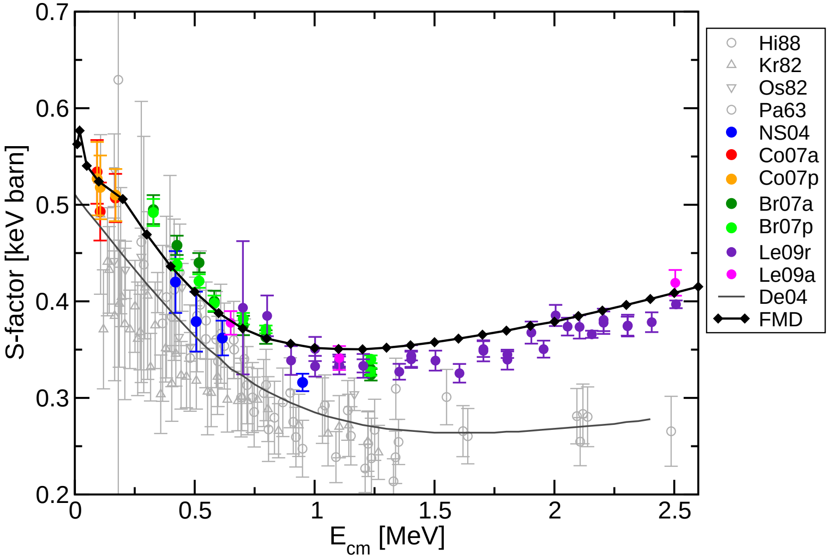
Microscopic Nuclear Structure and Reaction Calculations in the FMD Approach



Thomas Neff
Hans Feldmeier, Karlheinz Langanke

11th International Symposium on Nuclei in the Cosmos NIC XI

Heidelberg, Germany
July 19, 2010

Overview

Effective Nucleon-Nucleon interaction:

Unitary Correlation Operator Method

- **Short-range Central and Tensor Correlations**

Many-Body Method:

Fermionic Molecular Dynamics

- **Model**
- **Nuclear Structure Applications**

Reactions:

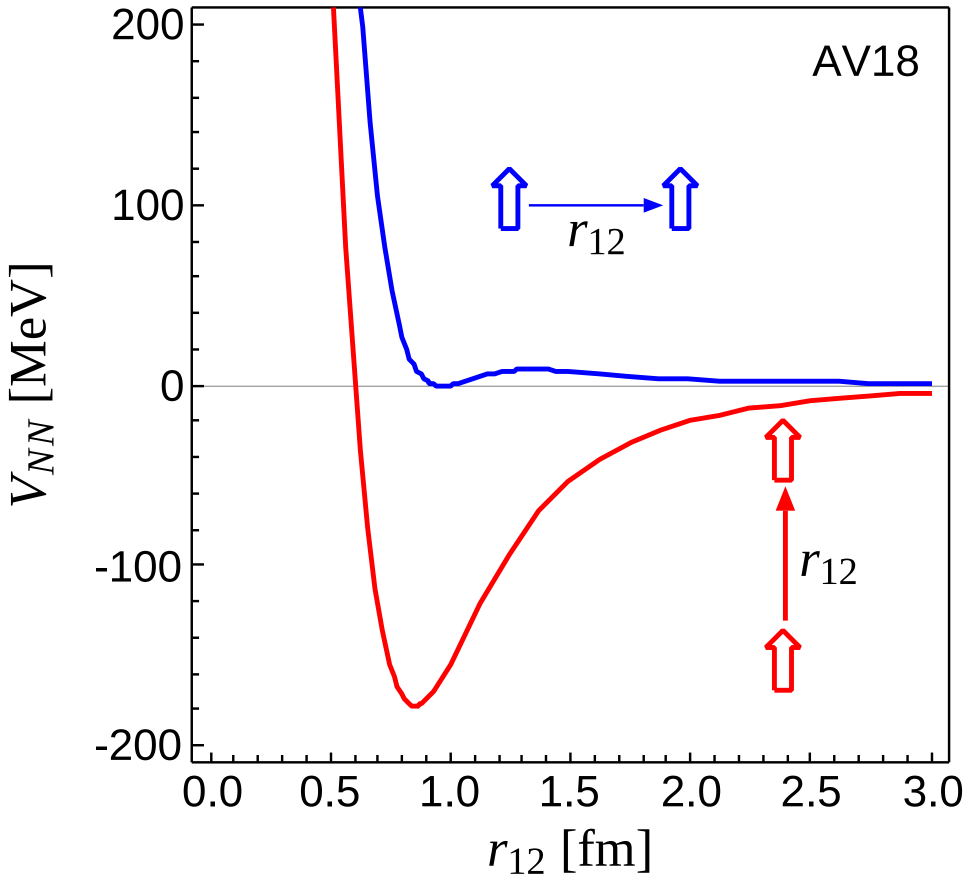
$^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ radiative capture

- $^7\mathrm{Be}$ **Bound States and Scattering Phase Shifts**
- **S-Factor**

Nuclear Force

Argonne V18 (T=0)

spins aligned parallel or perpendicular to the relative distance vector



- strong repulsive core: nucleons can not get closer than ≈ 0.5 fm

➨ **central correlations**

- strong dependence on the orientation of the spins due to the tensor force

➨ **tensor correlations**

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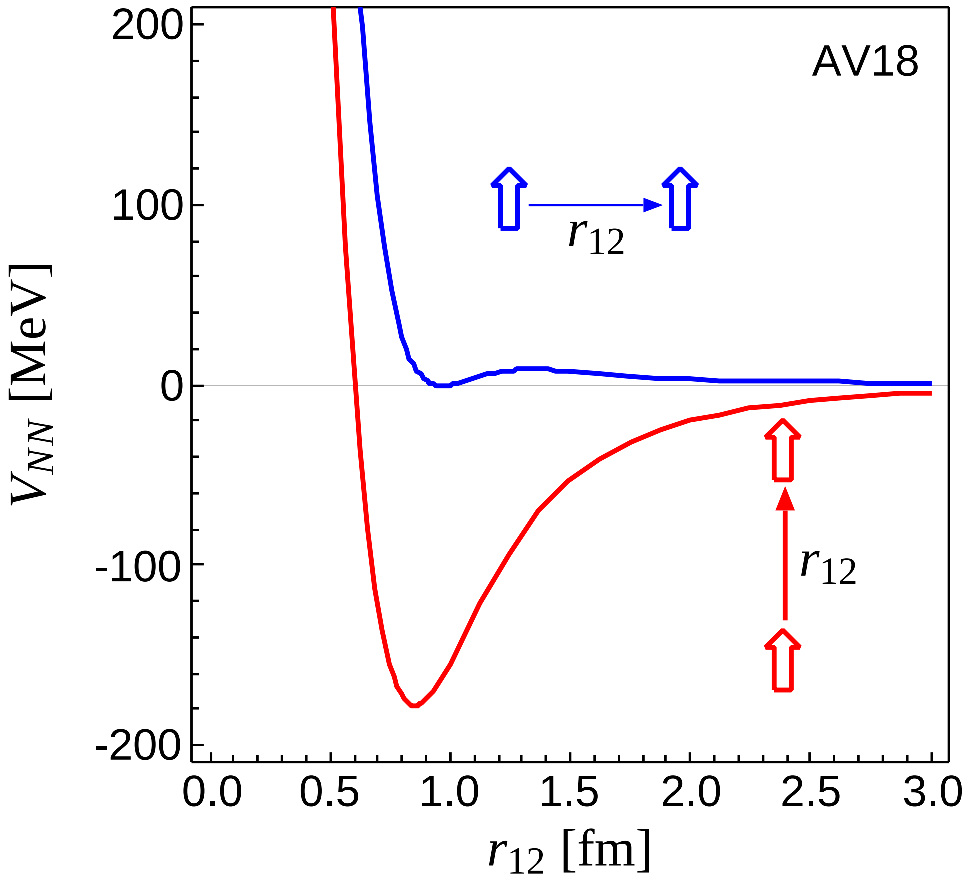

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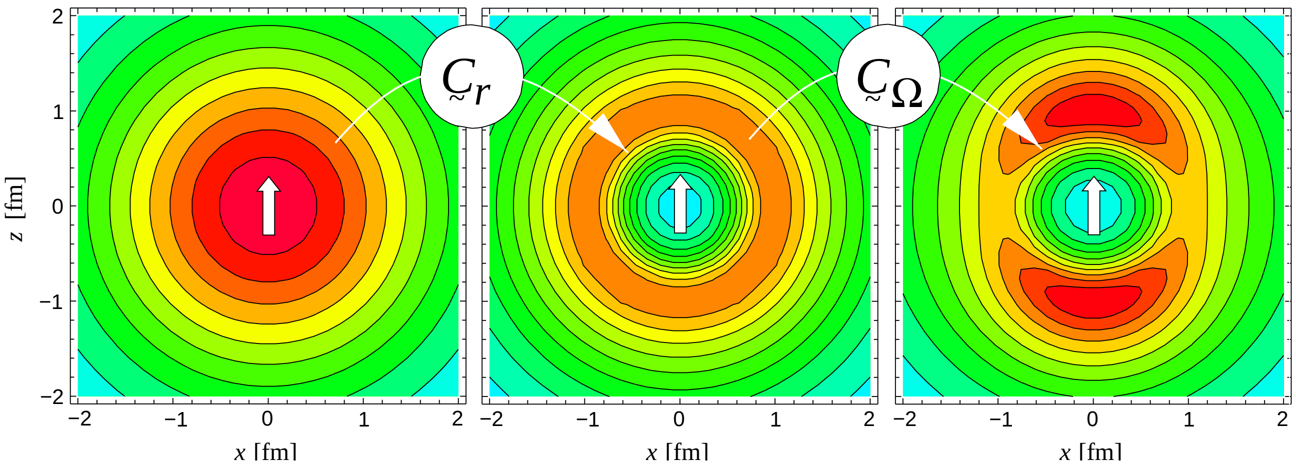
the nuclear force will induce **strong short-range correlations** in the nuclear wave function

Realistic Effective Interaction

two-body densities



$$\rho^{(2)}_{S,T}(\mathbf{r}_1 - \mathbf{r}_2) \quad S = 1, M_S = 1, T = 0$$

central correlator $\underset{\sim}{C}_{\mathbf{r}}$ shifts density out of the repulsive core

tensor correlator $\underset{\sim}{C}_{\mathbf{\Omega}}$ aligns density with spin orientation

Neff and Feldmeier, Nucl. Phys. **A713** (2003) 311

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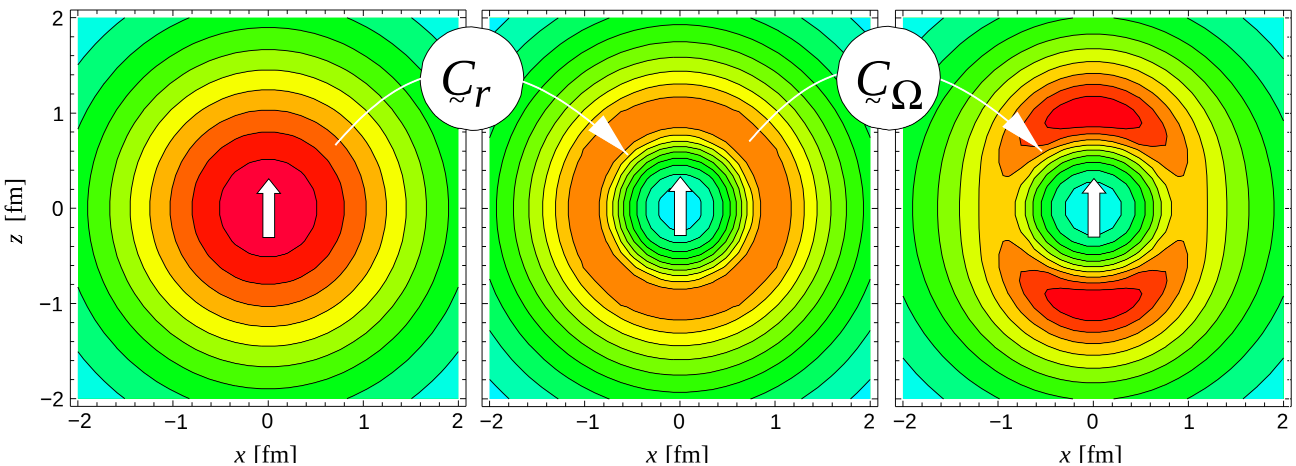

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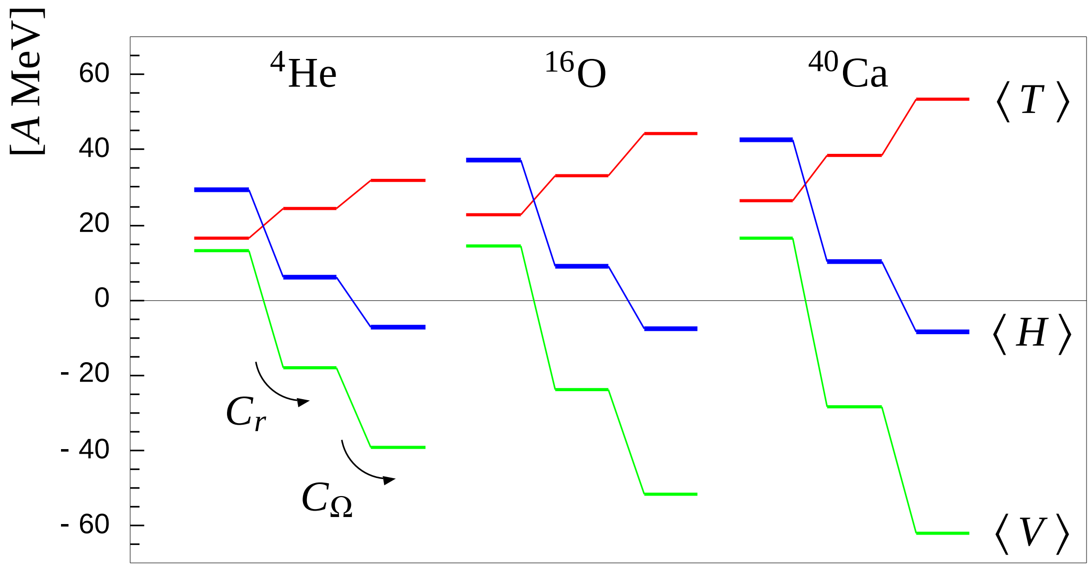
both central and tensor correlations are essential for binding

energies

$0\hbar\omega$ Harmonic Oscillator



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Fermionic Molecular Dynamics

Fermionic

Slater determinant

$$\left| Q \right\rangle = \underset{\sim}{\mathcal{A}} \Big(\left| q_1 \right\rangle \otimes \cdots \otimes \left| q_A \right\rangle \Big)$$

- antisymmetrized A-body state

Feldmeier, Schnack, Rev. Mod. Phys. **72** (2000) 655
Neff, Feldmeier, Eur. Phys. J Special Topics **156** (2008) 69

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single-particle states

$$\left\langle \mathbf{x} \middle| q \right\rangle = \exp\left\{ -\frac{(\mathbf{x}-\mathbf{b})^2}{2a} \right\} \otimes \left| \chi^{\uparrow}, \chi^{\downarrow} \right\rangle \otimes \left| \xi \right\rangle$$

- Gaussian wave-packets in phase-space (complex parameter **b** encodes mean position and mean momentum), spin is free, isospin is fixed
- width a is an independent variational parameter for each wave packet

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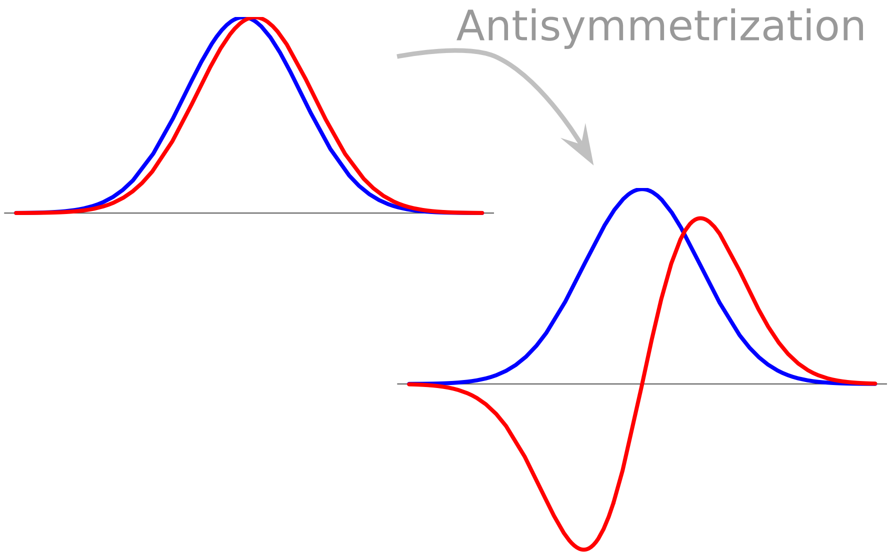

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PAV, VAP and Multiconfiguration

Projection After Variation (PAV)

- intrinsic state may break symmetries of Hamiltonian
- restore inversion, translational and rotational symmetry by projection on parity, linear and angular momentum

$$\underset{\sim}{P}^{\pi} = \frac{1}{2}(1 + \pi \underset{\sim}{\Pi})$$

$$\underset{\sim}{P}^{J}_{MK} = \frac{2J+1}{8\pi^2} \int d^3\Omega \, D^{J}_{MK}{}^{\star}(\Omega) \, \underset{\sim}{R}(\Omega)$$

$$\underset{\sim}{P}^{\mathbf{P}} = \frac{1}{(2\pi)^3} \int d^3X \; \exp\{-i(\underset{\sim}{\mathbf{P}} - \mathbf{P}) \cdot \mathbf{X}\}$$

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- full Variation after Angular Momentum and Parity Projection (VAP) for light nuclei

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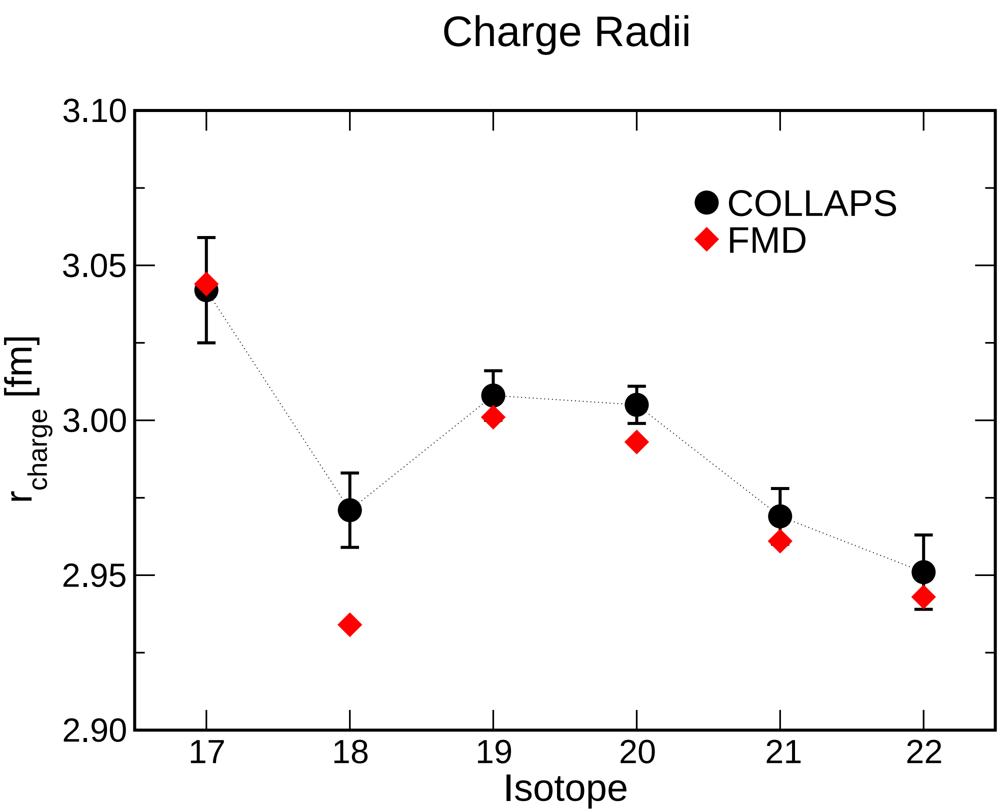
Multiconfiguration Calculations

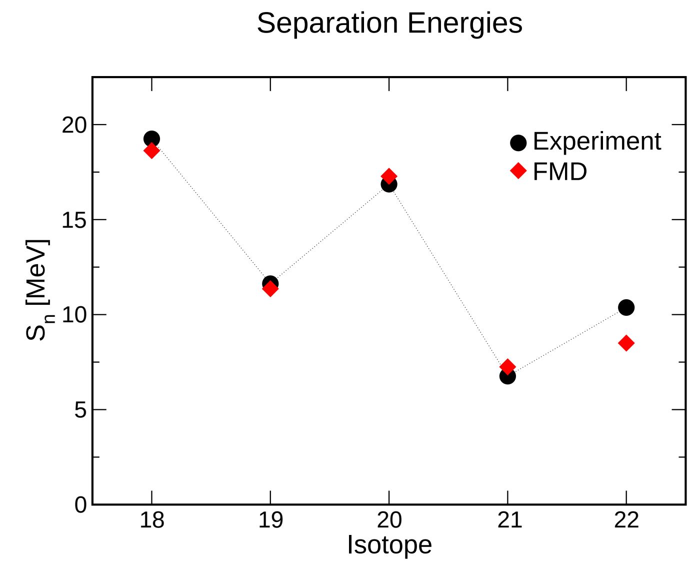
- **diagonalize** Hamiltonian in a set of projected intrinsic states

$$\left\{ \left| Q^{(a)} \right\rangle, \quad a = 1, \ldots, N \right\}$$

$$\sum_{K'b} \left\langle Q^{(a)} \right| \underset{\sim}{H} \underset{\sim}{P}^{J^\pi}_{KK'} \underset{\sim}{P}^{\mathbf{P}=0} \left| Q^{(b)} \right\rangle \cdot c^{\alpha}_{K'b} = E^{J^\pi \alpha} \sum_{K'b} \left\langle Q^{(a)} \right| \underset{\sim}{P}^{J^\pi}_{KK'} \underset{\sim}{P}^{\mathbf{P}=0} \left| Q^{(b)} \right\rangle \cdot c^{\alpha}_{K'b}$$

Example: Neon Isotopes





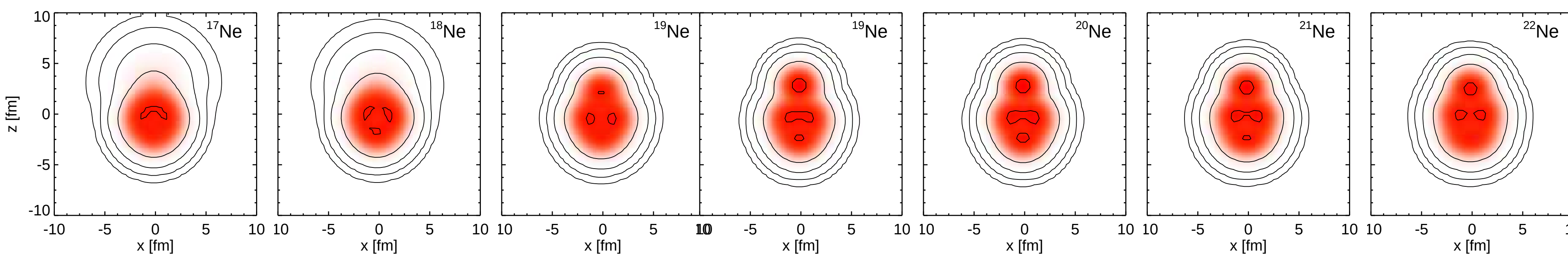
nuclear structure details responsible for peculiar behaviour of charge radii

Geithner, Neff, *et. al.*, Phys. Rev. Lett. **101** (2008) 252502

^{17,18}Ne: s^2/d^2 admixture

^{19}Ne: ^{3}He, α clustering

$^{20-22}$Ne: α clustering



Models

Potential models

- ^{4}He and ^{3}He are point-like particles
- interacting via an effective nucleus-nucleus potential fitted to bound state properties and phase shifts

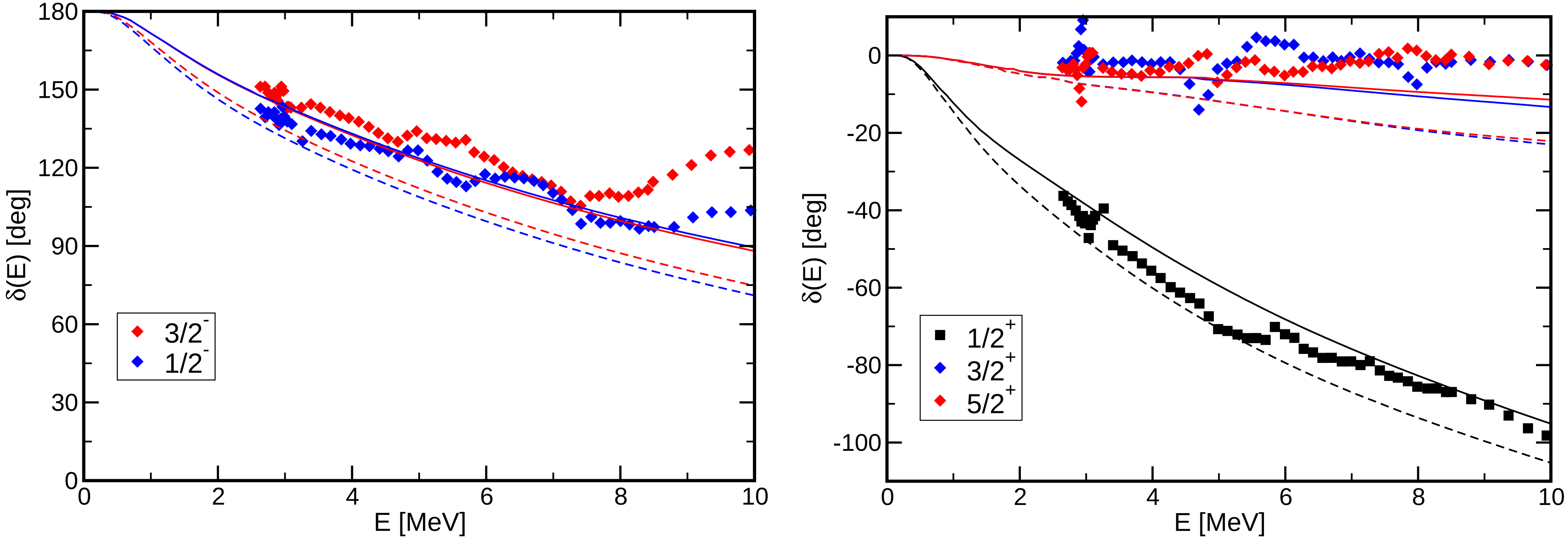
Microscopic Cluster Models

- antisymmetrized wave function built with ^{4}He and ^{3}He clusters
- polarization effects sometimes included by adding other channels like ^{6}Li plus proton
- interacting via an effective nucleon-nucleon potential, adjusted to describe bound state properties and phase shifts

Fermionic Molecular Dynamics

- antisymmetrized wave function built with ^{4}He and ^{3}He FMD clusters
- FMD wave functions obtained in variation after angular momentum projection on $1/2^-$, $3/2^-$, $5/2^-$, $7/2^-$ and $1/2^+$, $3/2^+$ and $5/2^+$ with radius constraint in the interaction region to include polarization effects
- interacting via realistic UCOM interaction that reproduces the nucleon-nucleon phase shifts

Bound and Scattering States



dashed lines – frozen configurations only, solid lines – FMD configurations in interaction region included

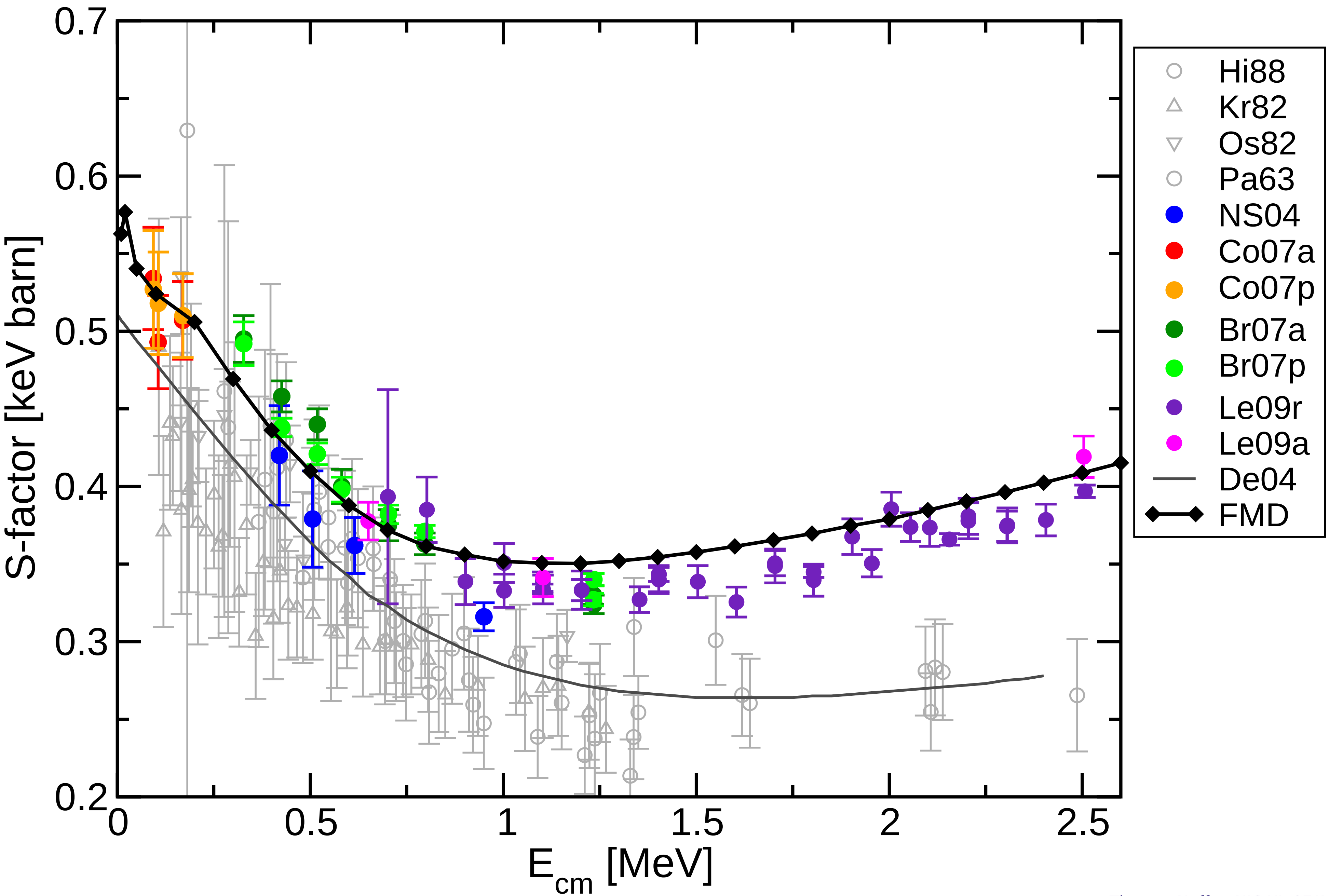
Bound states

	Experiment	FMD
$E_{3/2-}$	-1.59 MeV	-1.50 MeV
$E_{1/2-}$	-1.15 MeV	-1.49 MeV
r_{charge}	2.647(17) fm	2.67 fm

- Scattering phase shifts well described, polarization effects important
- splitting between $3/2^-$ and $1/2^-$ states too small, but centroid energy and charge radius well reproduced
- with frozen configurations only the $1/2^-$ is and the $3/2^-$ state is almost unbound – polarization effects are essential

$^3He(\alpha,\gamma)^7Be$

S-Factor



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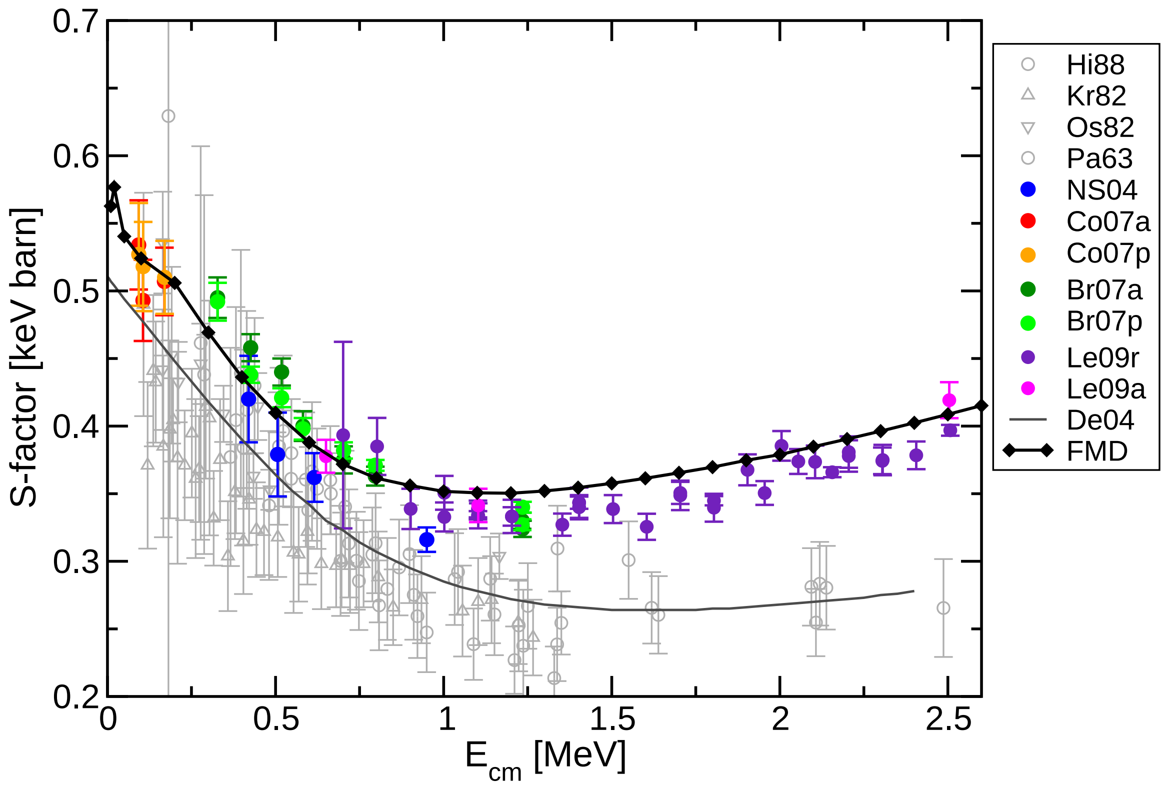

- dipole transitions from $1/2^{+}$, $3/2^{+}$, $5/2^{+}$ scattering states into $3/2^{-}$, $1/2^{-}$ bound states

- energy dependence and normalization of new high quality data well described
- cross section depends significantly on internal part of wave function, description as an “external” capture is too simplified

- numerics becomes difficult at very low energies, extrapolation to $E = 0$ therefore hard

Summary

Unitary Correlation Operator Method

- Explicit description of short-range central and tensor correlations
- Decouples low- and high-momentum modes

Fermionic Molecular Dynamics

- Microscopic many-body approach using Gaussian wave-packets
- Projection and multiconfiguration mixing
- Consistent description of well bound states with shell structure and loosely bound states of cluster or halo nature

$^{3}\mathrm{He}(\alpha,\gamma)^{7}\mathrm{Be}$ Radiative Capture

- Fully microscopic calculation with realisitic two-body interaction
- Bound states, resonance and scattering wave functions
- S-Factor: energy dependence and normalization reproduced

➨ analyze internal part of wave function, extrapolation to $E = 0$

➨ role of three-body forces ?

Thanks

to my Collaborators

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