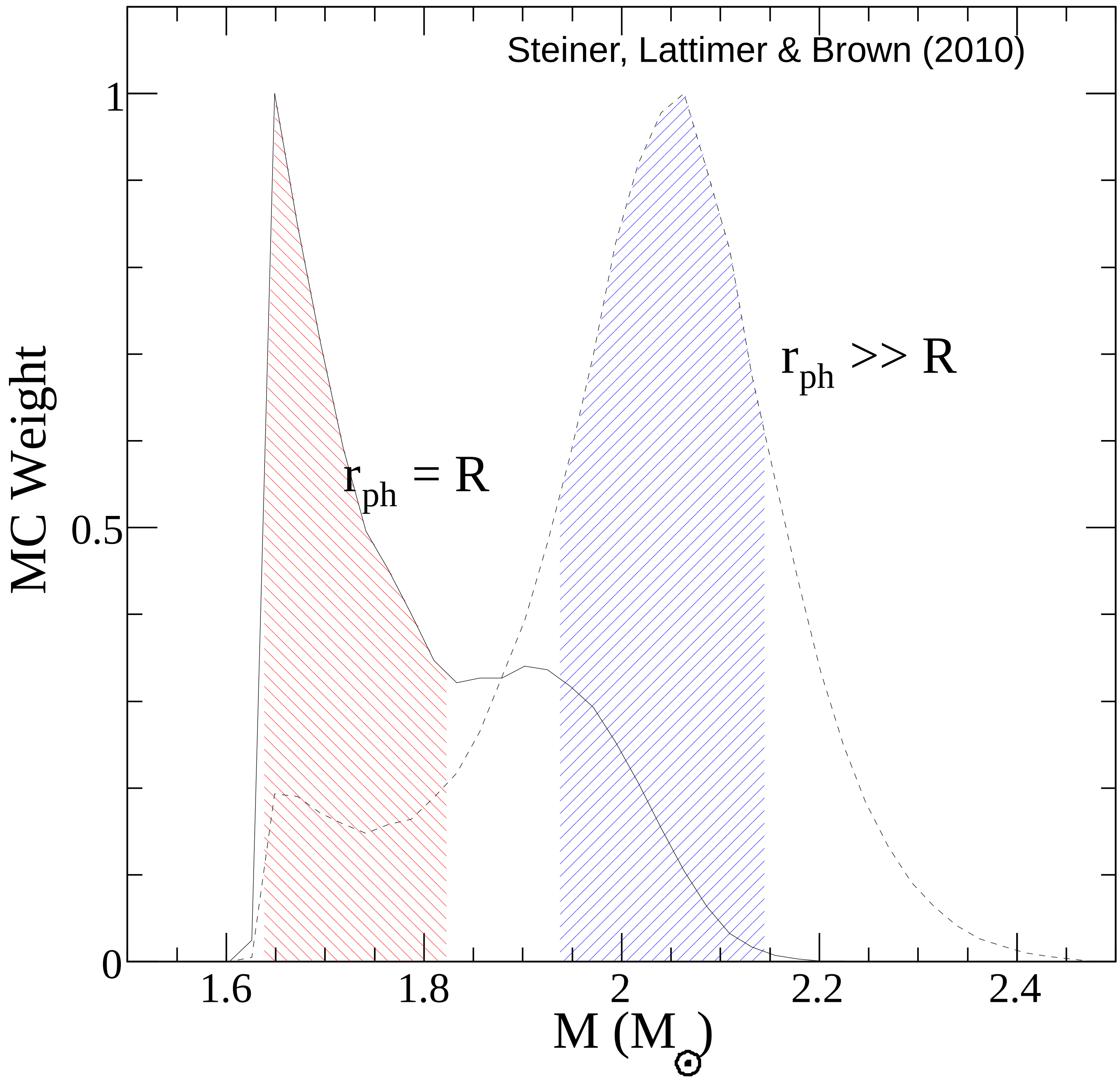
Inferred Model EOS Parameters



Steiner, Lattimer & Brown (2010)



Maximum Mass Probability Distributions



Conclusions

- In spite of large estimated errors for individual sources, when taken as an ensemble they predict a remarkably tight pressure-density relation.
- Nuclear symmetry energy is predicted to be very soft, $\gamma \simeq 0.3 \pm 0.1$, consistent with pure neutron matter results ($\gamma \simeq 0.3 \pm 0.3$). Incompatible with Shen et al. EOS.
- Estimated radii of 1.4 $M_\odot$ stars are $\simeq 11.3 \pm 0.3$ km.
- The maximum neutron star mass is estimated to be near or above 2 $M_\odot$.
- The estimated maximum mass and stiffness of the high-density EOS are sensitive to interpretations of photospheric radius expansion bursts.