Clusters in Dense Matter and the Equation of State

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11th Symposium on Nuclei in the Cosmos

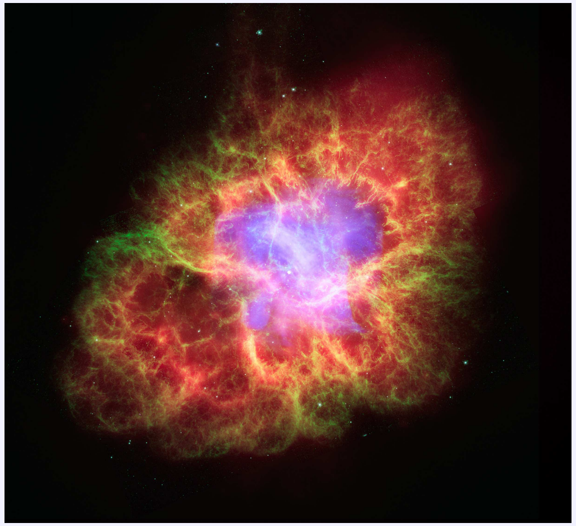
The Life and Death of Stars

when the era of nuclear fusion reactions ends:

- last phases in the life of a massive star $(8M_{\rm sun} \lesssim M_{\rm star} \lesssim 30M_{\rm sun})$

  $\Rightarrow$ core-collapse supernova

  $\Rightarrow$ neutron star or black hole



X-ray: NASA/CXC/J.Hester (ASU)
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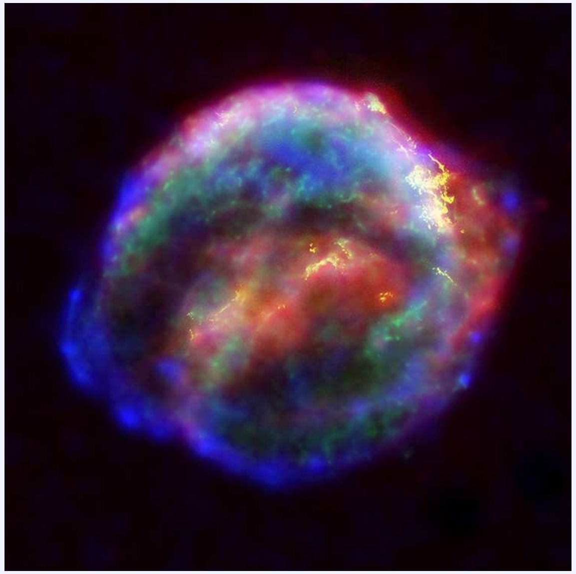
Infrared: NASA/JPL-Caltech/R.Gehrz (Univ. Minn.)



NASA/ESA/R.Sankrit & W.Blair (Johns Hopkins Univ.)

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- essential ingredient in
  astrophysical model calculations:

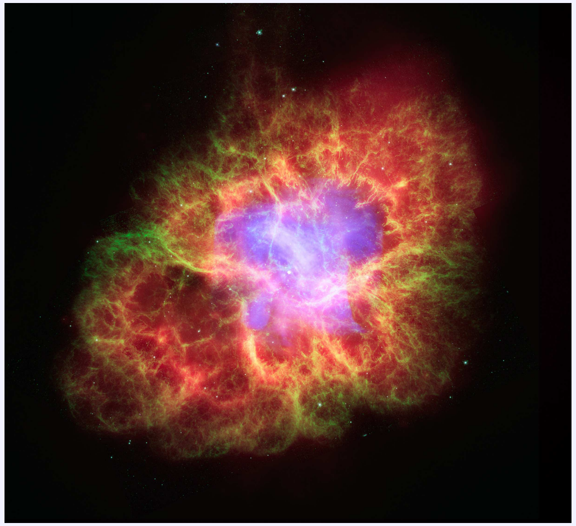
  equation of state (EoS) of dense matter

  $\Rightarrow$ dynamical evolution of supernova

  $\Rightarrow$ static properties of neutron star

  $\Rightarrow$ conditions for nucleosynthesis

  $\Rightarrow$ energetics, chemical composition,
  transport properties, . . .



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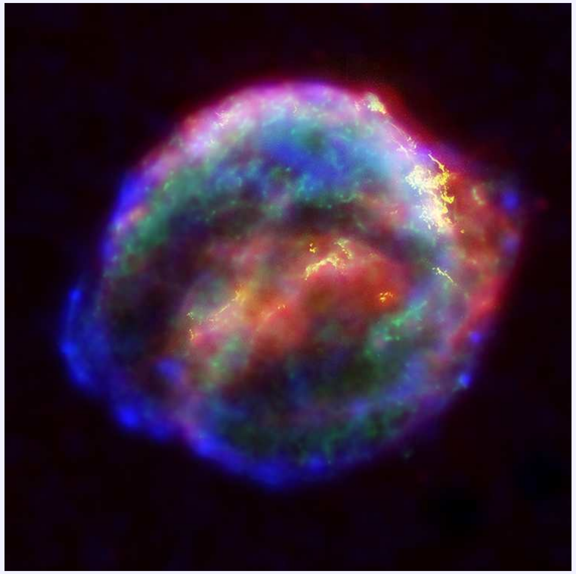

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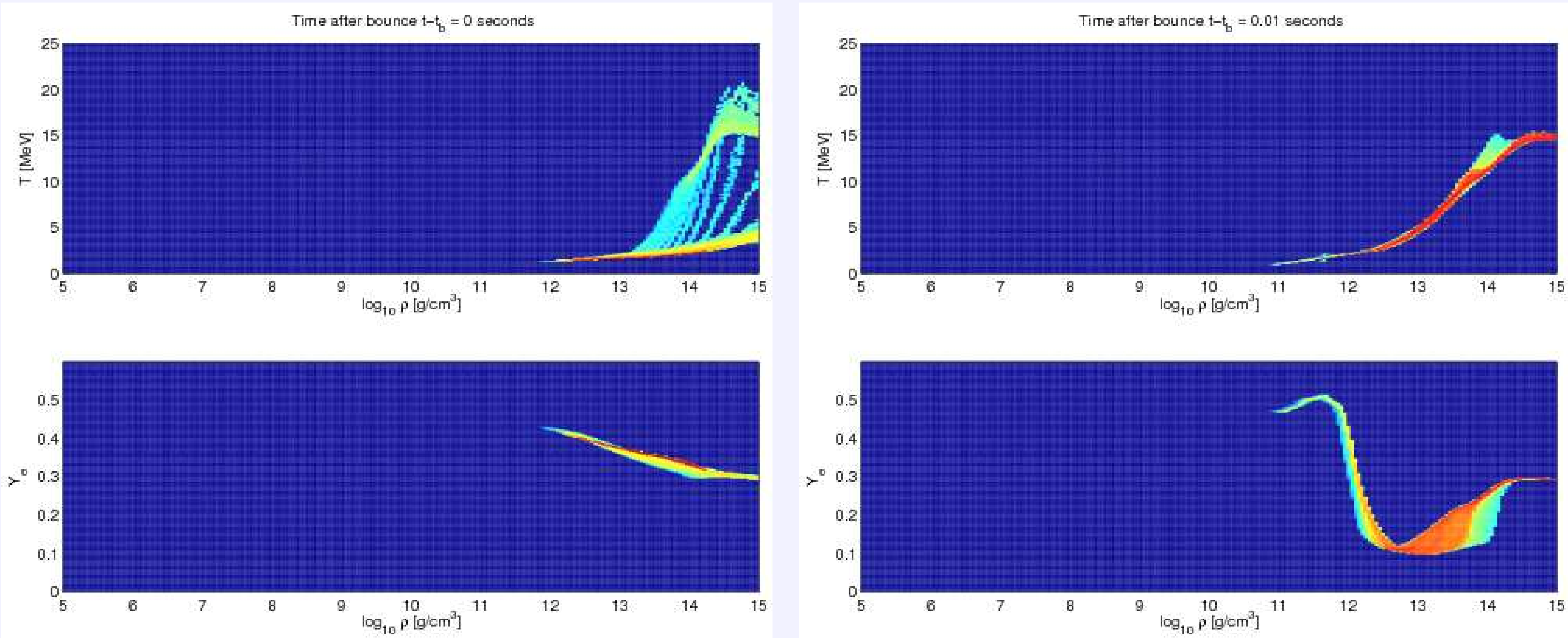
Thermodynamical Conditions

- **densities**: $10^{-9} \lesssim \varrho/\varrho_{\rm sat} \lesssim 10$ ($\varrho_{\rm sat} \approx 2.5 \cdot 10^{14}$ g/cm^3)
- **temperatures**: 0 MeV $\leq k_B T \lesssim 25$ MeV ($\hat{=}\ 2.9 \cdot 10^{11}$ K)
- **electron fraction**: $0 \leq Y_e \lesssim 0.6$



M. Liebendörfer, R. Käppeli, S. Scheidegger, Universität Basel

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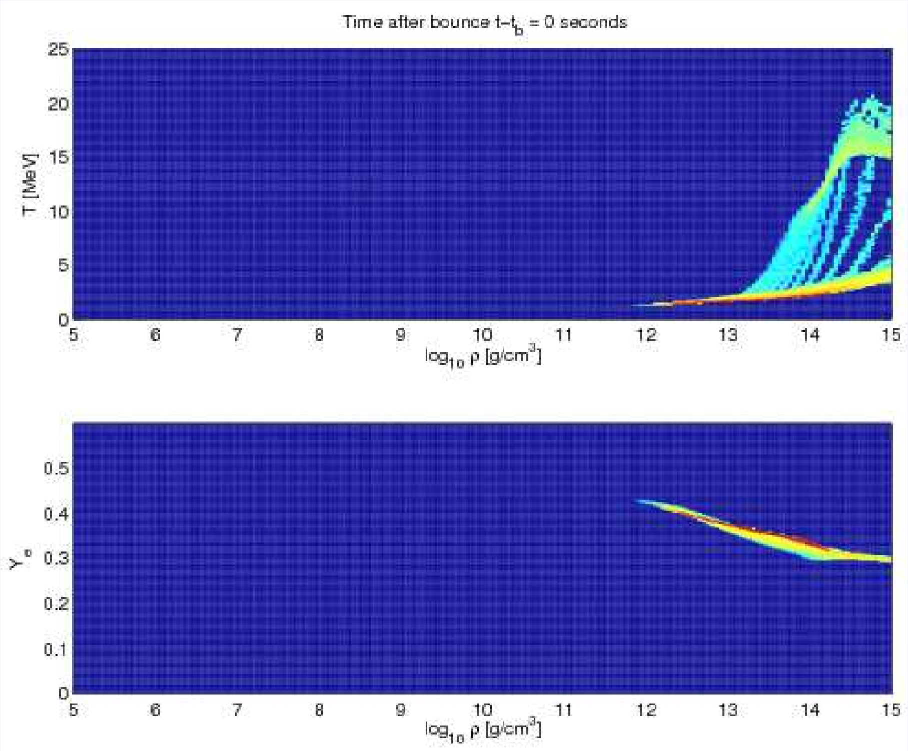
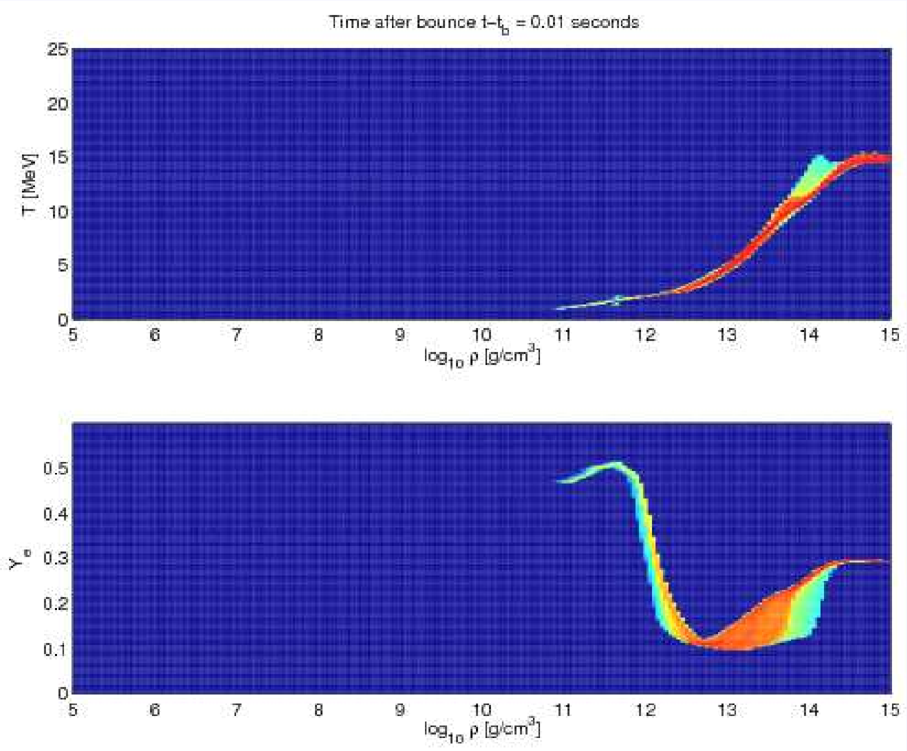
$\Rightarrow$ global theoretical description of matter properties is required





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Equation of State of Dense Matter

- many EoS developed in the past:

  from simple parametizations to sophisticated models

- many investigations of detailed aspects:

  often restricted to particular conditions (e.g. zero temperature)

- only few EoS used in astrophysical models: most well known
  - J.M. Lattimer, F.D. Swesty (Nucl. Phys. A 535 (1991) 331)
  - H. Shen, H. Toki, K. Oyamatsu, K. Sumiyoshi (Prog. Theor. Phys. 100 (1998) 1013)

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- most difficult problem:

  description of strongly interacting subsystem (hadronic or quark matter)
  in this talk: formation of "clusters" in nuclear matter

- in "standard" astrophysical EoS:

  only nucleons, α particle and representative heavy nucleus,
  suppression of cluster formation with phenomenological excluded-volume mechanism

$\Rightarrow$ consider **more microscopic model, more clusters**

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    problems:
    - properties of constituents independent of medium
    - interaction between particles
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  - **physical picture:**
    correlations of nucleons ⇒ formation of bound states
    problems:
    - treatment of three-, four-, . . . many-body correlations difficult
    - choice of interaction

⇒ combination of approaches?

Composition of Nuclear Matter II

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  - surface effects and long-range Coulomb interaction
  - inhomogeneous matter
  - formation of "pasta" phases/lattice structures

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interpolation between low-density and high-density limit needed

⇒ consider quantum statistical approach and generalized relativistic mean-field model

Quantum Statistical Approach I

- nonrelativistic finite-temperature Green's function formalism
- starting point: nucleon number densities $(\tau = p, n)$

  $n_\tau(T, \tilde{\mu}_p, \tilde{\mu}_n) = 2 \int \frac{d^3k}{(2\pi)^3} \int \frac{d\omega}{2\pi}\, f_\tau(\omega) S_\tau(\omega)$ with Fermi distribution $f_\tau(\omega)$

  and spectral function $S_\tau(\omega)$ depending on self-energy Σ_τ
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$\Rightarrow$ **generalized Beth-Uhlenbeck descripton** with

- ◦ medium dependent self-energy shifts/binding energies
- ◦ generalized scattering phase shifts from in-medium T-matrix

- T, n_p, $n_n \Rightarrow \tilde{\mu}_p$, $\tilde{\mu}_n \Rightarrow$ free energy $F(T, n_p, n_n)$ by integration $\left(\left.\frac{\partial(F/V)}{\partial n_\tau}\right|_{T, n_{\tau'}} = \tilde{\mu}_\tau\right)$

  $\Rightarrow$ thermodynamically consistent derivation of EoS

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medium modifications

- single nucleon properties
  - self-energy shift of quasiparticle energy
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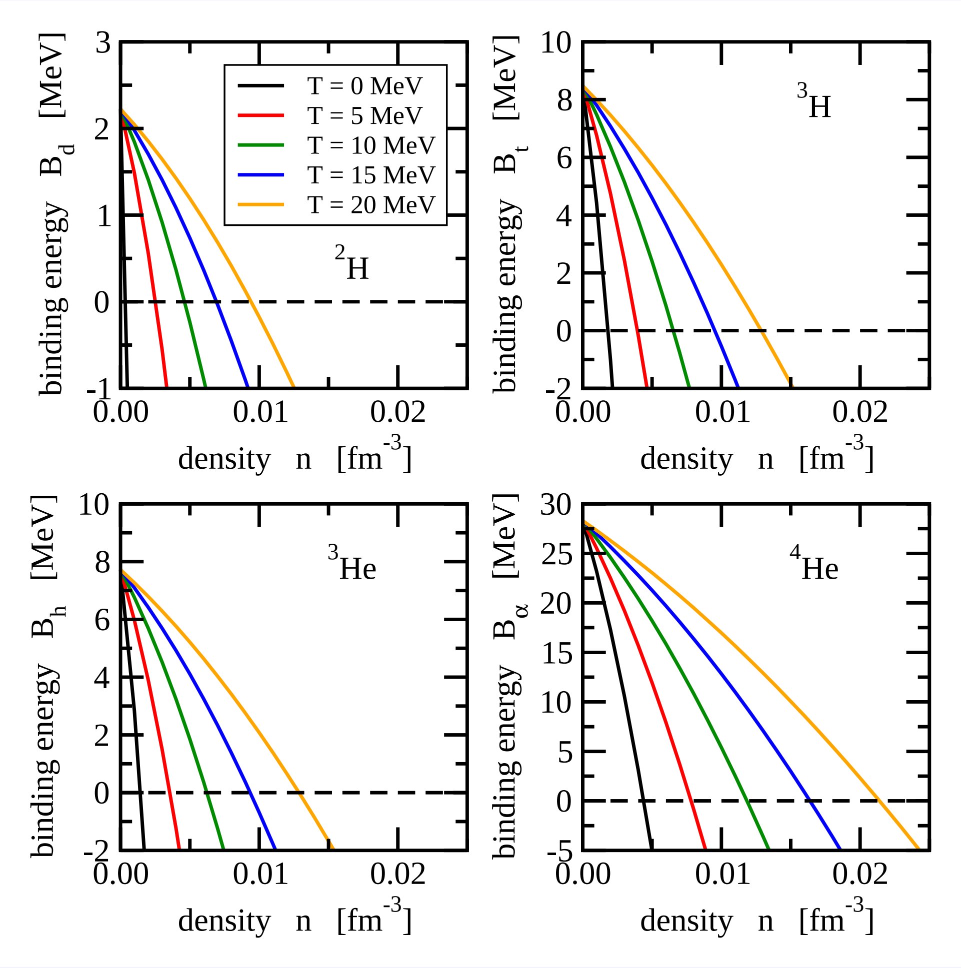
⇒ quasi-particles

symmetric nuclear matter



parametrization used in generalized RMF model

Generalized Relativistic Mean-Field (RMF) Model

- extended **relativistic Lagrangian density** of Walecka type
  with nucleons (ψ_p, ψ_n), deuterons (φ_d^μ), tritons (ψ_t), helions (ψ_h), α-particles (φ_α), mesons ($\sigma, \omega_\mu, \vec{\rho}_\mu$), electrons ($\psi_e$) and photons ($A_\mu$) as degrees of freedom
  - only minimal (linear) meson-nucleon couplings
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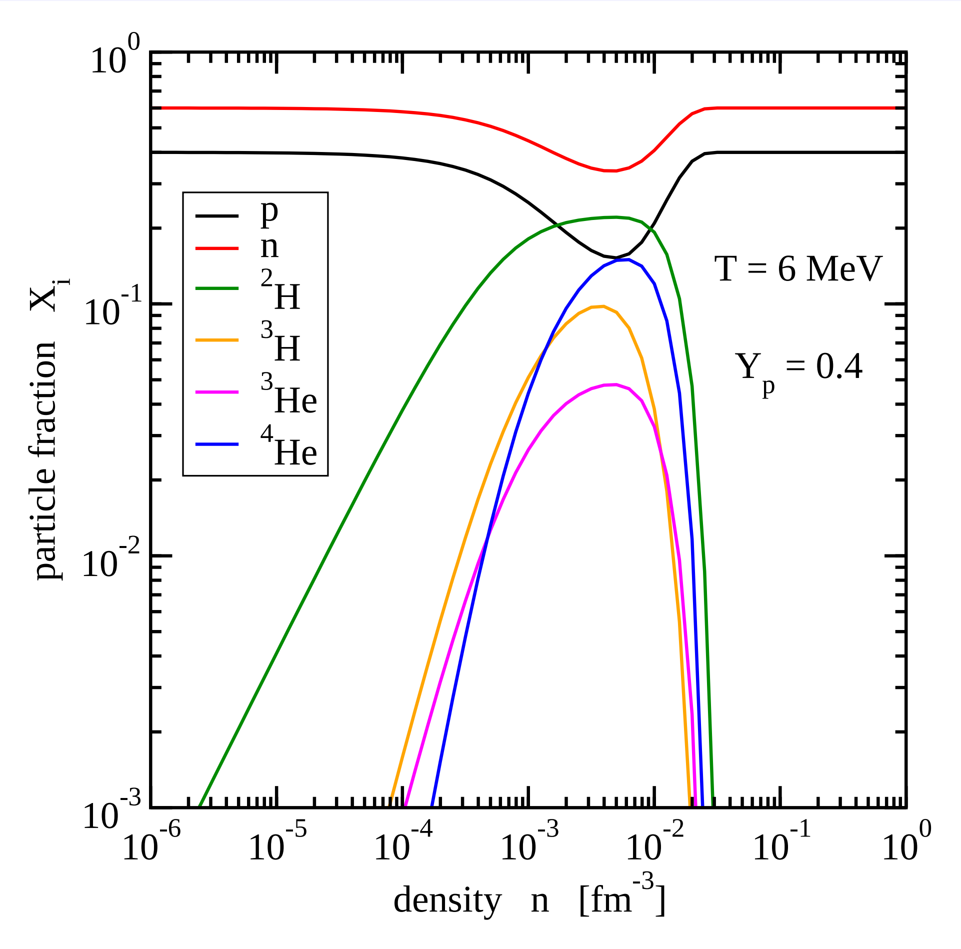
⇒ nucleon/cluster/meson/photon field equations,
solved selfconsistently in mean-field approximation
(classical meson/photon fields, Hartree approximation, no-sea approximation)

EoS with Light Clusters - Generalized RMF Model

- consider 2-, 3-, and 4-body correlations in the medium
  - presently only bound states (deuterons, tritons, helions, and alphas)
  - scattering contributions neglected so far
- Mott effect: clusters dissolve at high densities
- correct limits at low and high densities



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- Mott effect: clusters dissolve at high densities
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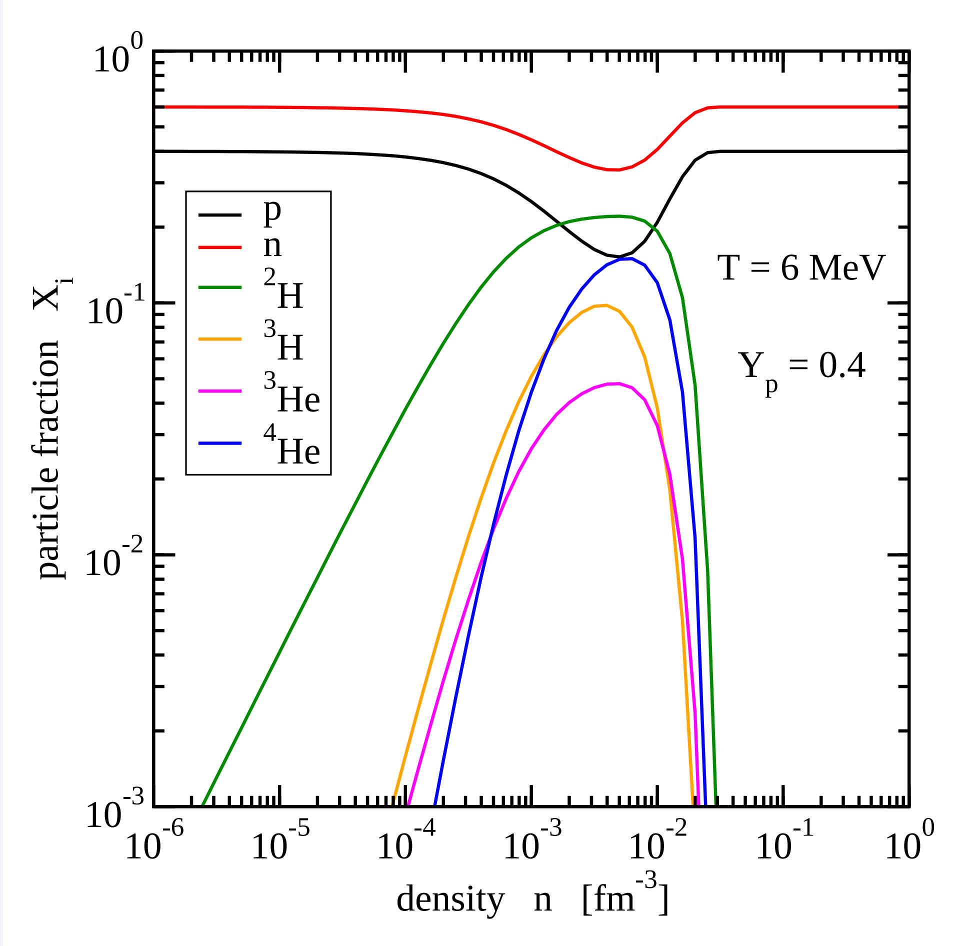
- no heavy clusters/phase transition included here
- medium dependence of couplings and binding energies

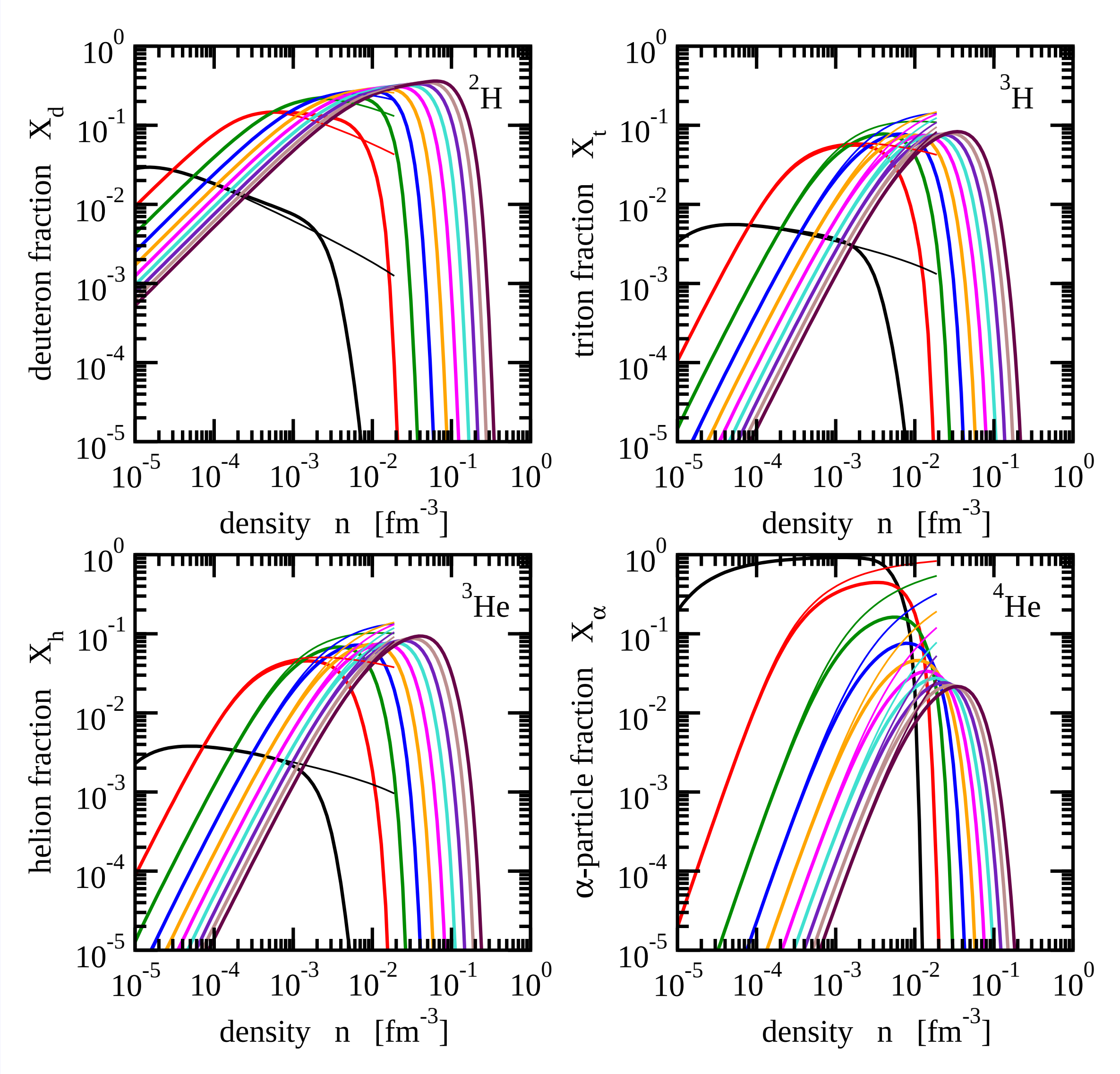
  ⇒ "rearrangement" contributions in self-energies and source densities essential for thermodynamical consistency

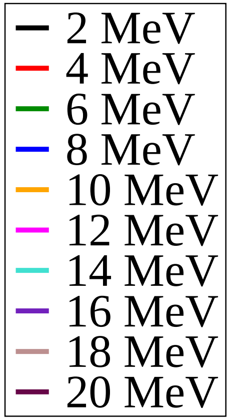


EoS with Light Clusters - Cluster Fractions

symmetric nuclear matter

generalized RMF model vs. NSE (thin lines)

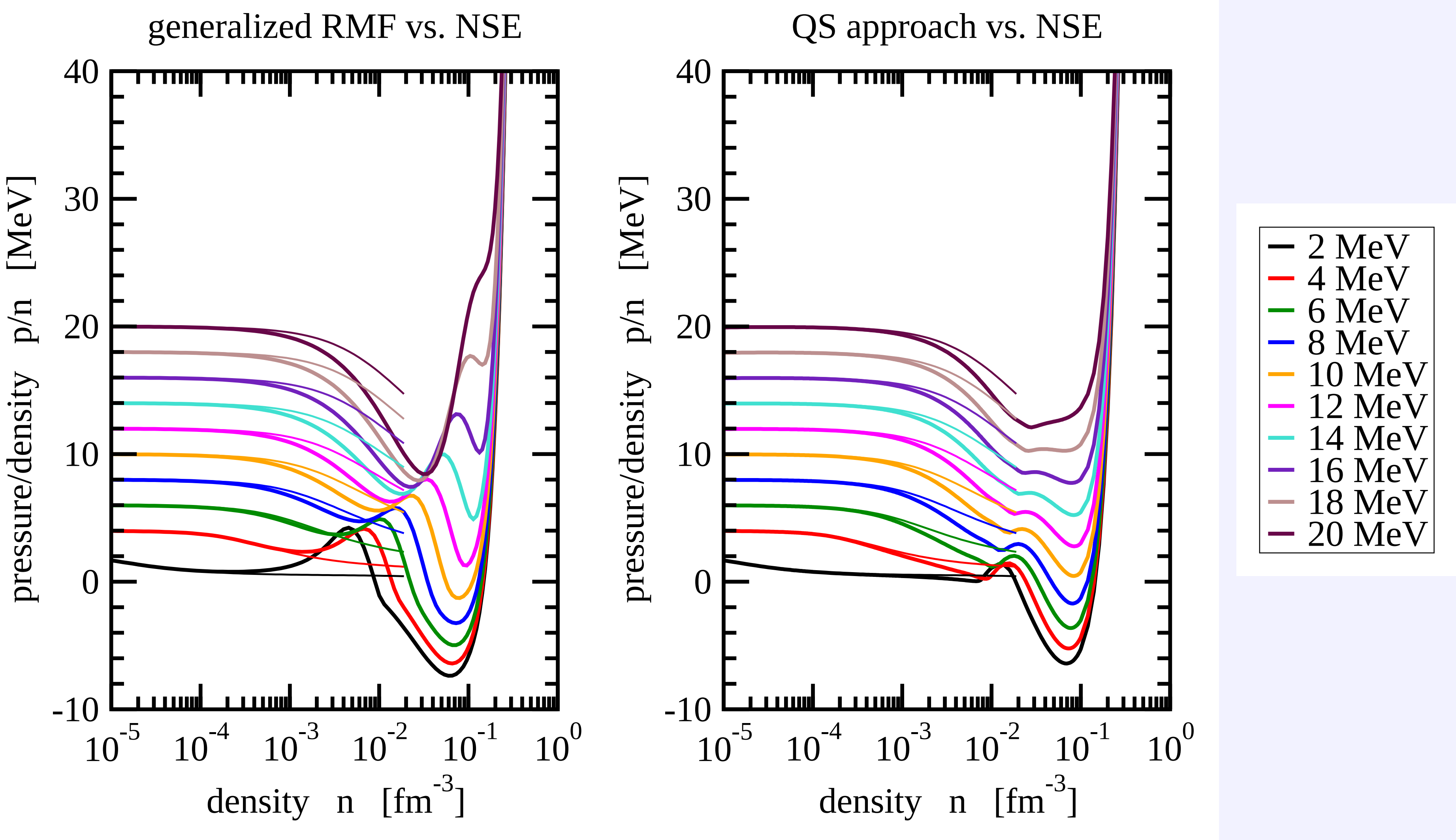




EoS with Light Clusters - Pressure/Density

symmetric nuclear matter

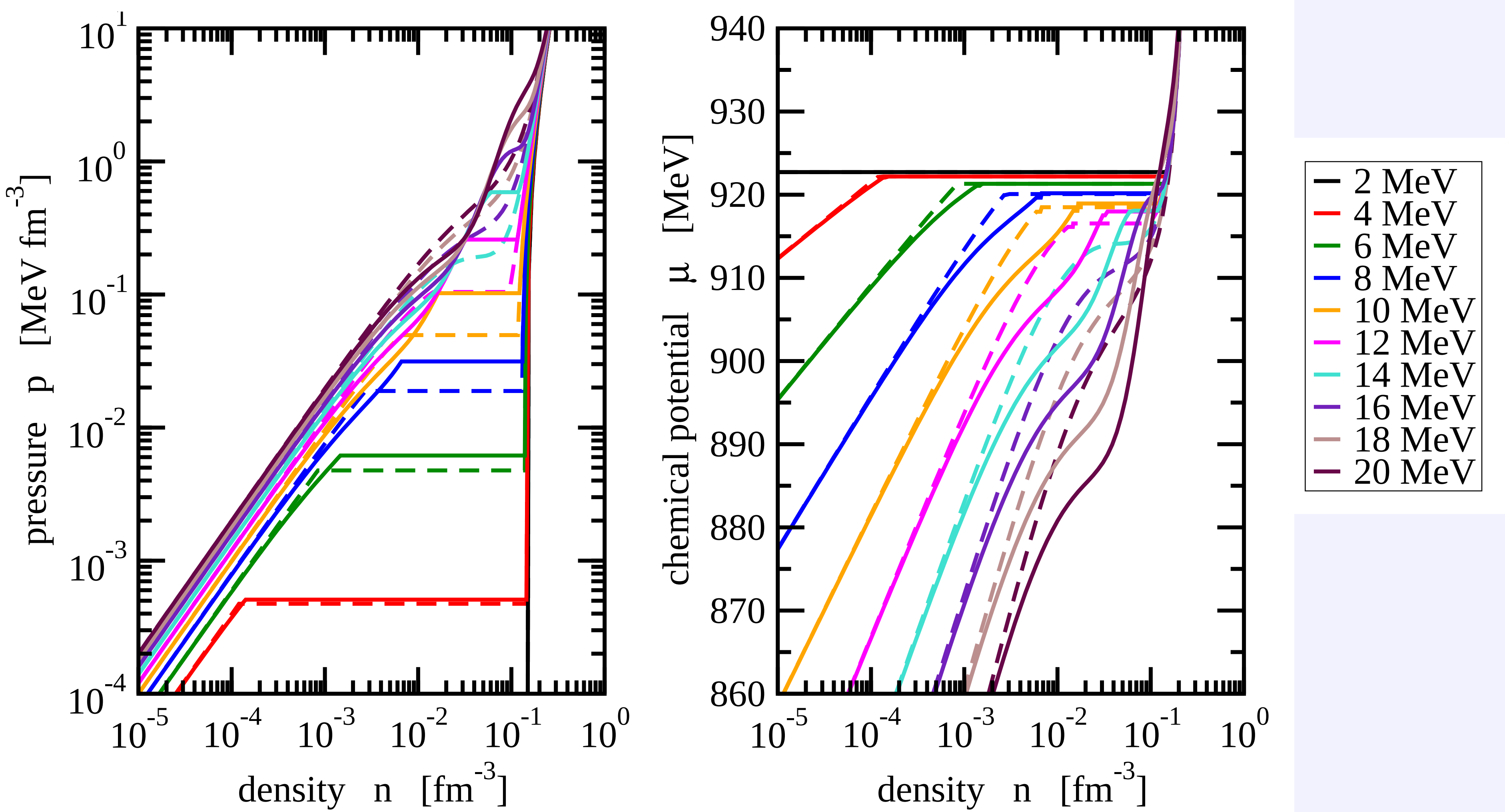
$\lim_{n \to 0}(p/n) = T$ (ideal gas)



Phase Transition - Pressure and Chemical Potential

symmetric nuclear matter (Maxwell construction sufficient)

RMF model without (dashed lines) and with (solid lines) clusters



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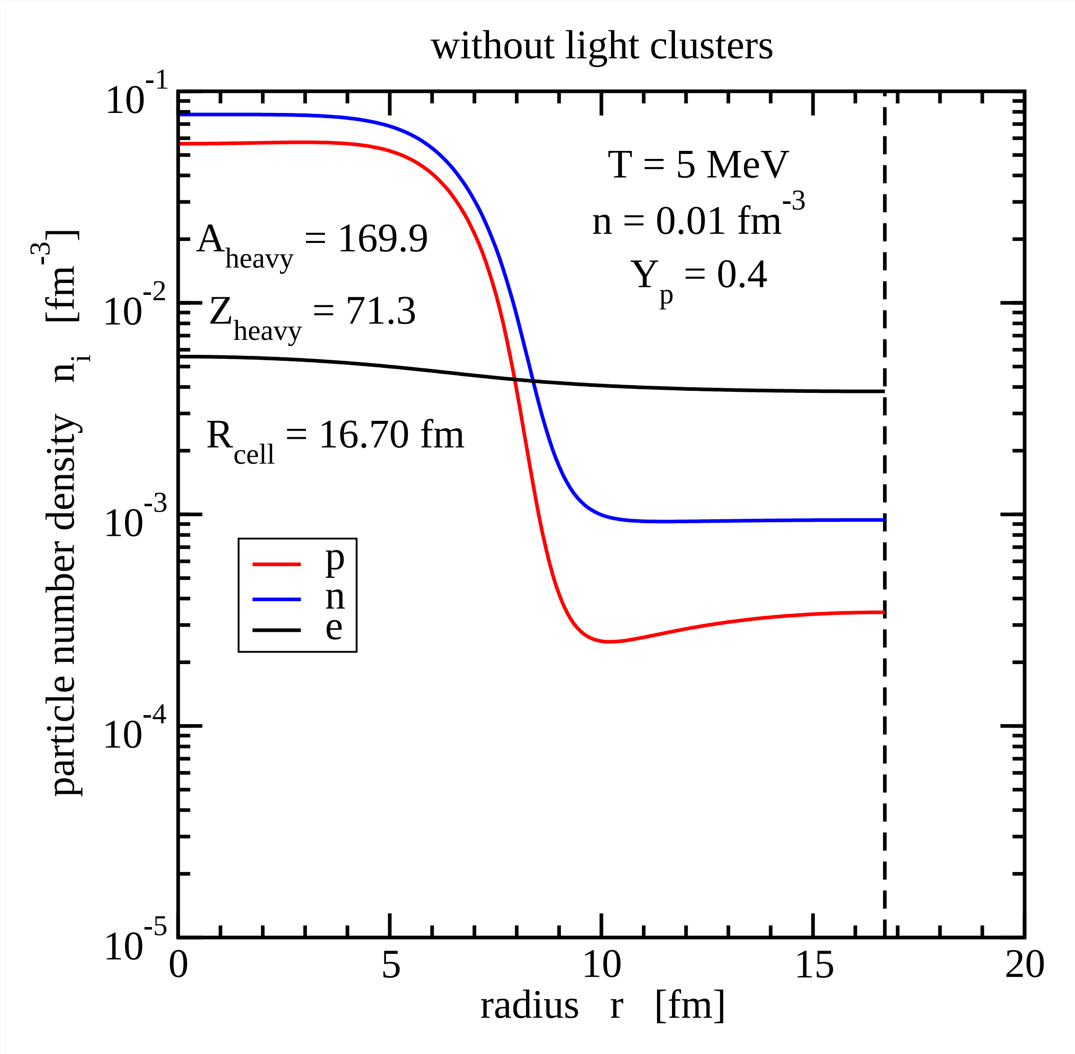
  - generalized RMF model
  - Thomas-Fermi approximation
  - electrons for charge compensation
  - heavy nucleus surrounded by gas of nucleons and light clusters



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  spherical Wigner-Seitz cell calculation
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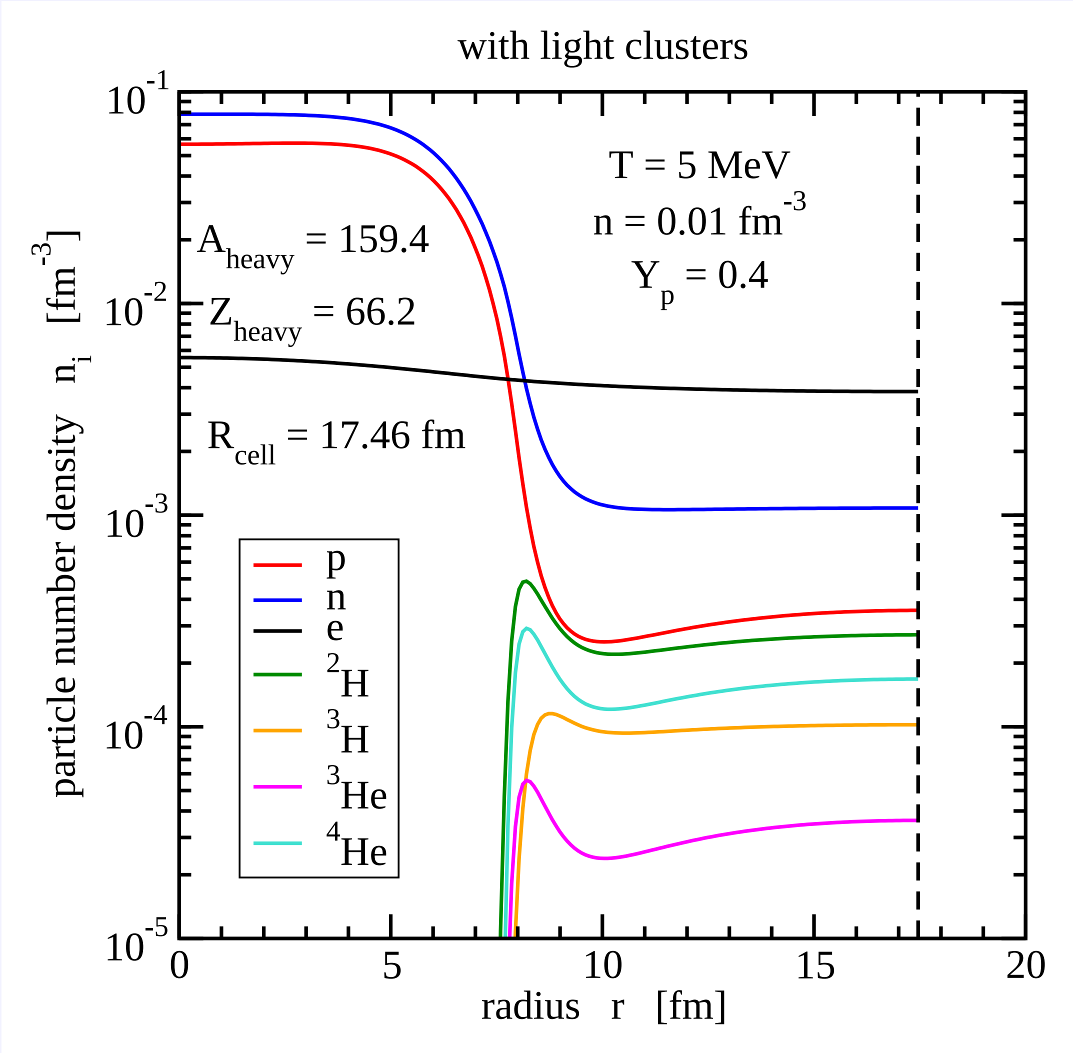
  - heavy nucleus surrounded by gas of nucleons and light clusters
- first self-consistent calculation with interacting nucleons, light clusters and electrons



Symmetry Energy I

- general definition for zero temperature:

$$E_s(n) = \frac{1}{2}\frac{\partial^2}{\partial\beta^2}\frac{E}{A}(n,\beta)\Big|_{\beta=0} \qquad \beta = \frac{n_n - n_p}{n_n + n_p}$$

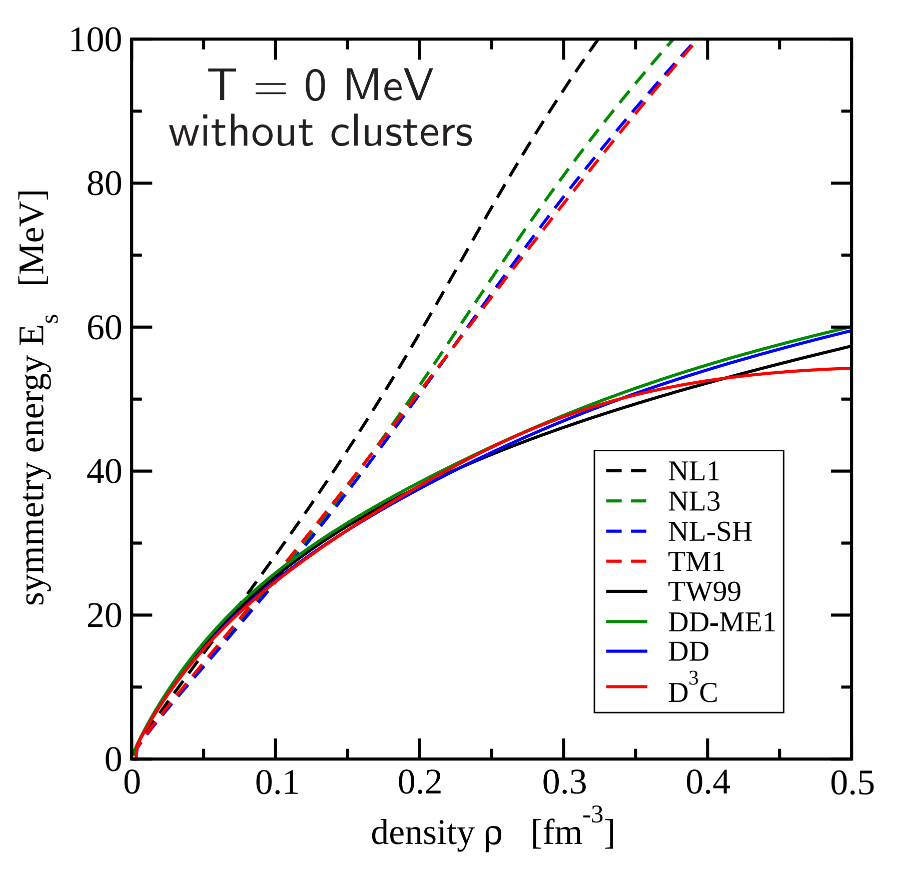
$\Rightarrow$ nuclear matter parameters

$$J = E_s(n_{\text{sat}}) \qquad L = 3n\frac{d}{dn}E_s\Big|_{n=n_{\text{sat}}}$$

- correlation: neutron skin thickness
  $\Leftrightarrow$ slope of neutron matter EoS ($\Leftrightarrow L$)

B. A. Brown, Phys. Rev. Lett. 85 (2000) 5296,

S. Typel, B. A. Brown, Phys. Rev. C 64 (2001) 027302



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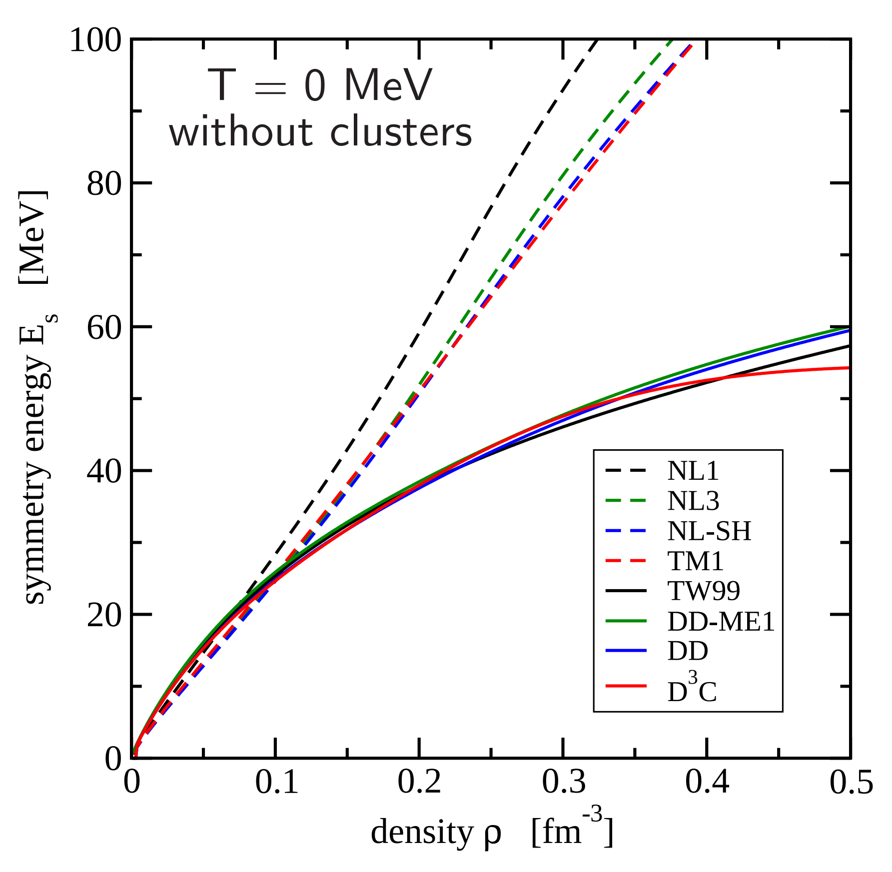


- with clusters and at finite temperatures:
  - use finite differences

$$E_{\text{sym}}(n) = \frac{1}{2}\left[\frac{E}{A}(n, 1) - 2\frac{E}{A}(n, 0) + \frac{E}{A}(n, -1)\right]$$

effects of cluster formation? experimental observation?

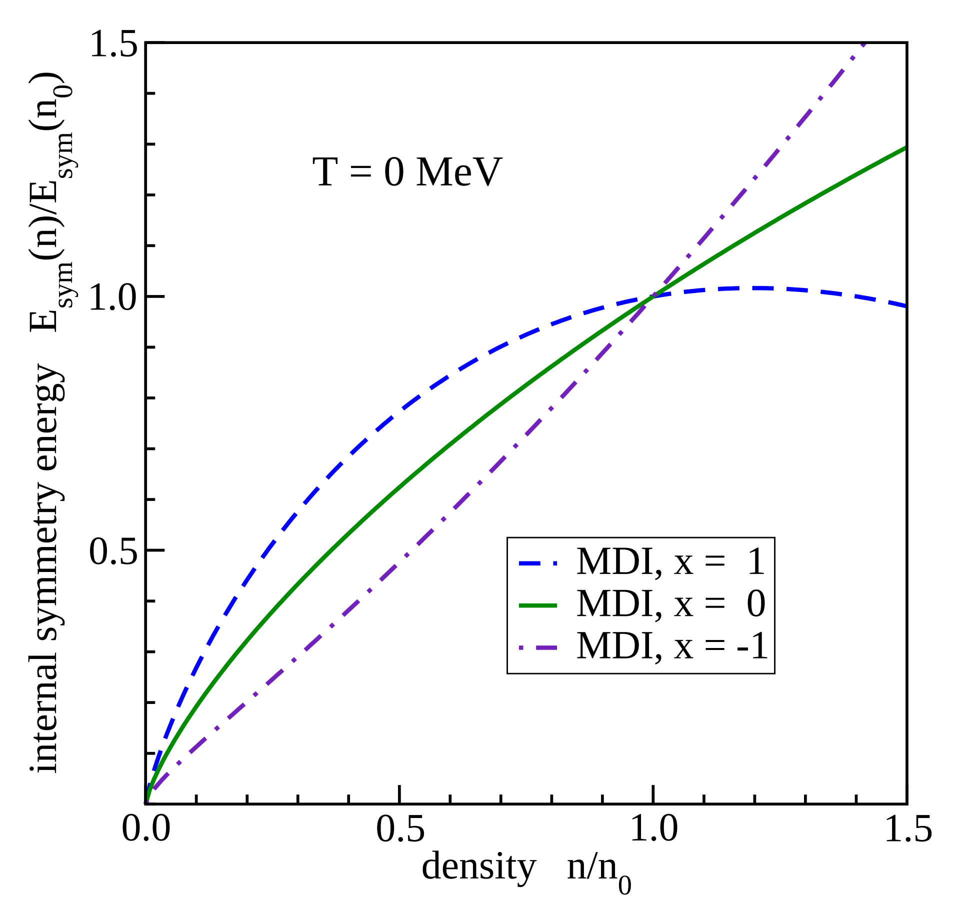
Symmetry Energy II

temperature $T = 0$ MeV

- mean-field models without clusters

  e.g. model with momentum-dependent interaction (MDI), parameter x controls density dependence of E_{sym}
  (B. A. Li et al., Phys. Rep. 464 (2008) 113)

  $\Rightarrow$ low-density behaviour not correct



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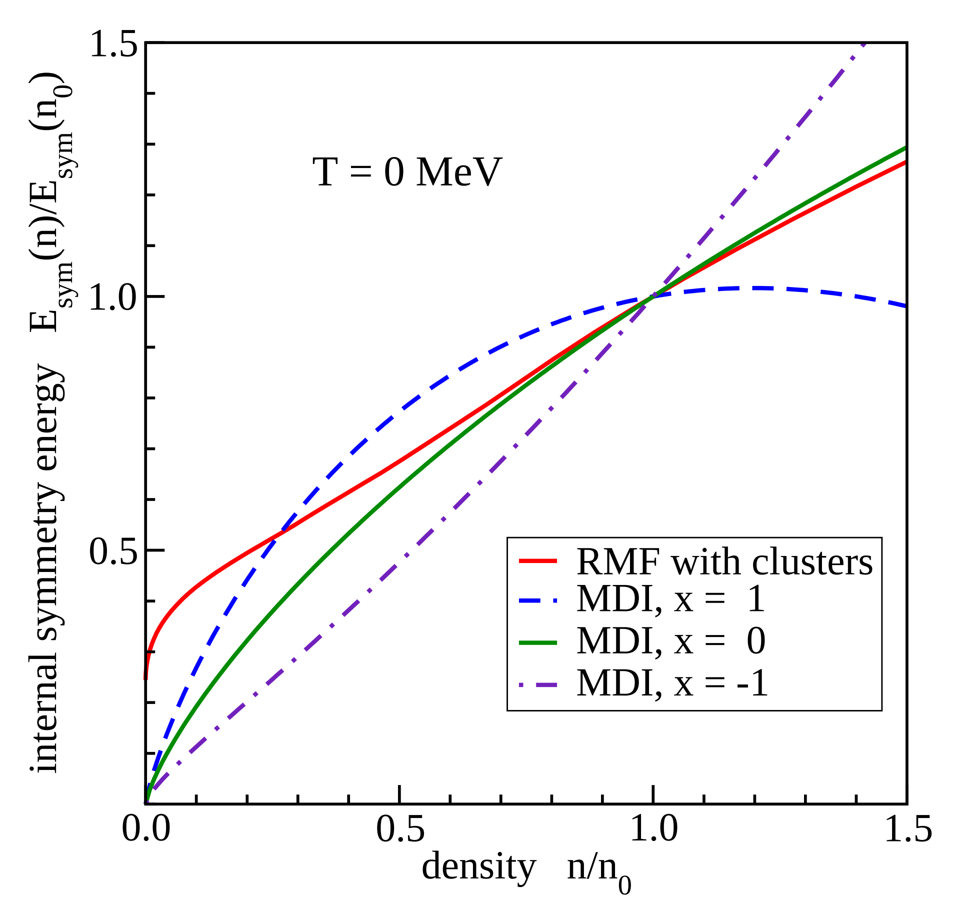
- RMF model with (heavy) clusters

  $\Rightarrow$ increase of $E_{\rm sym}$ at low densities due to formation of clusters

  $\Rightarrow$ finite symmetry energy in the limit $n \to 0$



Symmetry Energy III

finite temperature

- experimental determination of symmetry energy
  - heavy-ion collisions of ^{64}Zn on ^{92}Mo and ^{197}Au at $35\ A$ MeV
    temperature, density, free symmetry energy derived as functions of parameter $v_{\rm surf}$ (measures time when particles leave the source)
    (S. Kowalski et al., Phys. Rev. C 75 (2007) 014601)

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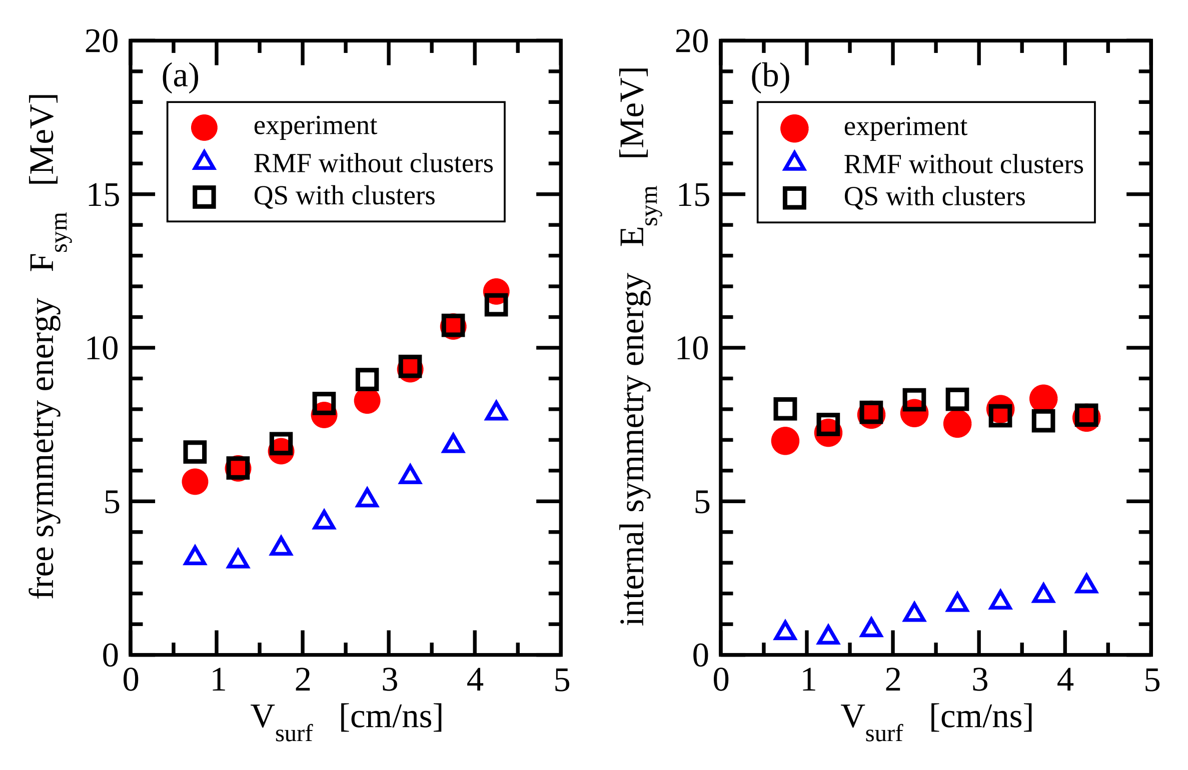
- symmetry energies in RMF calculation without clusters are too small

- very good agreement with QS calculation with light clusters



Summary and Outlook

- **theoretical models of EoS with clusters**
  - quantum statistical approach (QS)
  - generalized relativistic mean-field model (gRMF)
  - both thermodynamically consistent
  - correct limits at low and high densities
  - difference in details

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- **nuclear matter at low densities**
  - formation of clusters with medium dependent properties
  - modification of thermodynamical properties/symmetry energies
  - change of phase transition boundaries

for details see Phys. Rev. C **81**, 015803 (2010) and Phys. Rev. Lett. **104**, 202501 (2010)

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- **future**
  - further improvement of RMF parametrization (low-density limit)
  - application to astrophysical models

  ⇒ CompStar (compstar-esf.org) initiative:
  repository of modern EoS for astrophysical applications