3D Stellar Models

Casey A. Meakin

University of Arizona

With:

David Arnett (U. Arizona)

Tuguldur Sukhbold (U. Arizona)

Frank Timmes (ASU)

Why 1D?

- Stellar models have historically been restricted to 1D due to **computational cost**. The hydrodynamics known to exist in stellar interiors were highly approximated, e.g., convectively unstable regions were treated as simply adiabatic. Each moment in time was treated as **time independent**: in hydrostatic equilibrium. Evolution was followed by crudely connecting one model to the next by accounting for nuclear evolution.

Why 1D?

- 2D and 3D computational fluid dynamics (CFD) is a relatively new field, driven by the exponentially increasing availability of computational resources known as Moore's Law: The beginning of every decade has ~16 times the computational capability as the beginning of the last, and 256 times the capability of 20 years ago, 4096 times the capability of 30 years ago, and so on.

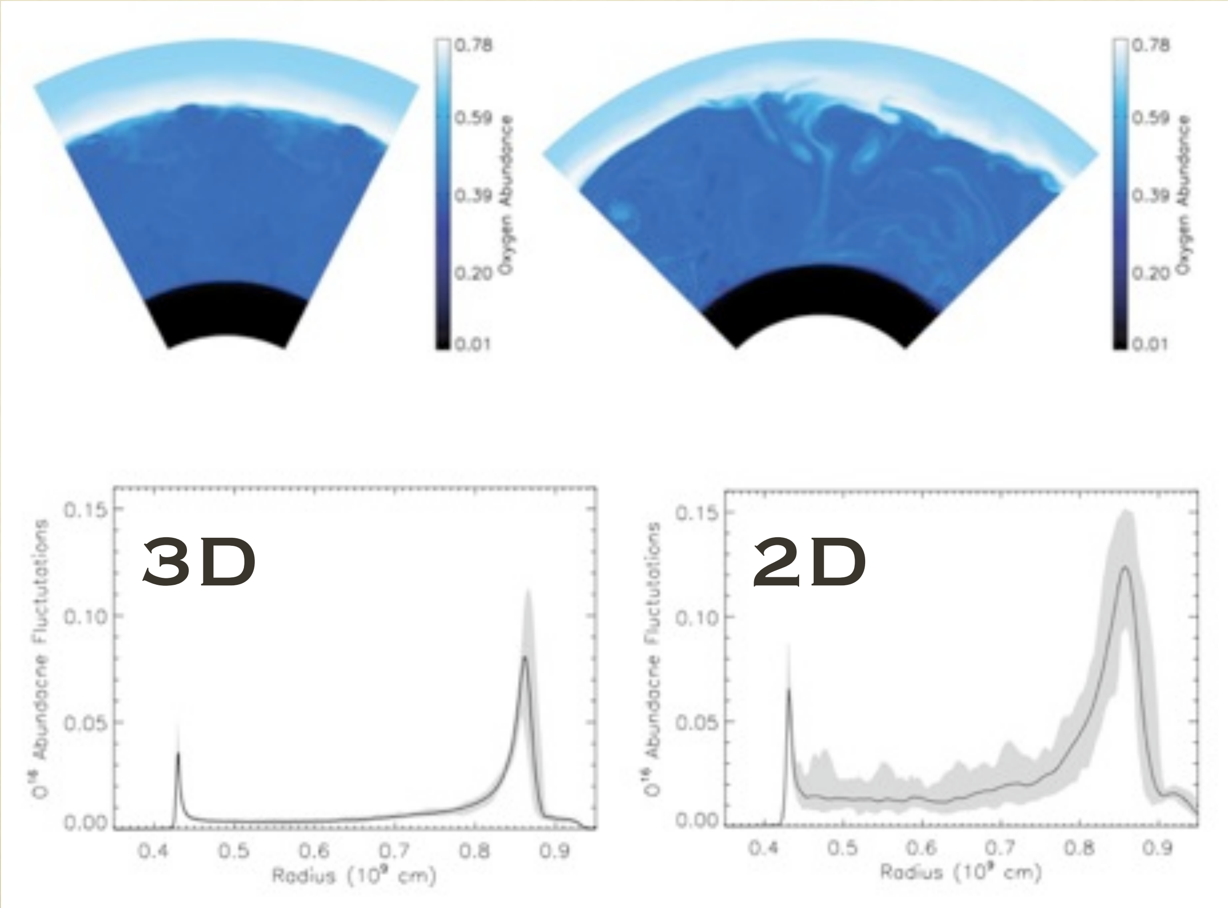
- Today's computational resources are ~1.7 million times more powerful than Martin Schwarzschild's (say in 1958).

Why 3D?

- the development of hydrodynamic instabilities are intrinsically three dimensional.
- Lack of knowledge regarding the mixing rates attending these instabilities undermines our ability to interpret stellar abundance data and develop a predictive stellar model.
- Interpreting stellar abundance data and developing a predictive stellar model are capabilities sought after by many astrophysicists.

Why not 2D?

- The restriction of spatial dimensionality to 2 significantly modifies the development and character of instabilities, including their transport properties.



Why not 2D?

- Some have argued that 2D allows you to achieve higher Reynold's number for the same computational cost: this is misleading for two reasons.

1.) 2D and 3D turbulence are qualitatively different, e.g., Kolmogorov's theory doesn't apply to 2D turbulence.

2.) 3D is cheaper for the same number of degrees of freedom:
larger zone edge sizes $\longrightarrow$ larger CFL time step.

321D

- Even given Moore's Law we can't simulated stellar evolution on a hydrodynamic timescale.
- Therefore we need to be able to capture the cumulative impact of turbulent flow on stellar structure, possibly as an algorithm to be used in a 1D stellar evolution code.
- Following John Lattanzio (sp?) we call this 321D, meaning "projecting" 3D to 1D. Keeping this goal in mind provides focus for analyzing 3D flows.

321D

- Reynold's decomposition (using time & angle average) provides a classical framework for 321D.

$$\begin{aligned} \varphi &= \varphi_0 + \varphi' \\ \langle \varphi \rangle &= \varphi_0 \\ \langle \varphi' \rangle &= 0 \end{aligned}$$

$$\langle \varphi(r) \rangle = \frac{\int \int \varphi(r, \theta, \phi, t) d\Omega dt}{\int \int d\Omega dt}$$

321D

- For instance, consider the one-point correlation tensor.

$$\mathcal{R} = \mathbf{u} \otimes \mathbf{u} = \begin{pmatrix} u_r u_r & u_r u_\theta & u_r u_\phi \\ u_\theta u_r & u_\theta u_\theta & u_\theta u_\phi \\ u_\phi u_r & u_\phi u_\theta & u_\phi u_\phi \end{pmatrix}$$

321D

- For instance, consider the one-point correlation tensor.

$$\mathcal{R} = \mathbf{u} \otimes \mathbf{u}$$

$$\partial_t \langle \rho \mathcal{R} \rangle + \nabla \cdot \langle \rho \mathcal{R} \mathbf{u}_0 \rangle = -\nabla \cdot \langle \rho \mathcal{R} \mathbf{u}' \rangle + \langle \mathcal{P}_{\mathrm{F}} \rangle + 2 \langle p' \mathcal{S} \rangle + \langle \mathcal{G} \rangle + \langle \mathcal{V} \rangle + \langle \mathcal{H} \rangle$$

$$\mathcal{P}_{\mathrm{F}} = -\left(\nabla \otimes \mathbf{F}_p + [\nabla \otimes \mathbf{F}_p]^T \right)$$

$$\mathcal{G} = \rho' \mathbf{u}' \otimes \mathbf{g} + [\rho' \mathbf{u}' \otimes \mathbf{g}]^T$$

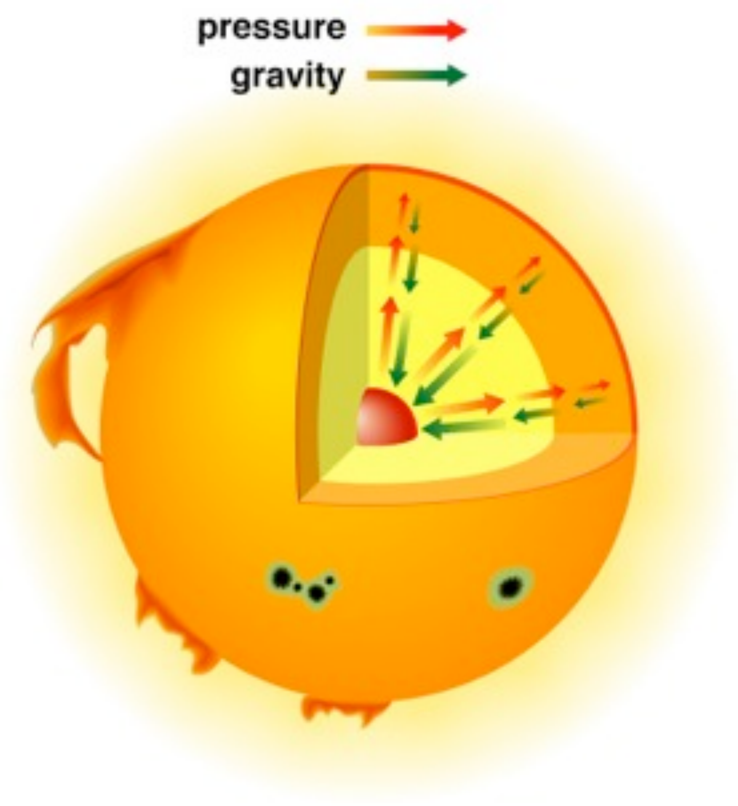
$$\mathbf{F}_p = p' \mathbf{u}'.$$

$$\mathcal{V} = \mu \left(\mathbf{u}_0 \otimes \nabla^2 \mathbf{u}_0 + [\mathbf{u}_0 \otimes \nabla^2 \mathbf{u}_0]^T \right) + \mu \left(\mathbf{u}' \otimes \nabla^2 \mathbf{u}' + [\mathbf{u}' \otimes \nabla^2 \mathbf{u}']^T \right)$$

$$\left(\nabla \otimes \mathbf{u}' + [\nabla \otimes \mathbf{u}']^T \right) = 2\mathcal{S}$$

321D

- The Current approach.



$$
\begin{aligned}
\frac{\partial r}{\partial m} &= \frac{1}{4\pi r^2 \rho} \\
\frac{\partial P}{\partial m} &= -\frac{Gm}{4\pi r^4} \boxed{- \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t}} \\
\frac{\partial L}{\partial m} &= \epsilon_n - \epsilon_\nu + \boxed{\epsilon_g} \\
\frac{\partial T}{\partial m} &= -\frac{GmT}{4\pi r^4 P}\nabla \\
D_t X_i &= \dot{X}_{i,\mathrm{nuc}} - \nabla \cdot \mathbf{J}_i \,, \quad i = 1, \ldots, N_{\mathrm{iso}}.
\end{aligned}
$$

321D

- THE CURRENT APPROACH.

$$\left(\frac{\partial X_n}{\partial t}\right)_m = \left(\frac{\partial}{\partial m}\right)_t \left[(4\pi r^2 \rho)^2 D \left(\frac{\partial X_n}{\partial m}\right)_t\right] + \left(\frac{dX_n}{dt}\right)_{\rm nuc}, \quad (7)$$

$$\left(\frac{\partial \omega}{\partial t}\right)_m = \frac{1}{i}\left(\frac{\partial}{\partial m}\right)_t \left[(4\pi r^2 \rho)^2 i\nu \left(\frac{\partial \omega}{\partial m}\right)_t\right] - \frac{2\omega}{r}\left(\frac{\partial r}{\partial t}\right)_m \left(\frac{1}{2}\frac{d\ln i}{d\ln r}\right) \quad (46)$$

And a variety of processes have been identified and mixing efficiencies have been formulated within this framework:

$$\boxed{D \sim vL, \text{ or } D \sim L^2/t}$$

$$D = D_{\rm conv} + D_{\rm sem} + f_c \times (D_{\rm DSI} + D_{\rm SHI} + D_{\rm SSI} + D_{\rm ES} + D_{\rm GSF}), \quad (53)$$

$$\nu = D_{\rm conv} + D_{\rm sem} + D_{\rm DSI} + D_{\rm SHI} + D_{\rm SSI} + D_{\rm ES} + D_{\rm GSF}. \quad (54)$$

From Heger, Langer, Woosley (2000)

3D Stellar Models

Some examples of stellar astrophysics impacted by stellar mixing

1. Trans-Fe element production by the s-process (slow neutron capture nucleosynthesis) in double shell burning giants

2. Core collapse supernova and gamma ray burst progenitor structures

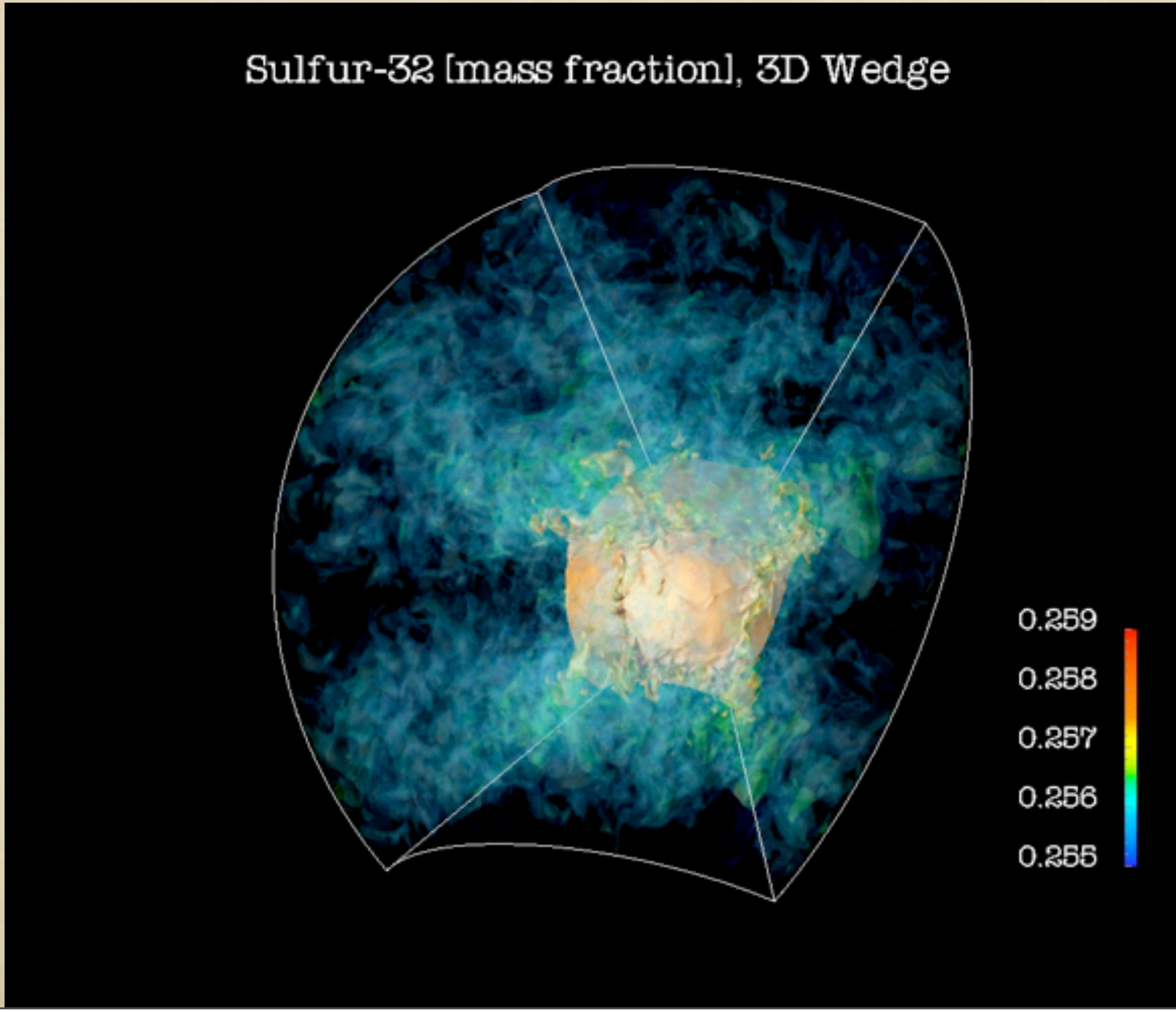
3. Pre-ignition convection in Type Ia progenitors

3D Stellar Models

A Numerical Laboratory

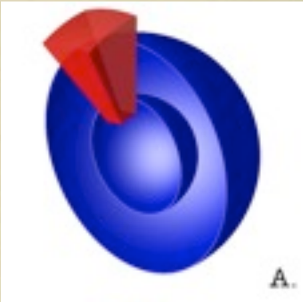
3D Stellar Models

A Numerical Laboratory



3D Stellar Models

A Numerical Laboratory



Internal Waves

Convective Boundary Adjustment

Nuclear "Flame"

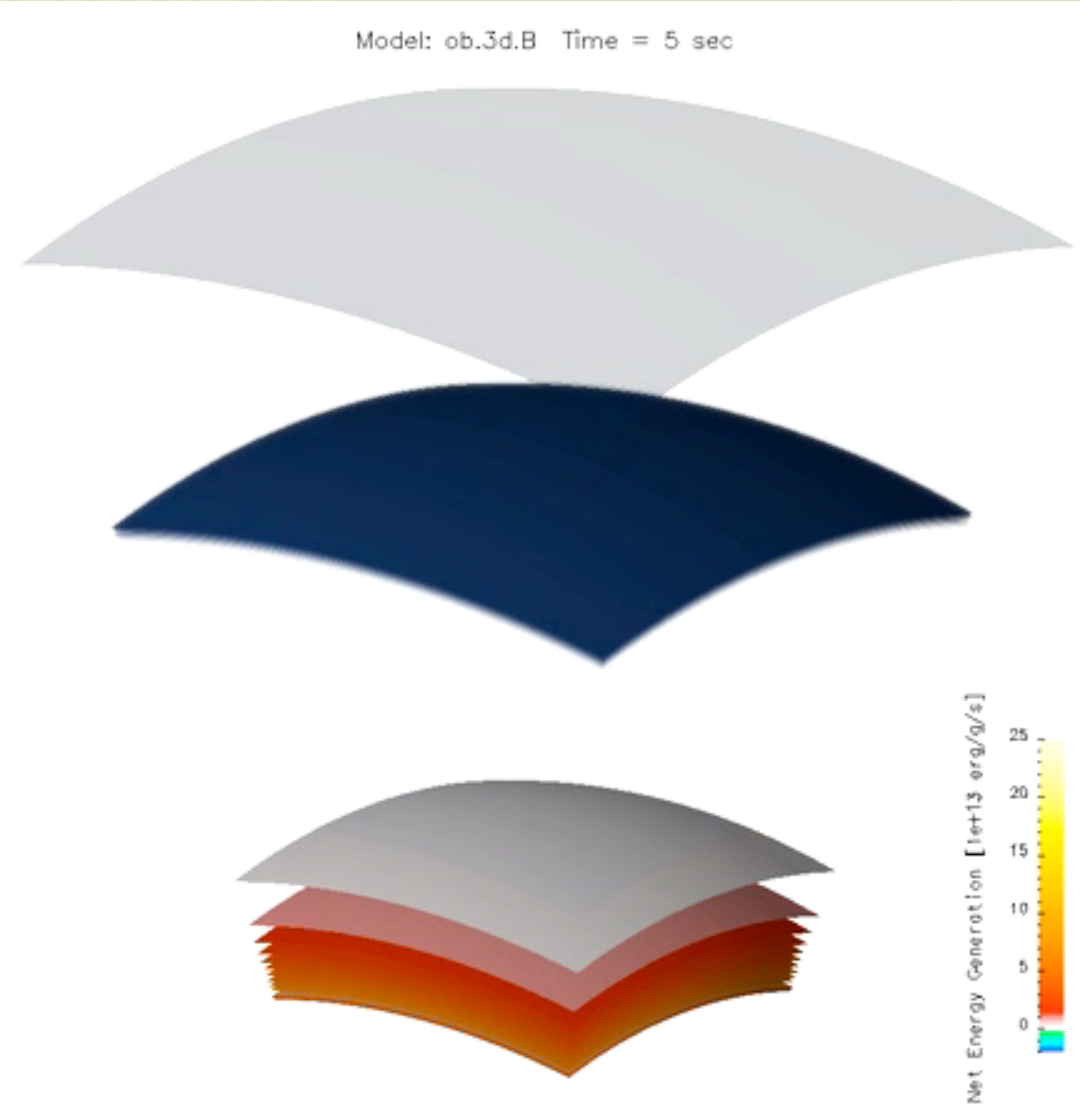
3D Stellar Models

A Numerical Laboratory

Internal Waves

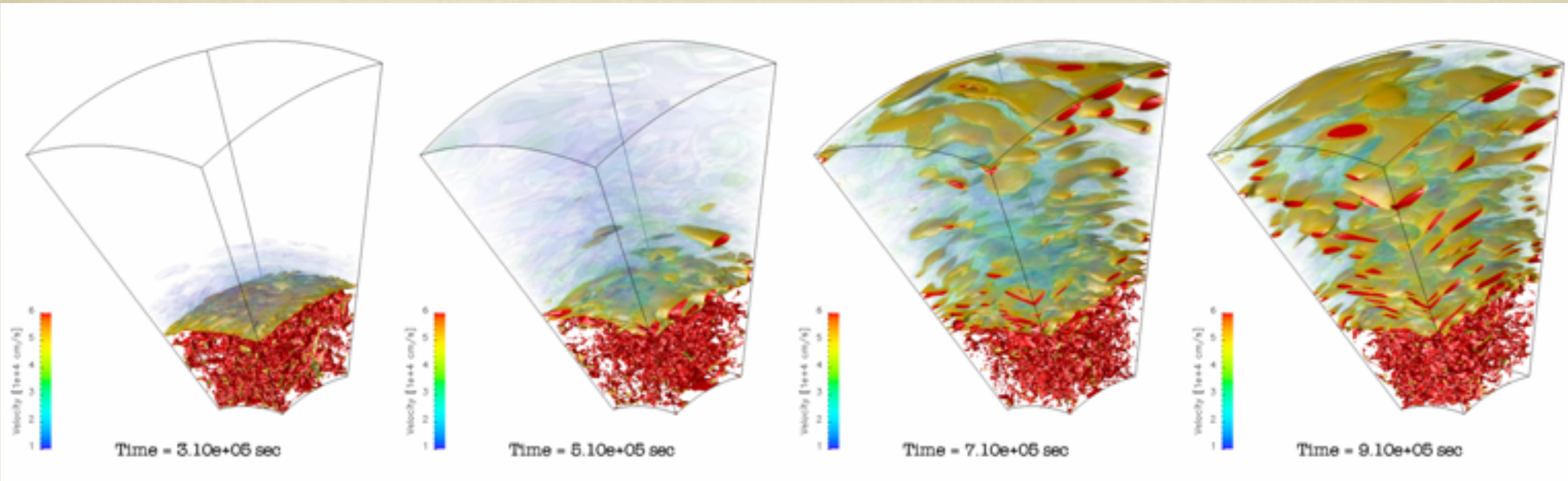
Convective Boundary Adjustment

Nuclear "Flame"



3D Stellar Models

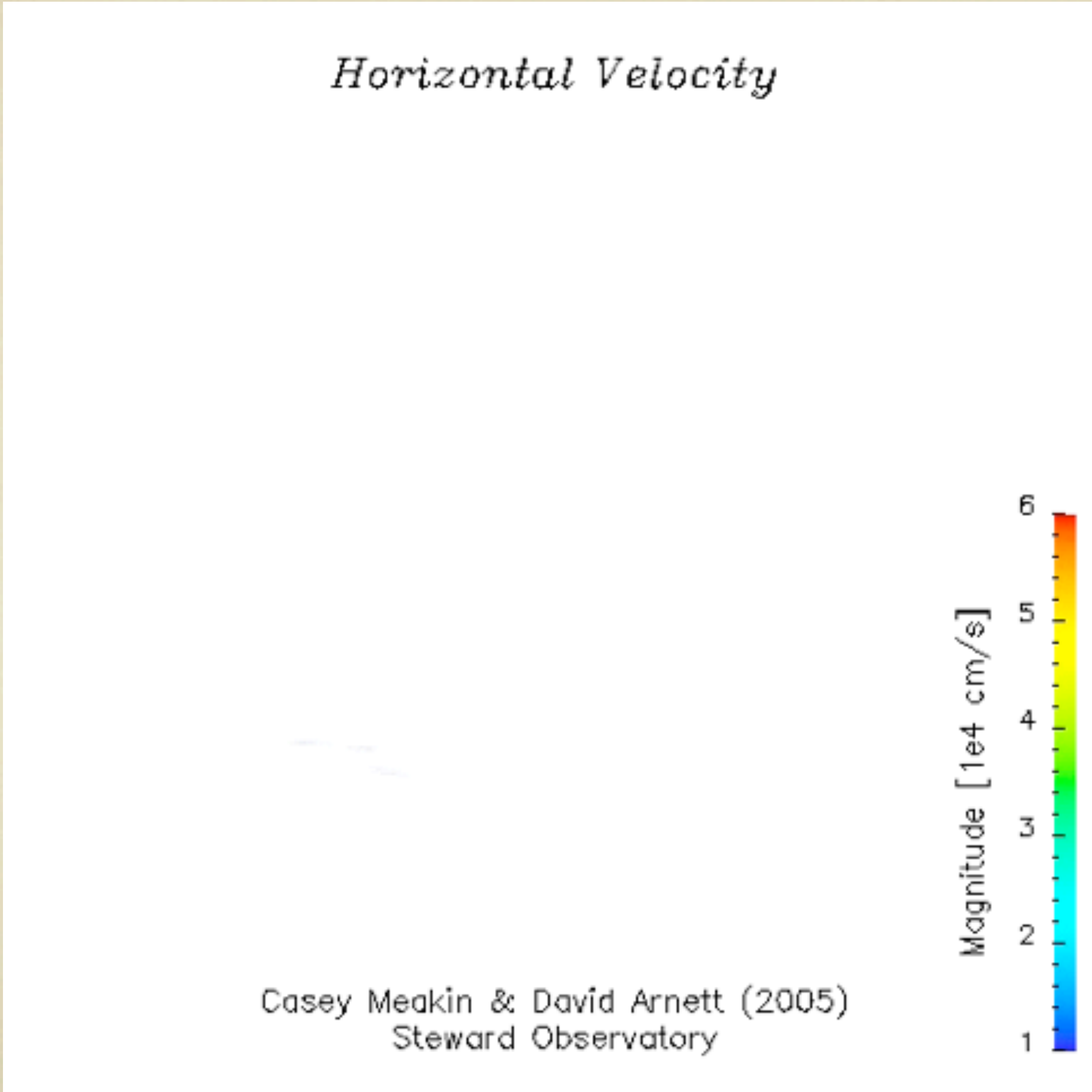
Internal Wave Physics



Main Sequence Core Convection

3D Stellar Models

Internal Wave Physics



3D Stellar Models

Multiple Shell Interactions

Convection during last hour before core collapse

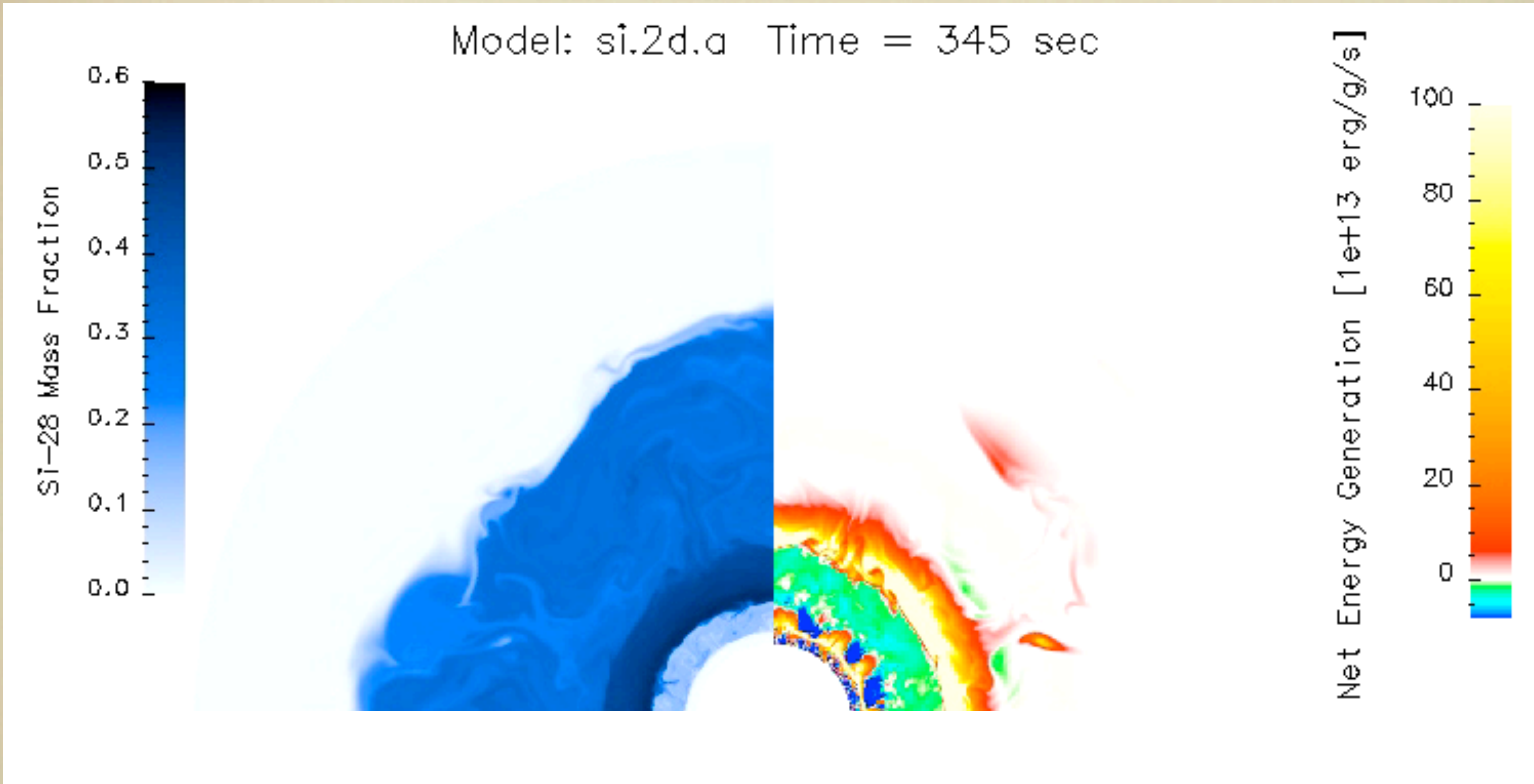
Oxygen Burning

Neon/Carbon Burning

C. Meakin, PhD Thesis

3D Stellar Models

Multiple Shell Interactions



Convection during last hour before core collapse

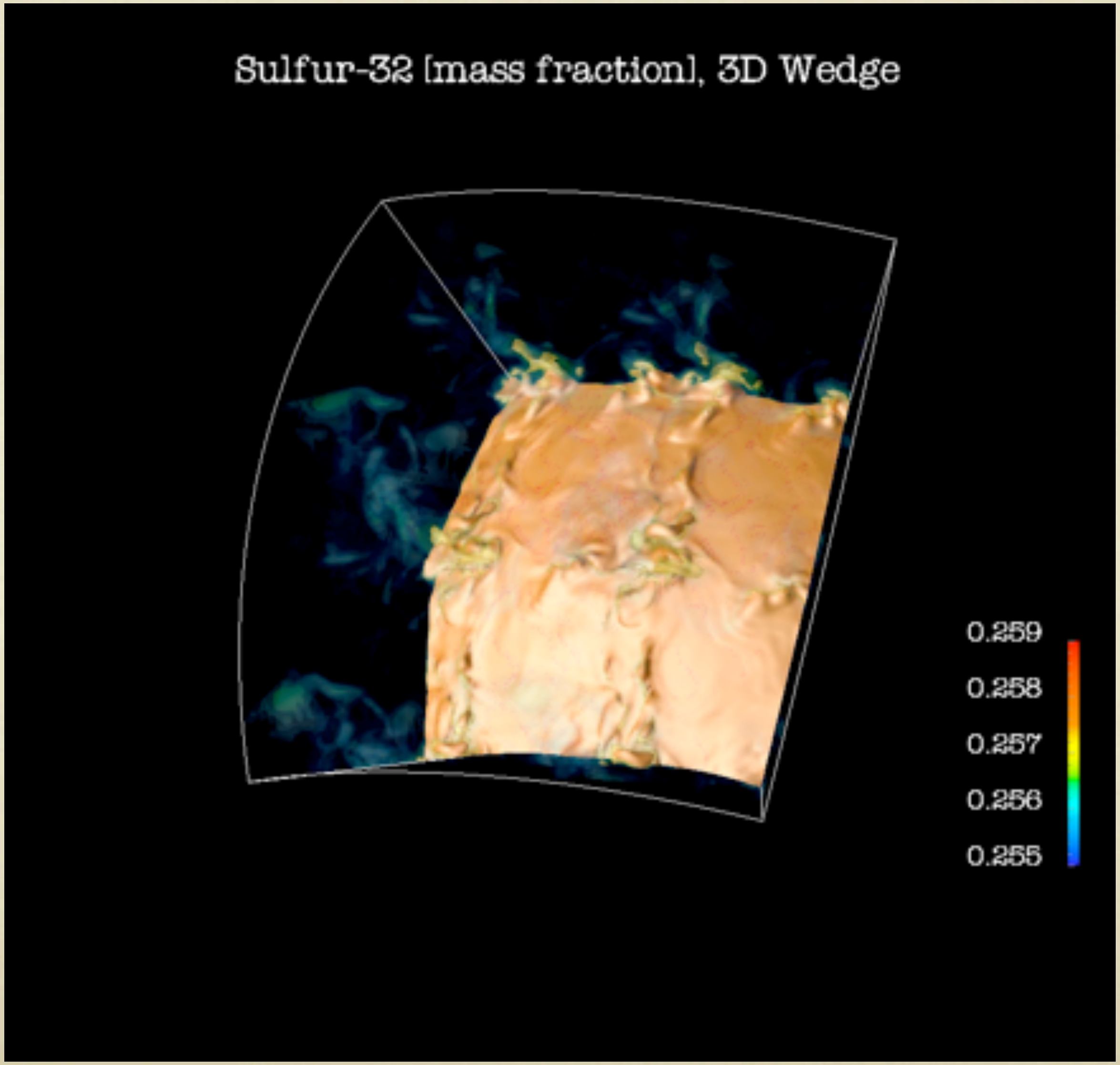
Oxygen Burning

Neon/Carbon Burning

C. Meakin, PhD Thesis

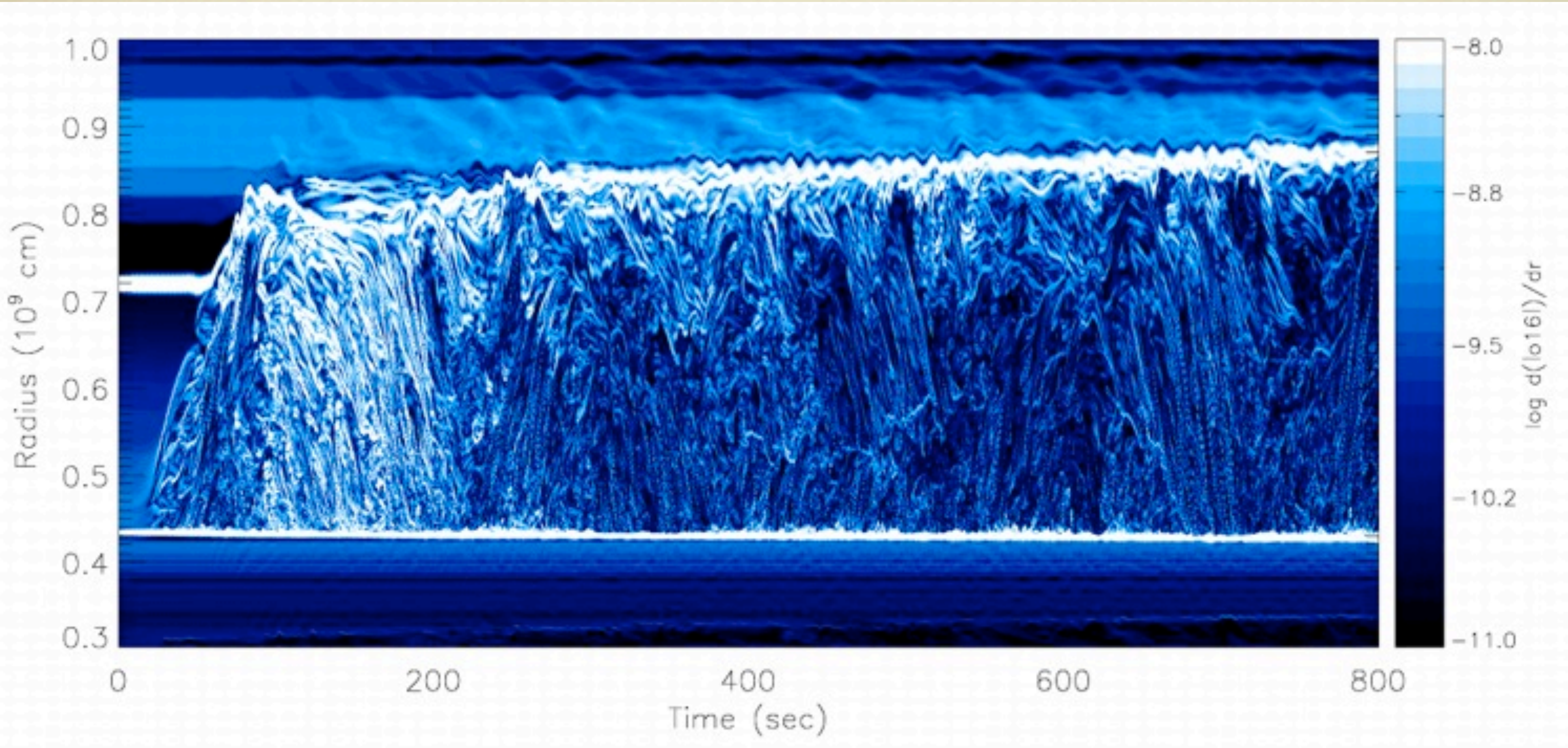
3D Stellar Models

Boundary Layer Mass Entrainment



3D Stellar Models

Boundary Layer Mass Entrainment



3D Stellar Models

Conclusions

★ While I discussed primarily my own work, several other groups are investigating the interiors of stars in 3D. These include:

1. Colorado Group - ASH code:
   Solar models, internal rotation, Dynamo

2. Stony Brook & LBNL - Maestro Code:
   Pre-ignition convection in CO WD

3. Munich - Mocak & Muller - PPM Code:
   Core He and C flash

4. Herwig & Woodward et al. - PPM Code:
   He shell flash convection

3D Stellar Models

Conclusions

★ Stellar evolution is a crucial input to many areas of astrophysical research.

★ Our current best models suffer severe deficiencies in treating turbulent transport and mixing which dominate uncertainties in most cases (together with mass loss).

★ Numerical simulations of turbulent flows and related interdisciplinary studies (e.g., geophysical) are beginning to provide ever deeper insight into how to improve this situation and lead to discoveries.

★ While there is still significant work ahead, the future looks bright for moving beyond back of the envelope, mixing-length style treatments of stellar evolution.

★Precision observational data is arriving just in time to begin testing our increasingly more sophisticated modeling of stellar interiors, e.g., wide eclipsing binary data and astero-seismic data.