The Duflo-Zuker approach to nuclear masses

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NIC Heidelberg 23 July 2010

Preliminaries

"One mass formula stands above all others..."
"However, this does not mean that with Duflo-Zuker we have reached the end of history.." Quoted from
D. Lunney, J. M. Pearson and C. Thibault RMP **75** (2003) 1021; who refer to

J. Duflo and A.P. Zuker, Phys. Rev. C **52** (1995) R23; [DZ28 RMSD$\approx$ 350 keV]

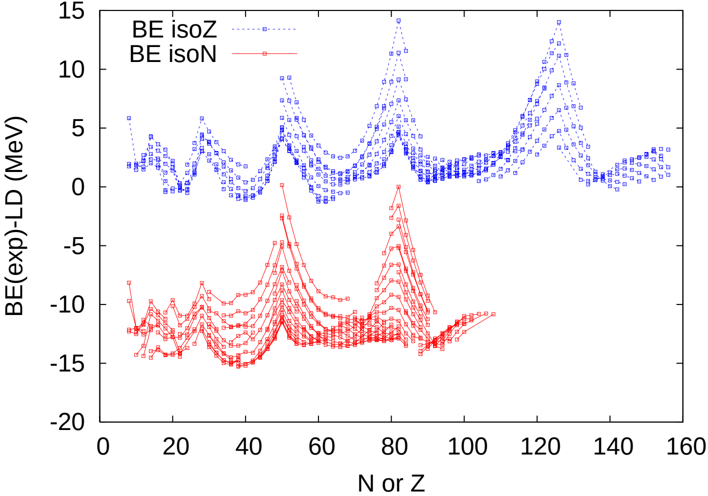
In the mean time little has changed, but now there is a second paper dealing with DZ10

(J. Duflo, 1996 unpublished, http://amdc.in2p3.fr/web/dz.html; RMSD$\approx$ 550 keV)

J. Mendoza-Temis, J. G. Hirsch and A. P. Zuker Nuc. Phys. A **843** (2010) 14-36.

DZ10 is an invaluable summary of the DZ approach. It does not point to the end of history, but to a (Three Body) follow up of the story.

What has to be explained. BE: Shell effects + LD



$$\mathrm{LD} = 15.5A - 17.8A^{2/3} - 28.6\frac{4T(T+1)}{A} + 40.2\frac{4T(T+1)}{A^{4/3}} - \frac{.7Z(Z-1)}{A^{1/3}}.$$

DZ10 Stucture and evolution

There are 10 terms:

1. Leading, contains basic shell effects. Goes asymptotically as A.
2. Surface, contains basic shell effects. Goes asymptotically as $A^{2/3}$.
3. Asymmetry, $T(T+1)/A$
4. Surface asymmetry,$T(T+1)/A^{4/3}$
5. Pairing, $mod(N,2)+mod(Z,2)$
6. Coulomb, $Z(Z-1)/A^{1/3}$
7. Cubic spherical “correlation”
8. Surface cubic spherical “correlation”
9. Quartic spherical correlation
10. Quartic deformation

Fits yield

- For the first six “macroscopic” terms, RMSD=2.88 MeV (LD RMSD=2.35 MeV) Looks bad, but
- For the first nine terms, RMSD=717 keV.
- For the ten terms, RMSD=567 keV

The master term

The DZ approach starts by identifying the collective term resposible for the bulk energy of nuclear matter and the basic shell effects, suggested by
M. Dufour and A.P. Zuker, Phys. Rev. C **54** (1996) 1641.

Calling m_p the number of particles in the major HO shell of principal quantum number p of degeneracy $D_p = (p+1)(p+2)$,

$$M_A = \frac{\hbar\omega}{\hbar\omega_0}\left(\sum_p \frac{m_p}{\sqrt{D_p}}\right)^2 \longrightarrow \frac{A^{1/3}}{\langle r^2\rangle}\left(\sum_p \frac{m_p}{\sqrt{D_p}}\right)^2 =$$

$$\frac{1}{\rho}\left(\sum_p \frac{m_p}{\sqrt{D_p}}\right)^2 \asymp \frac{1}{A^{1/3}}(p_f+2)^4 \approx (3/2)^{4/3}A \qquad (1)$$

Possible variant

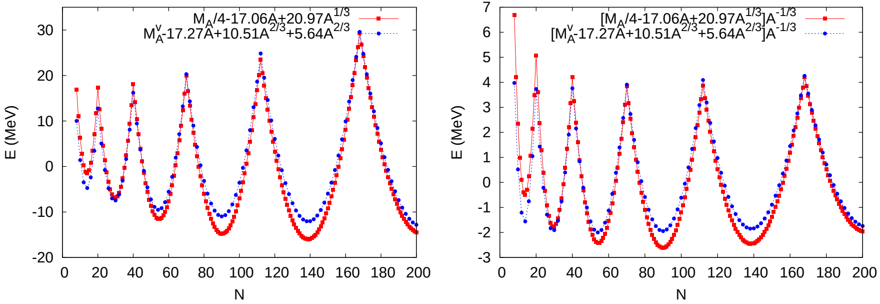
$$\frac{m_p}{\sqrt{D_p}} \longrightarrow \frac{m_p}{\sqrt{D_p}}(1+\frac{\alpha}{\sqrt{D_p}}) \equiv \frac{m_p}{\sqrt{D_p}}u_p \qquad (2)$$

Master shell effects. $A^{1/3}$ scaling

The two variants lead to different asymptotics

$M_A \asymp 17.05A - 20.87A^{1/3}$

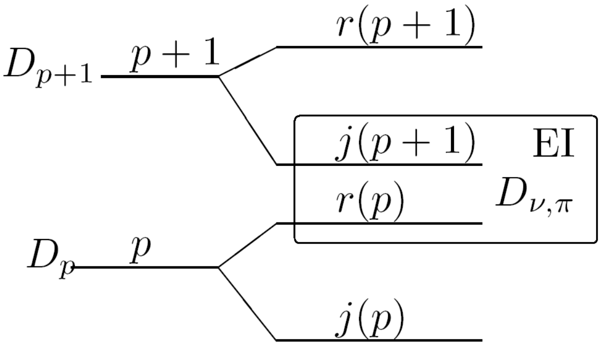
$M_A^v \asymp 17.27A - 10.57A^{2/3} - 5.13A^{1/3}$



Master shell effects produced by M_A and M_A^v for $t = N - Z = 0$. They are parabolic segments bounded by HO closures at $N = 8$, 20, 40, 70, 112 and 168, that scale asymptotically as $A^{1/3}$

The HO-EI transition

Transform HO closures into extruder-intruder (EI) ones at N,Z=28,50,82 and 126



Restrict subshell structure to j_p and r_p. Relevant operators must be linear, quadratic and cubic forms involving

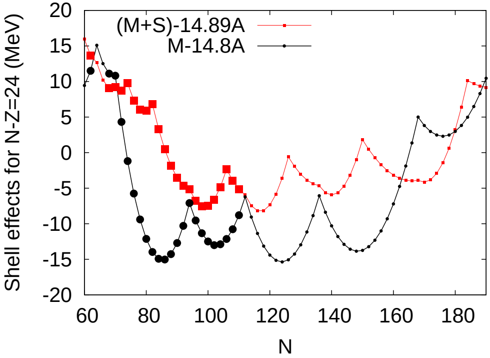
$$m_p = m_{j(p)} + m_{r(p)} \quad \text{and} \quad s_p = \left[(p m_{j_p} - 2 m_{r_p})/(2(p+2))\right]$$

The leading DZ10 term

By now it is clear that HO-EI transition must involve three body (3b) (Huge open problem) [A. P. Zuker PRL **90** 042502; A. Schwenk and A. P. Zuker, Phys. Rev. C **74** 061302(R) (2006).]

DZ28 used some 12 2b terms. DZ10 uses a single leading one

$$M + S = M + \sum_p [u^{(1)} s_p + u^{(2)} m_p s_p / \sqrt{D_p}]$$



The HO-EI transition for $N - Z = 24$ even-even nuclei

DZ10 correlations SM in EI spaces.

Use $\bar{m} = D - m, \quad m^{(2)} = m(m-1)$

$$|\bar{0}\rangle = (1 + \sum_k \hat{A}_k)|0\rangle \Longrightarrow E = \langle 0|H_m|0\rangle + \langle 0|H_M \hat{A}_2|0\rangle \Longrightarrow$$

$$\text{terms of type : macro;} \quad \frac{m_v \bar{m}_v}{D_v \rho}; \quad \frac{m_v \bar{m}_v (m_v - \bar{m}_v)}{D_v^2 \rho}; \quad \frac{m_v^{(2)} \bar{m}_v{}^{(2)}}{D_v^3 \rho}$$

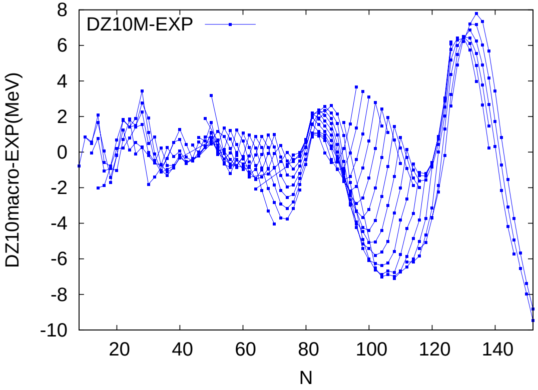
Problems

- $s_2 \equiv \frac{m_v \bar{m}_v}{D_v \rho}$ Not there. Why?
- $s_3 \equiv \frac{m_v \bar{m}_v (m_v - \bar{m}_v)}{D_v \rho}$ There, Why? Why wrong scaling?
- $s_4 \equiv \frac{m_v^{(2)} \bar{m}_v{}^{(2)}}{D_v^2 \rho}$ Expected. Why wrong scaling?
- $d_4 \equiv \frac{m_v^{(2)} \bar{m}_v{}^{(2)}}{D_v^2 \rho}$ Deformation with 4n4p jumps. Right scaling. OK.

No s_2; s_3 and s_4 scale wrong. Deformation d_4 scales right.

The DZ scaling problem

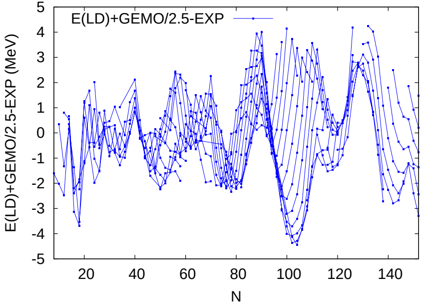
What should really be corrected scales atrociously.

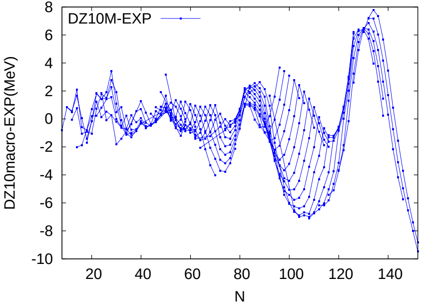


Explanation I: wrong HO-EI transition.
Explanation II: wrong asymptotics.
Leading $M+S$ has $A^{1/3}$. Surface M/ρ has no $A^{1/3}$. Therefore we are trying to correct huge $A^{1/3}$ with $A^{2/3}$.
Rather than correct directly study independently determined H_m

GEMO masses

How does DZ10 monopole relate to “true” H_m ?
J. Duflo and A. P. Zuker, Phys. Rev. C **59**, 2347R (1999).
GEMO is H_m fitted to $cs \pm 1$. Hence it describes strictly shell effects.
Calculate them.
Subject GEMO to a 2.5 contraction and add LD.

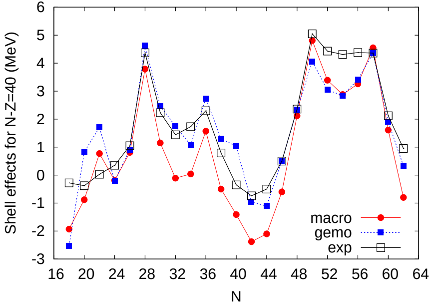


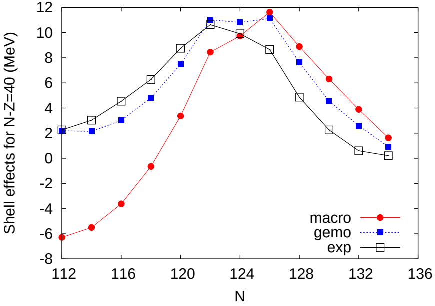


Beware of y-axis scales. Comment on RMSD
Comment on majestic cubics.

Predicted monopole shell effects for t=8,40

corrected asymptotics, 2.5 compression

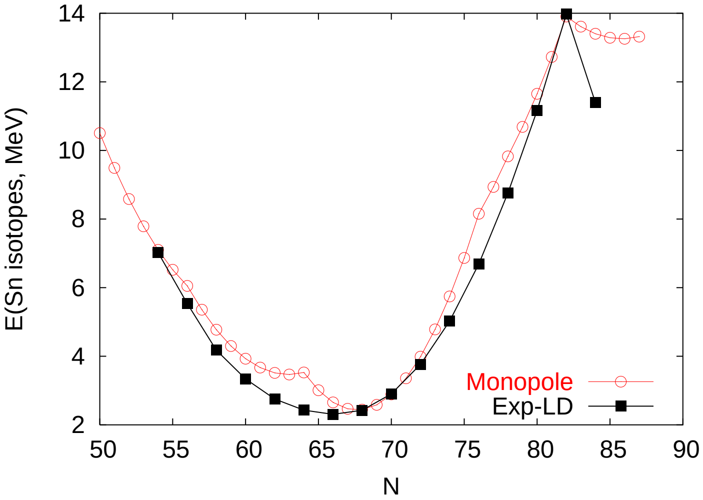




- If a closure exists, it is there, but;
- If it is there, it does not necessarily exist.

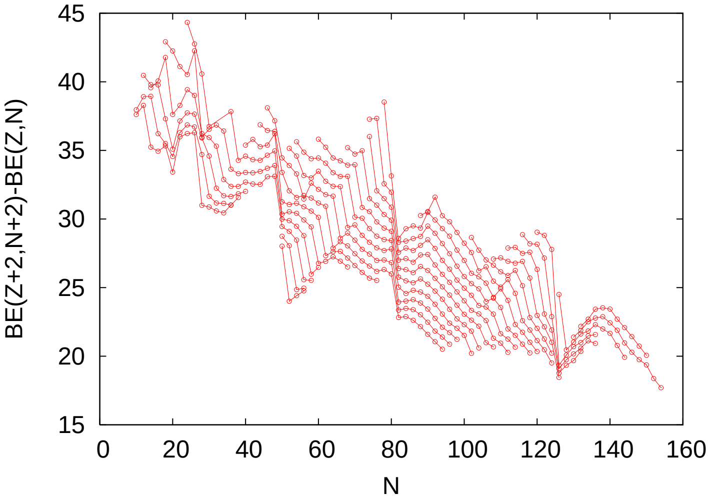
$N = 22$, $Z = 14$ erased closure. $N = 28$, $Z = 20$ and $N = 36$, $Z = 28$ good closures. $N = 40, Z = 32$ erased closure. $N = 50$, $Z = 42$ and $N = 58$, $Z = 50$ good closures.

More of the same for Sn

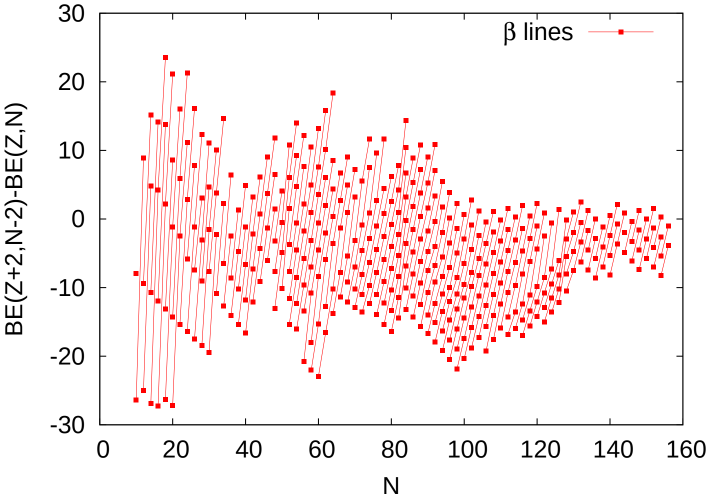


Comment on contraction factor

Alpha lines



Beta lines


β lines
BE(Z+2,N-2)-BE(Z,N)
N
30
20
10
0
-10
-20
-30
0
20
40
60
80
100
120
140
160