



Science & Technology
Facilities Council



Cosmic Ray Acceleration

Tony Bell

University of Oxford

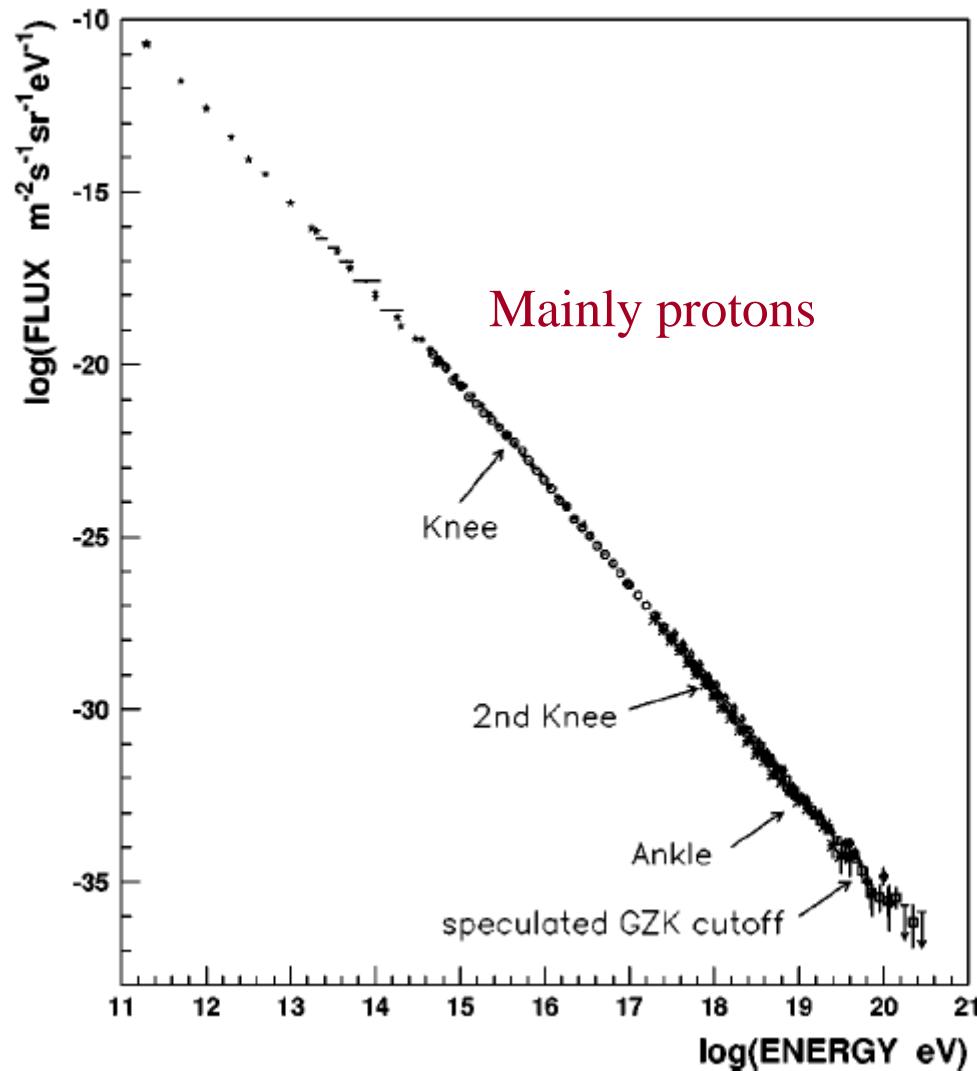
with Brian Reville & Klara Schure

SN1006: A supernova remnant 7,000 light years from Earth

X-ray (blue): NASA/CXC/Rutgers/G.Cassam-Chenai, J.Hughes et al; Radio (red): NRAO/AUI/GBT/VLA/Dyer, Maddalena & Cornwell; Optical (yellow/orange): Middlebury College/F.Winkler, NOAO/AURA/NSF/CTIO Schmidt & DSS

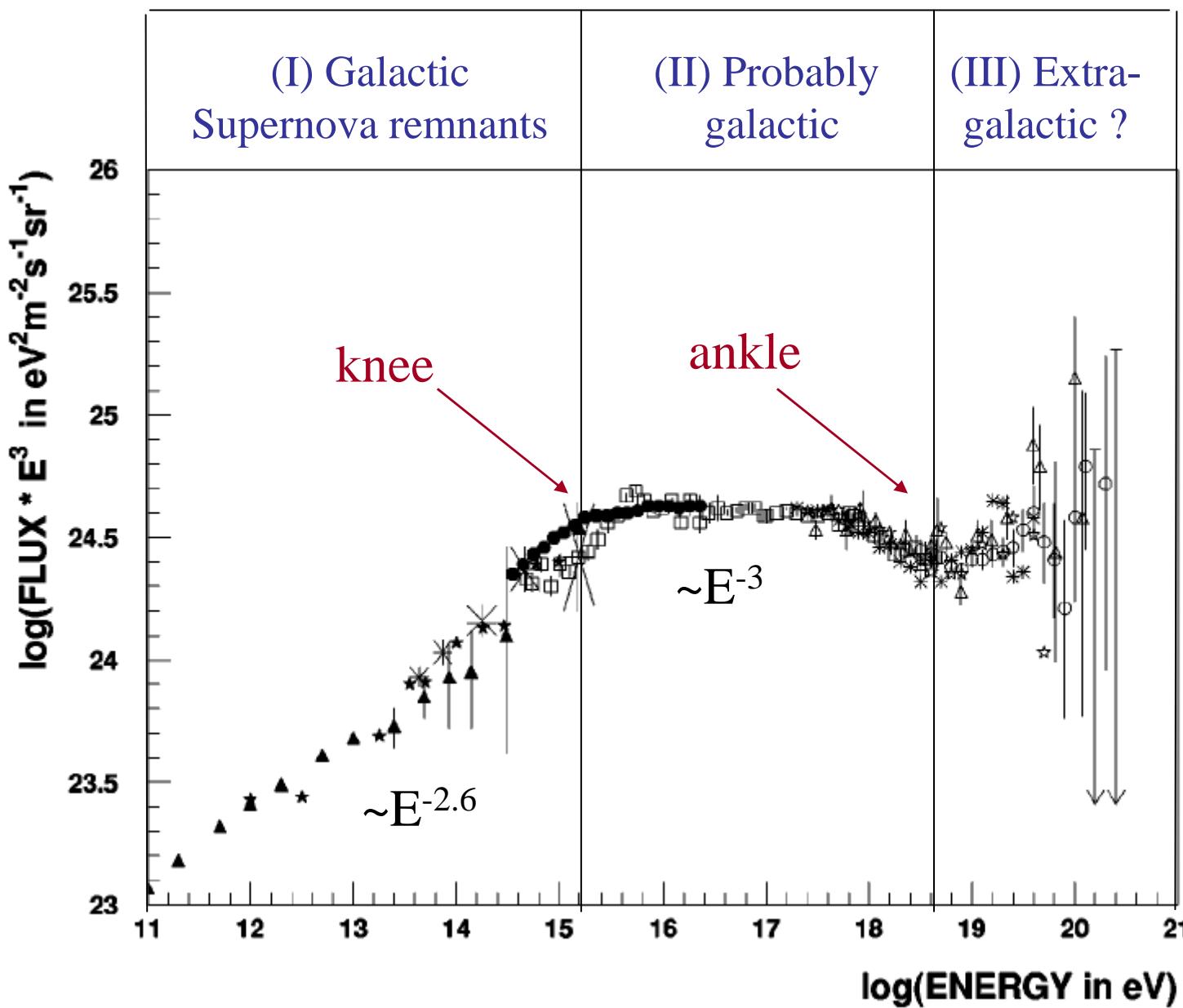
Cosmic ray spectrum arriving at earth

(Nagano & Watson 2000)

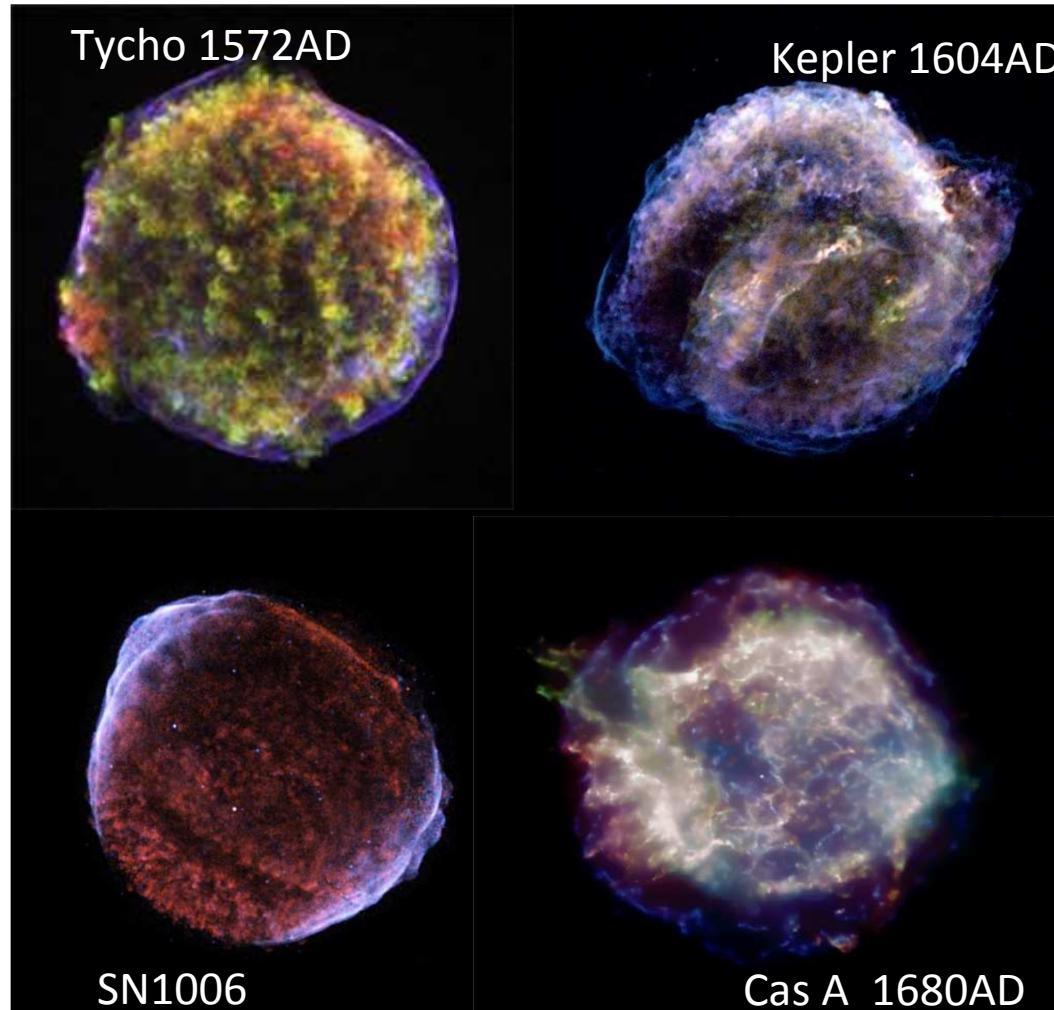


CR populations

(Nagano & Watson 2000)



Historical shell supernova remnants



Chandra observations

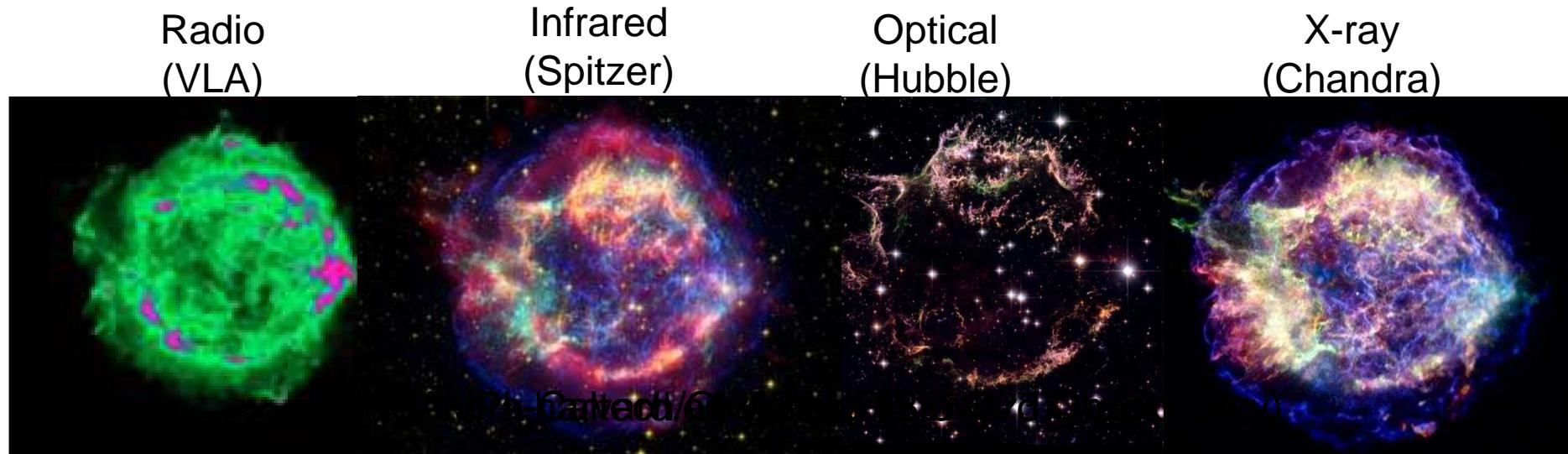
NASA/CXC/Rutgers/
J.Hughes et al.

NASA/CXC/Rutgers/
J.Warren & J.Hughes et al.

NASA/CXC/NCSU/
S.Reynolds et al.

NASA/CXC/MIT/UMass Amherst/
M.D.Stage et al.

Cassiopeia A



[chandra.harvard.edu/photo/
0237/0237_radio.jpg](http://chandra.harvard.edu/photo/0237/0237_radio.jpg)

NASA/JPL-Caltech/
O Krause (Steward Obs)

NASA/ESA/
Hubble Heritage
(STScI/AURA))

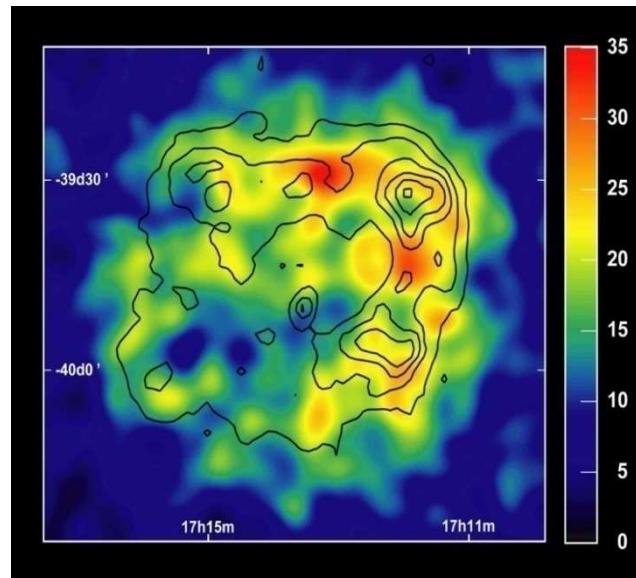
NASA/CXC/MIT/UMass Amherst/
M.D.Stage et al.

Mixture of line radiation
& synchrotron continuum

Synchrotron in magnetic field $\sim 0.1\text{-}1\text{ mG}$
Radio ($h\nu \sim 10^{-5}\text{ eV}$): electron energy $\sim 1\text{ GeV}$
X-ray ($h\nu \sim 10^3\text{ eV}$): electron energy $\sim 10\text{ TeV}$

HESS: γ -rays directly produced by TeV particles

SNR RX J1713.7-3946



Aharonian et al
Nature (2004)

Arrival directions of highest energy cosmic rays

Auger (2007)

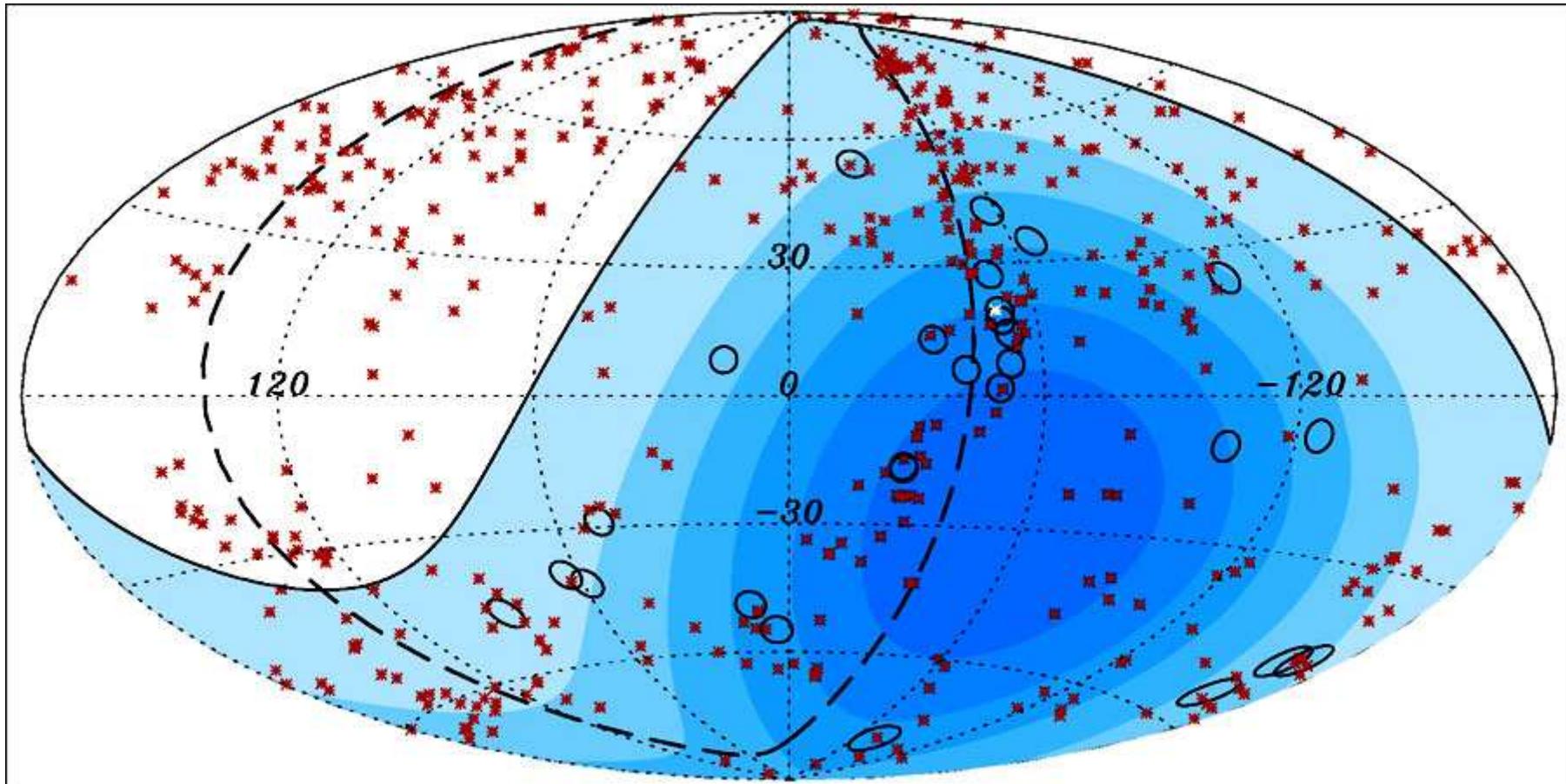
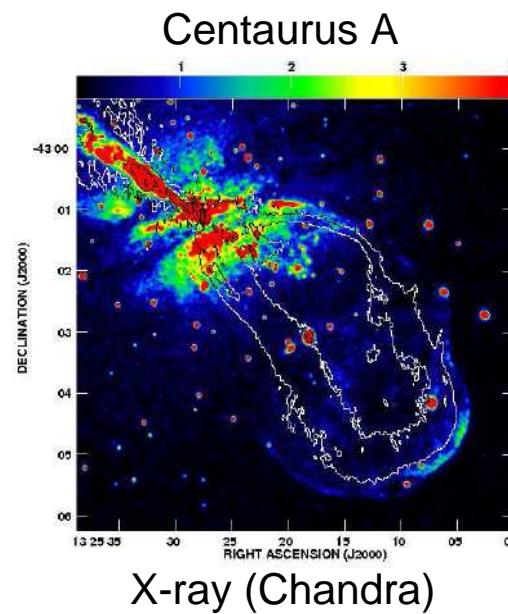
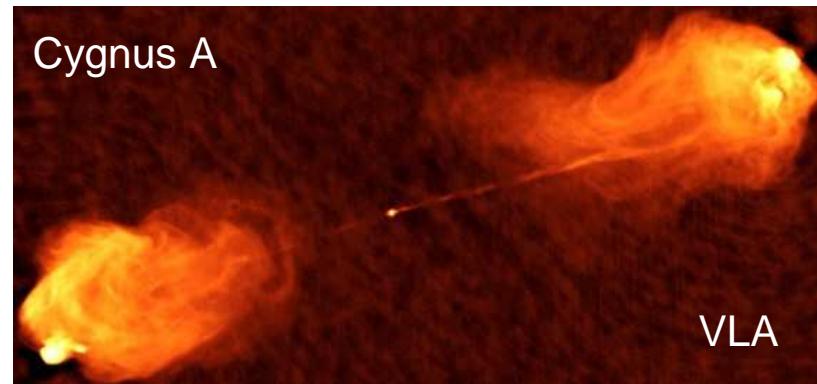


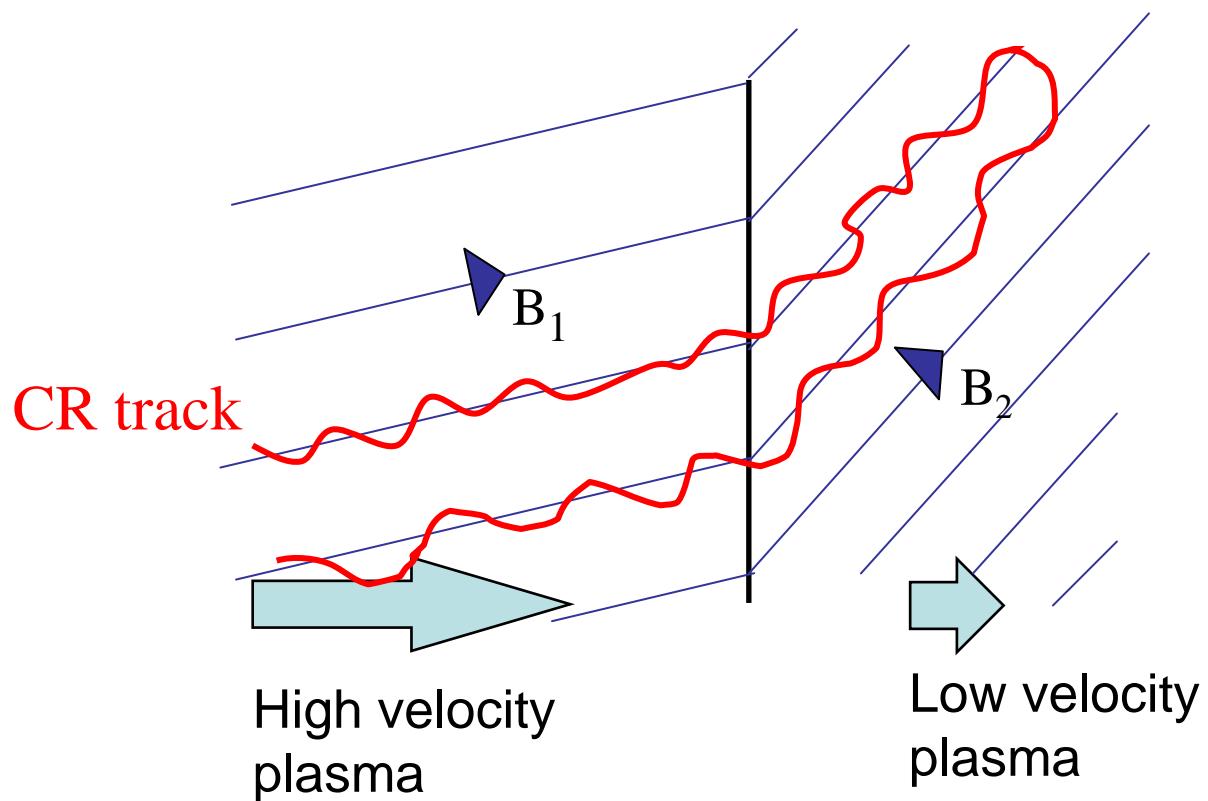
Fig. 2. Aitoff projection of the celestial sphere in galactic coordinates with circles of radius 3.1° centered at the arrival directions of the 27 cosmic rays with highest energy detected by the Pierre Auger Observatory. The positions of the 472 AGN (318 in the field of view of the Observatory) with redshift $z \leq 0.018$ ($D < 75$ Mpc) from the 12th edition of the catalog of quasars and active nuclei (12) are indicated by red asterisks. The solid line represents the border of the field of view (zenith angles smaller than 60°). Darker color indicates larger relative exposure. Each colored band has equal integrated exposure. The dashed line is the supergalactic plane. Centaurus A, one of our closest AGN, is marked in white.

Active galaxies



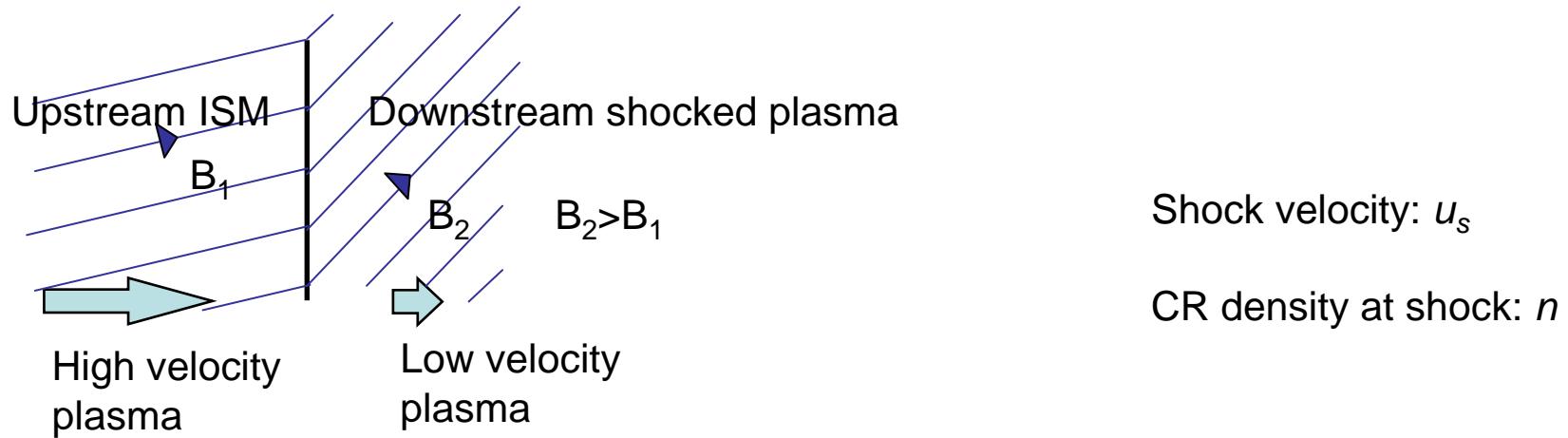
Cosmic ray acceleration by shocks

Cosmic ray acceleration



Due to scattering, CR recrosses shock many times
Gains energy at each crossing

Shock acceleration energy spectrum



Fractional CR loss per shock crossing

$$\frac{\Delta n}{n} = -\frac{u_s}{c}$$

Fractional energy gain per shock crossing

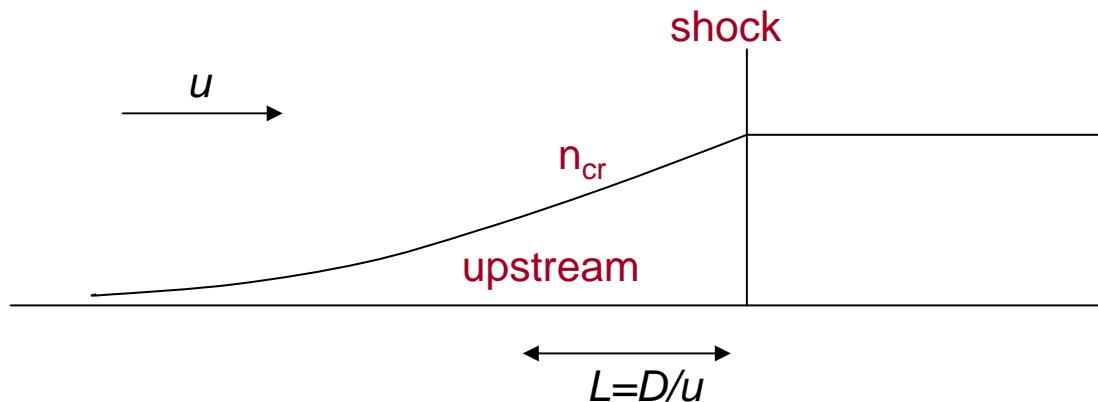
$$\frac{\Delta \epsilon}{\epsilon} = \frac{u_s}{c}$$

Differential energy spectrum

$$N(\epsilon)d\epsilon \propto \epsilon^{-2}d\epsilon$$

Maximum CR energy

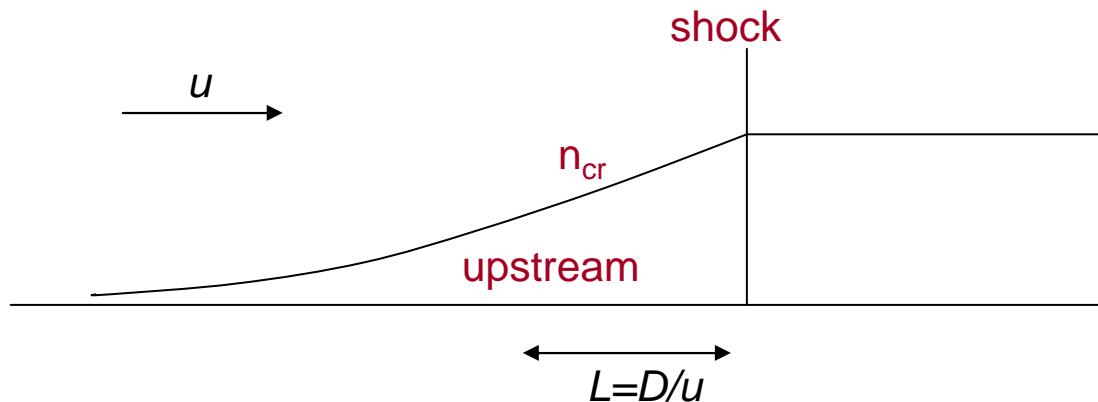
CR upstream of shock



Exponential density $n_{cr} = n_0 e^{ux/D}$

Scaleheight $L = \frac{D}{u}$

CR acceleration time



Average time per shock crossing: $\Delta t = \frac{4L}{u_{shock}}$

Energy gain per crossing: $\frac{\Delta \mathcal{E}}{\mathcal{E}} = \frac{u_{shock}}{c}$

$$\frac{\Delta \mathcal{E}}{\Delta t} = \frac{\mathcal{E}}{4} \frac{u_{shock}^2}{D_{upstream}}$$

Time needed for acceleration (Lagage & Cesarsky)

$$\tau = \frac{4D_{upstream}}{u_{shock}^2} + \frac{4D_{downstream}}{(u_{shock}/4)^2}$$

downstream time

Maximum CR energy

(Lagage & Cesarsky)

$$\text{Acceleration time: } \tau = \frac{8D}{u_{\text{shock}}^2}$$

$$\text{Smallest diffusion coefficient: } D = \frac{r_g c}{3}$$

$$\text{Maximum CR energy: } \frac{3}{8} e u_{\text{shock}} B R$$

Typically for young SNR

$$\text{Max CR energy} \quad \left(\frac{E}{eV} \right) = 3 \times 10^{13} \left(\frac{u_{\text{shock}}}{5000 \text{ km s}^{-1}} \right)^2 \left(\frac{B}{3 \mu\text{G}} \right) \left(\frac{t}{300 \text{ yr}} \right)$$

Appears that: Acceleration too slow to get to PeV

CR precursor too large

Magnetic field too small

Hillas diagram

(condition on uRB)

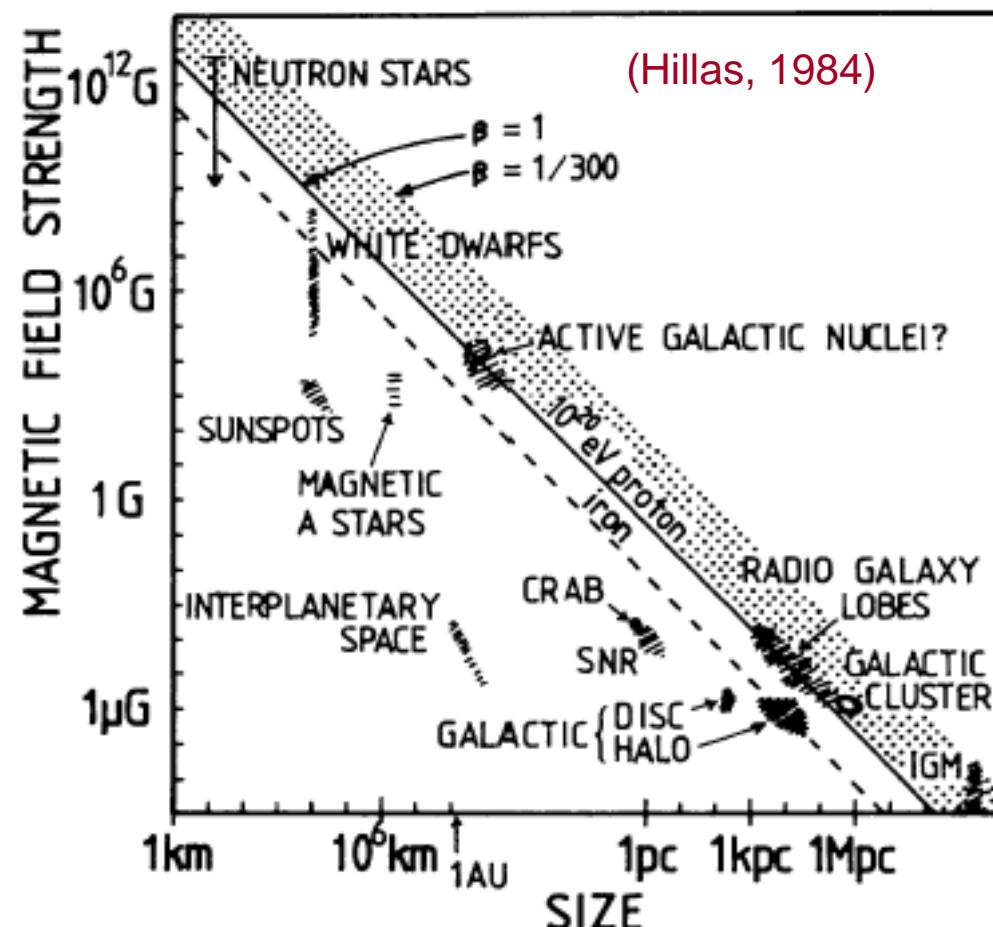
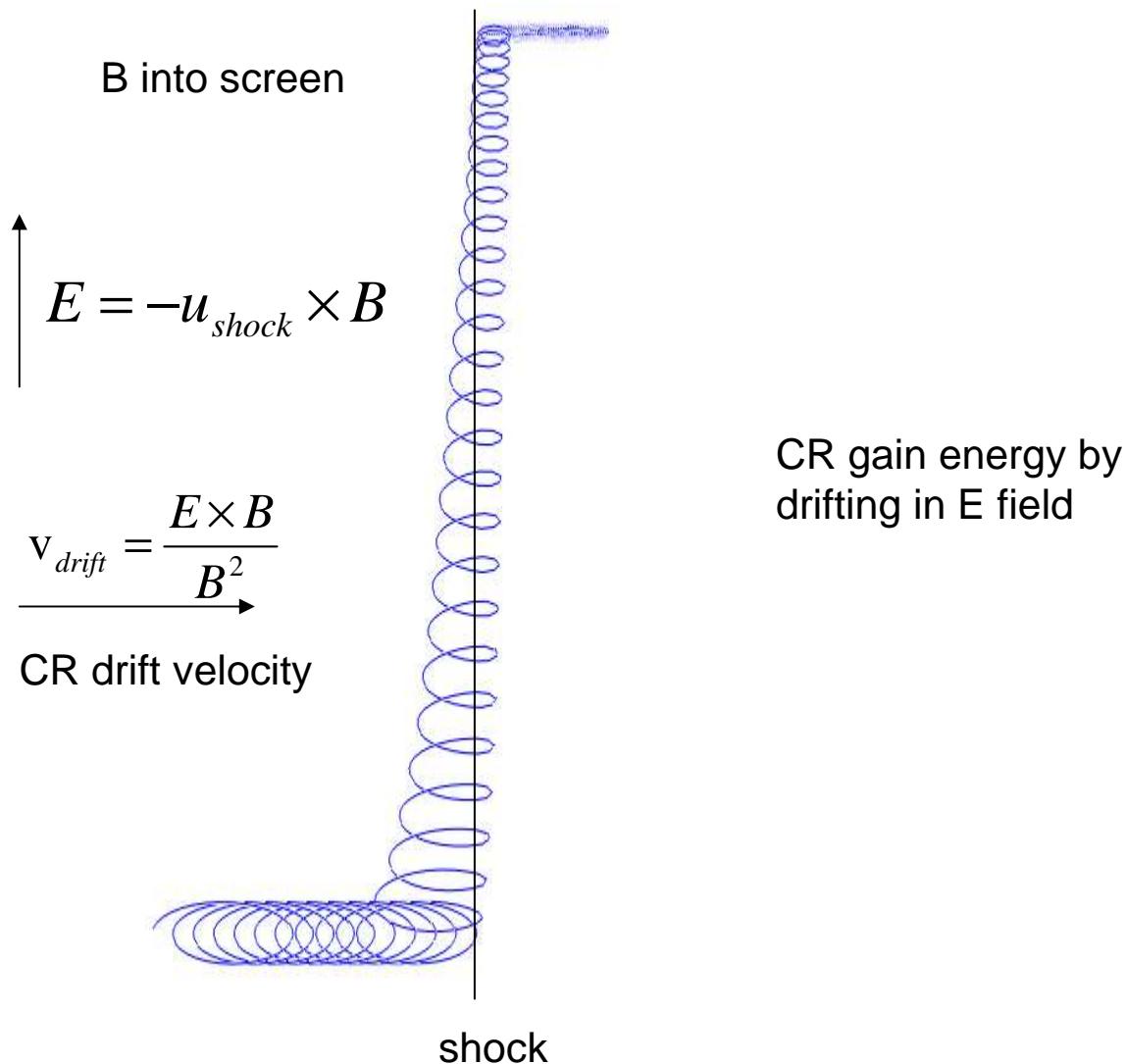


Figure 1 Size and magnetic field strength of possible sites of particle acceleration. Objects below the diagonal line cannot accelerate protons to 10^{20} eV.

Perpendicular shocks

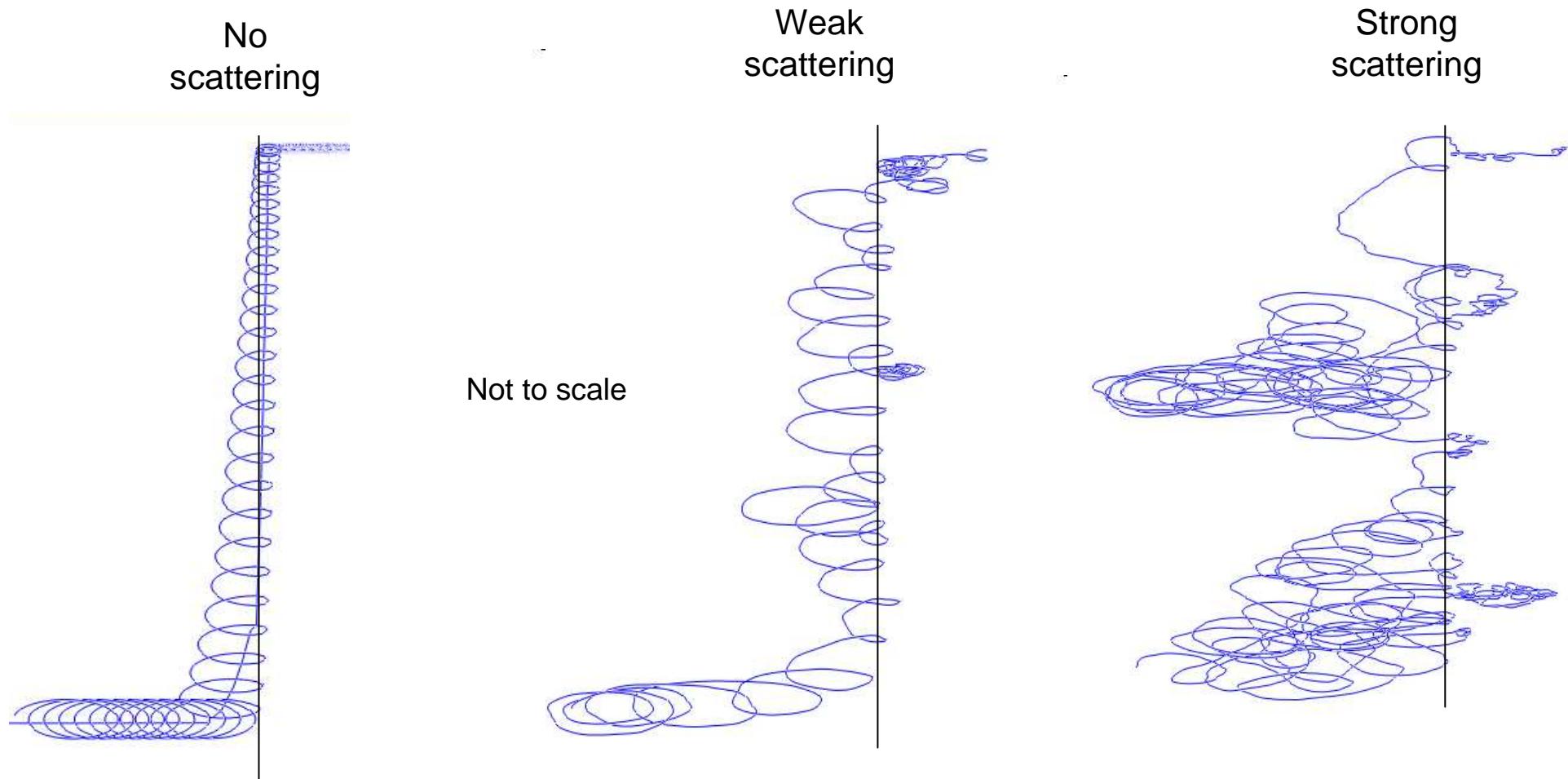
(Jokipii 1982, 1987)

CR trajectory at perpendicular shock (no scattering)

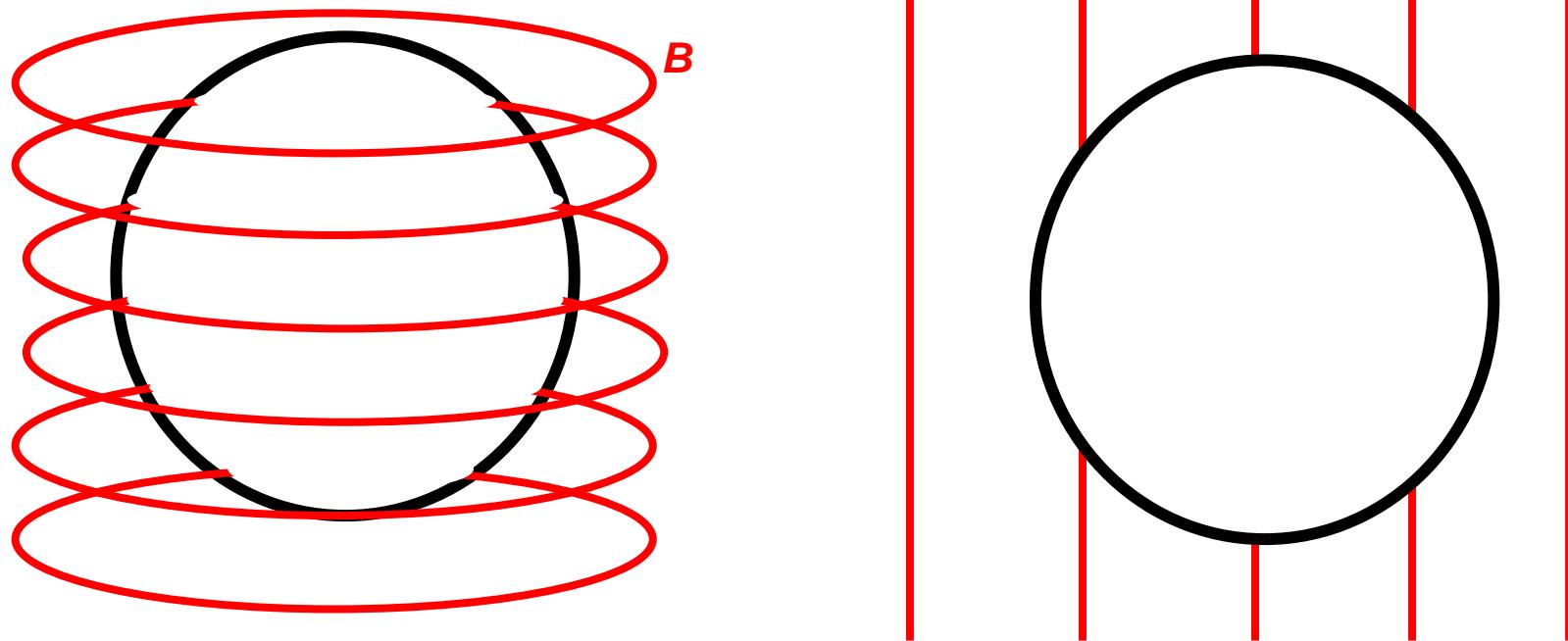


CR acceleration at perpendicular shock: with scattering

Diffusive shock theory applies
Provided gyrocentre diffuses over distances
greater than Larmor radius during shock transit
Same power law (see later)



CR acceleration at perpendicular shock

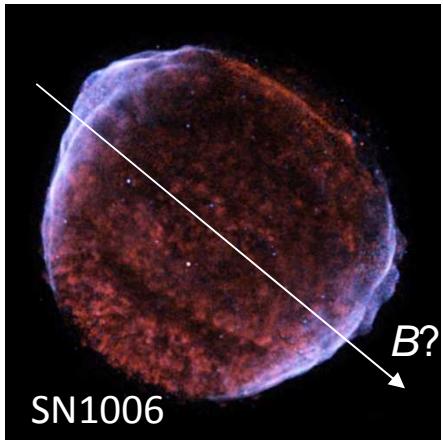


$$\mathbf{E} = -\mathbf{u}_{shock} \times \mathbf{B}$$

Transit between pole & equator: energy gain $\sim eER = eu_{shock}BR$

Hillas parameter as with parallel shock: similar max CR energy

The case of SN1006



Polar x-ray synchrotron emission?

(Rothenflug et al 2004)

At perpendicular shocks

- Acceleration is faster – potentially higher CR energy
- CR energy limited to $euBR$ (Hillas) by space rather than time
- CR scattering frequency has to be in right range

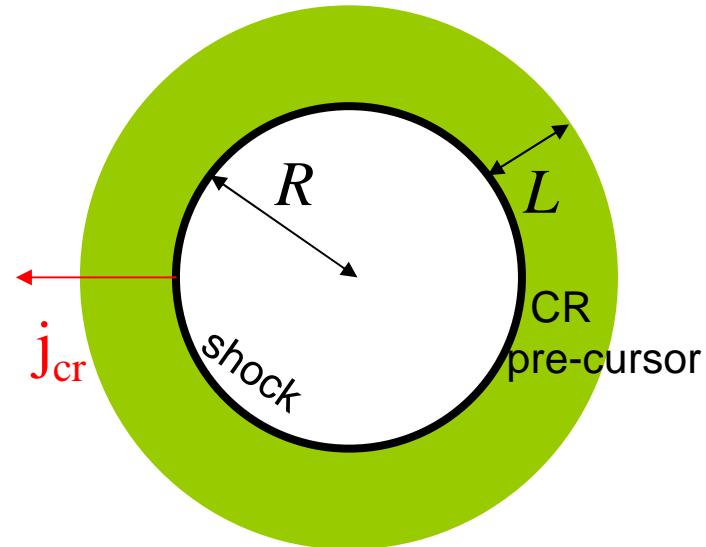
Room for discussion!

Magnetic field

- 1) Need larger field
- 2) Structured on scale of CR Larmor radius

$$\left(\frac{E}{eV}\right) = 3 \times 10^{13} \left(\frac{u_{shock}}{5000 \text{ km s}^{-1}}\right)^2 \left(\frac{B}{3 \mu\text{G}}\right) \left(\frac{t}{300 \text{ yr}}\right)$$

Electric currents carried by CR and thermal plasma



Density of 10^{15} eV CR: $\sim 10^{-12} \text{ cm}^{-3}$
Current density: $j_{\text{cr}} \sim 10^{-18} \text{ Amp m}^{-2}$

CR current must be balanced by current carried by thermal plasma

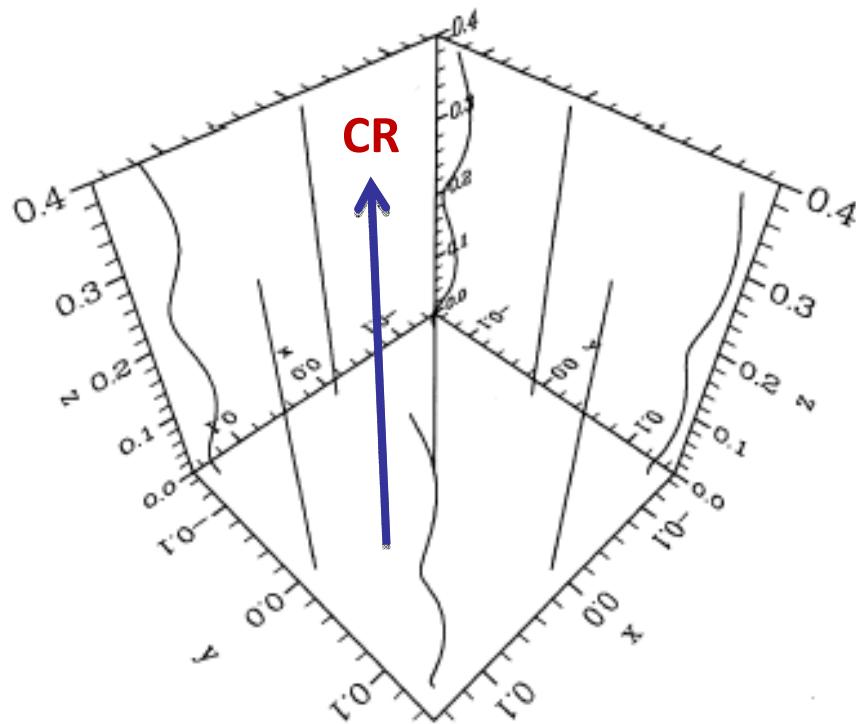
$$\dot{j}_{\text{thermal}} = -\dot{j}_{\text{cr}}$$

$j_{\text{thermal}} \times B$ force acts on plasma to balance $j_{\text{cr}} \times B$ force on CR

Streaming instabilities amplify magnetic field

Lucek & Bell (2000)

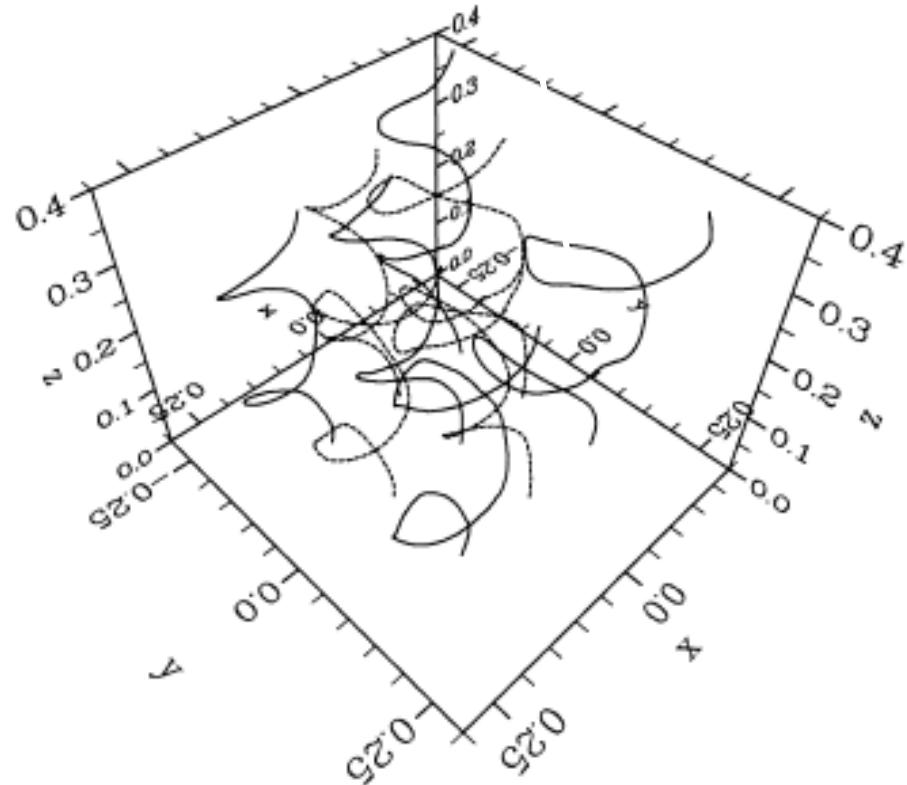
B field lines, $t = 0$



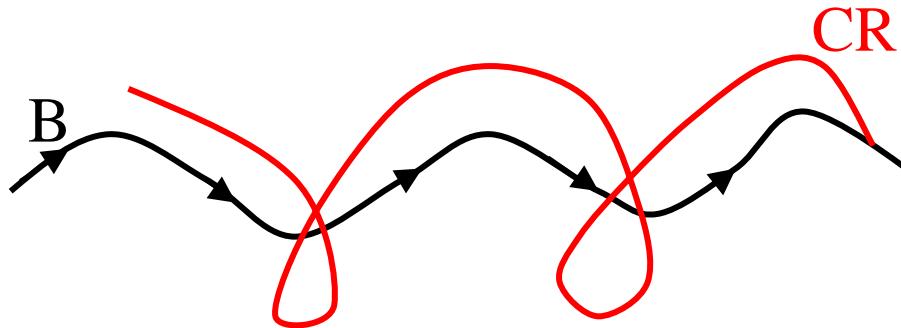
CR treated as particles

Thermal plasma as MHD

B field lines, $t = 2$



Stretching field lines



1) CR currents set plasma in motion

$$\rho \frac{d\mathbf{u}}{dt} = -\mathbf{j}_{CR} \times \mathbf{B}$$

2) Motion stretches field lines

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})$$

Need CR current to drive turbulence on scale of CR Larmor radius r_g in time t

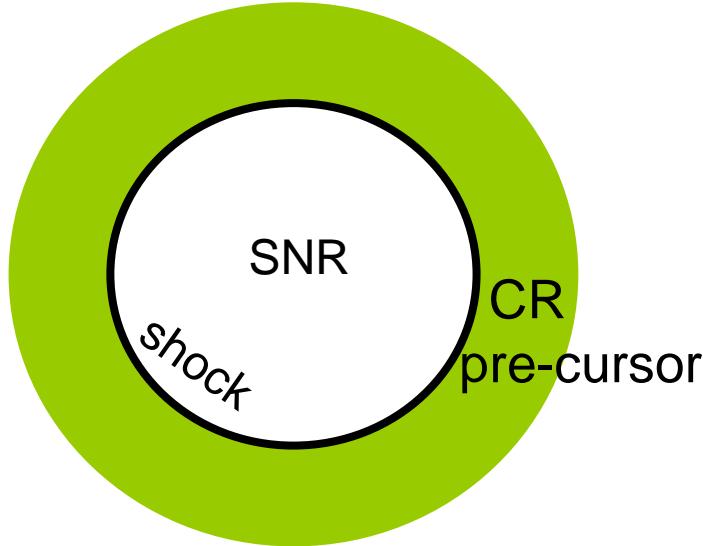
$$\frac{1}{2} \frac{j_{CR} B t^2}{\rho} > \frac{p_{CR}}{eB}$$

CR current carries energy flux \mathcal{Q}_{CR}

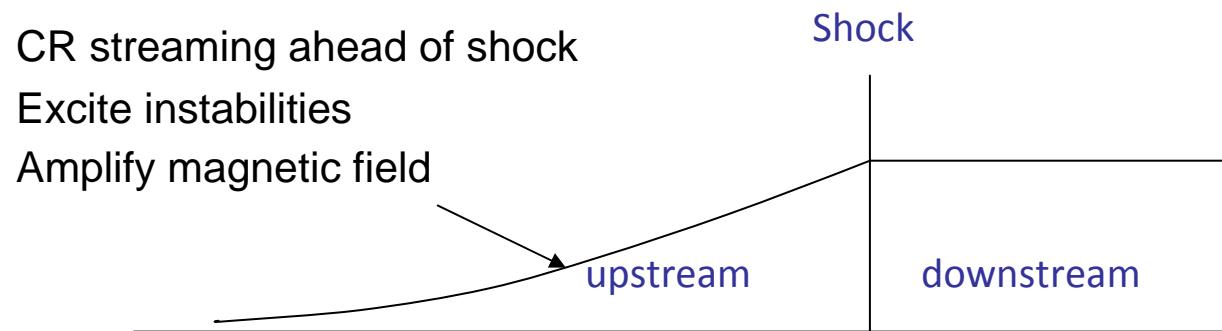
$$\frac{\mathcal{Q}_{CR}}{\rho u_{shock}^3} > 6 \left(\frac{E_{CR}}{\text{PeV}} \right)^2 \left(\frac{B}{3 \mu\text{G}} \right)^{-2} \left(\frac{u_{shock}}{5000 \text{km s}^{-1}} \right)^{-3} \left(\frac{t}{300 \text{yr}} \right)^{-2}$$

CR-driven instability amplifies magnetic field

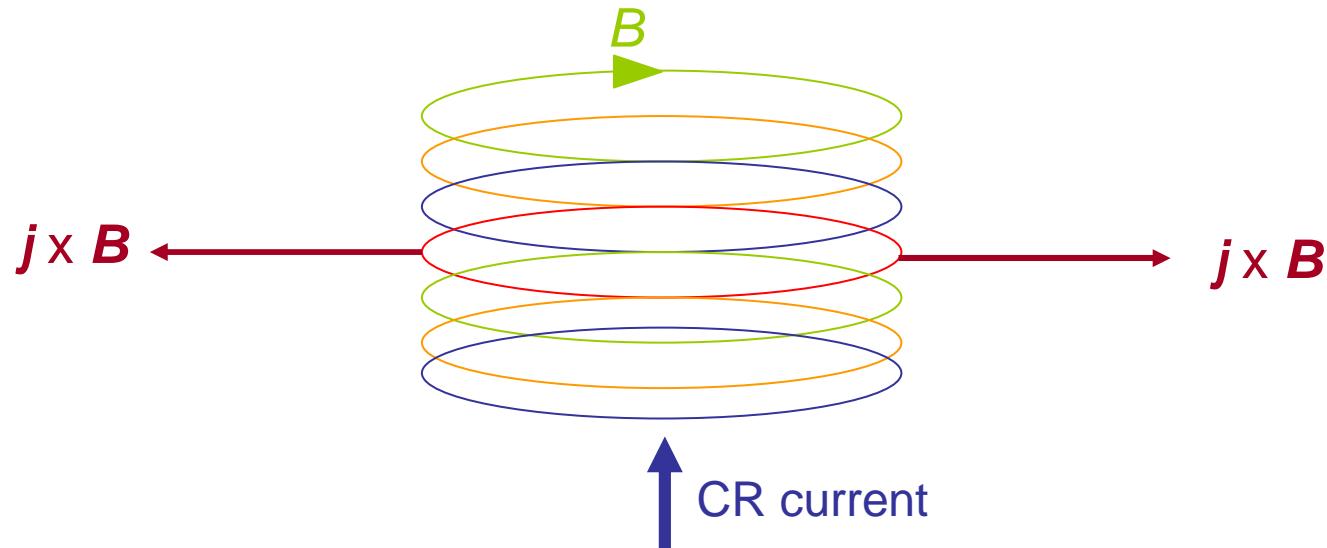
$$\rho \frac{d\mathbf{u}}{dt} = -\mathbf{j}_{CR} \times \mathbf{B}$$
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})$$



Streaming CR excite instabilities



Simplest form: expanding loops of B



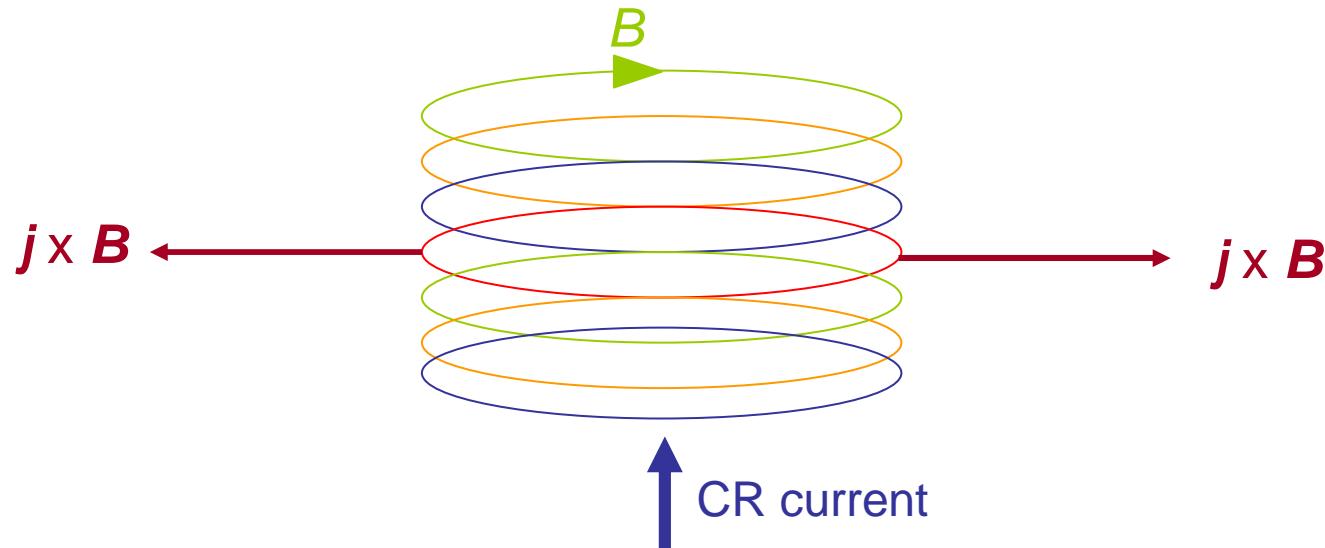
$j \times B$ expands loops

→ stretches field lines

→ more B

→ more $j \times B$

Simplest form: expanding loops of B

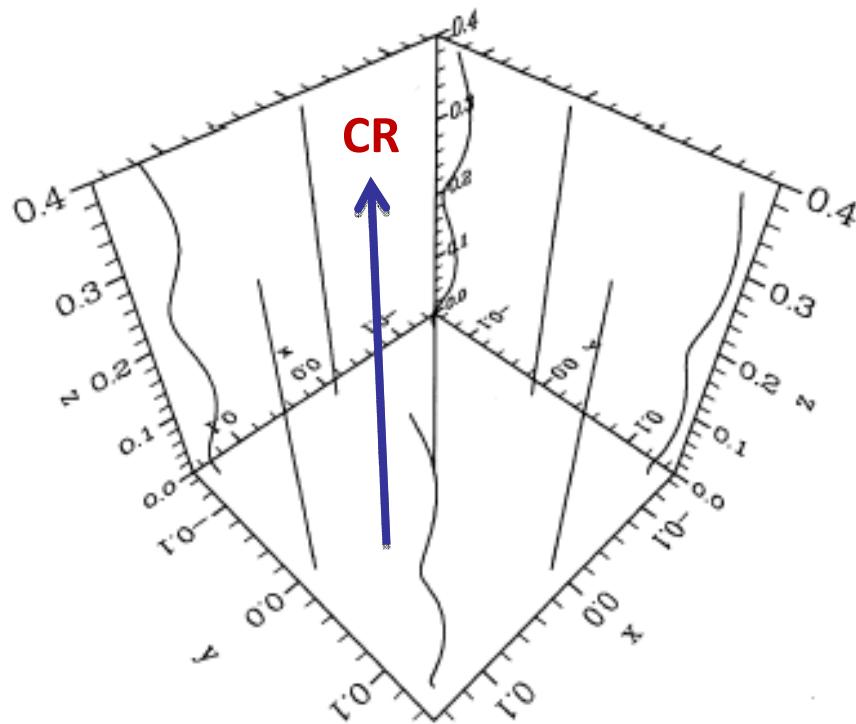


$j \times B$ expands loops
→ stretches field lines
→ more B
→ more $j \times B$

Streaming instabilities amplify magnetic field

Lucek & Bell (2000)

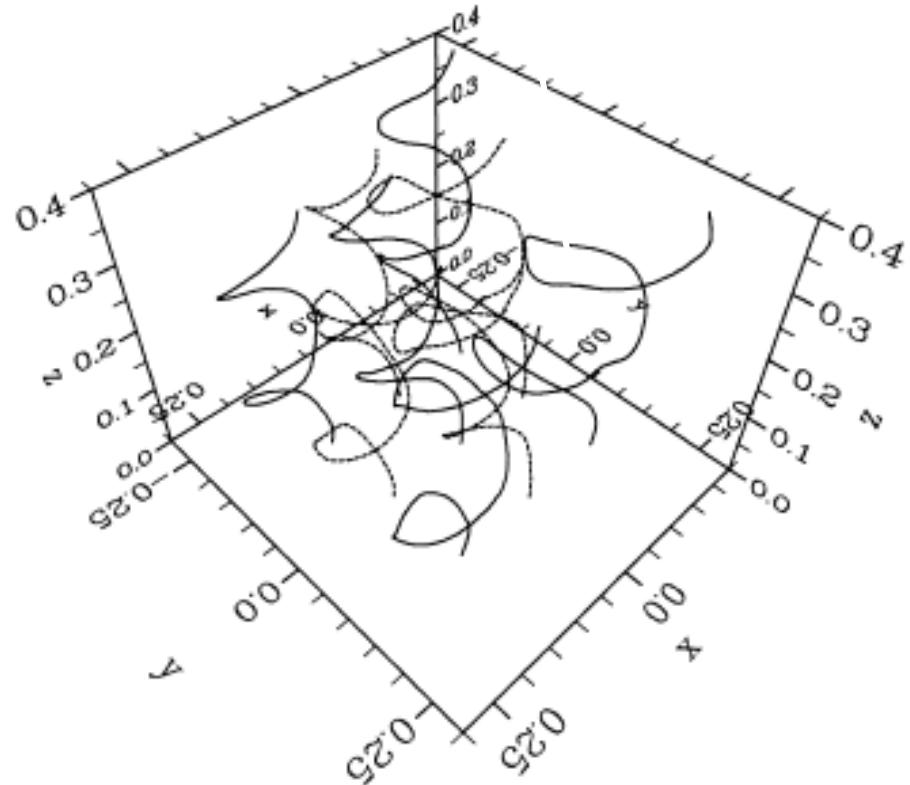
B field lines, $t = 0$



CR treated as particles

Thermal plasma as MHD

B field lines, $t = 2$

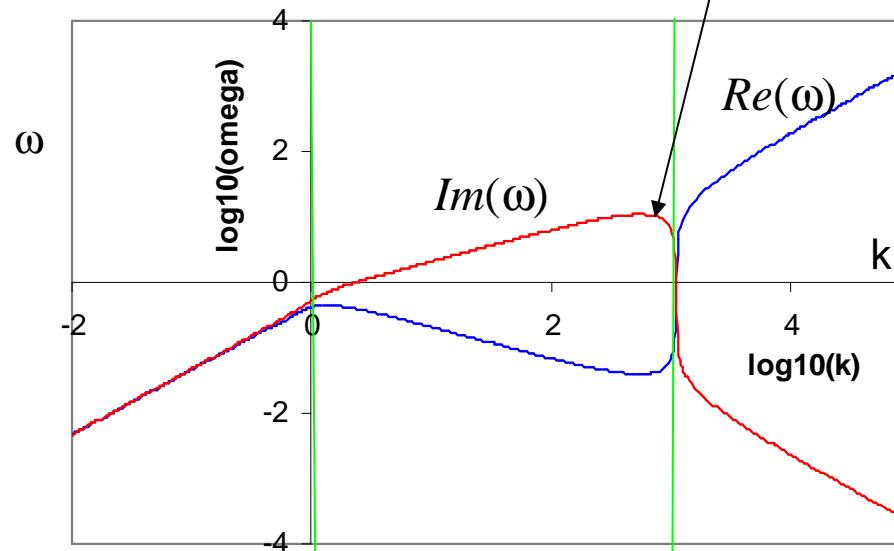


Dispersion relation

Max growth rate

$$\gamma_{\max} = j_{CR} \left(\frac{\mu_0}{2\rho} \right)^{1/2}$$

Red line is growth rate



k in units of r_g^{-1}
 ω in units of u_s^{-2}/cr_g

Wavelength longer than Larmor radius
 CR follow field lines.
 $j \times B$ drives weak instability

$$\rho \frac{d\mathbf{u}}{dt} = -\mathbf{j}_{CR} \times \mathbf{B}$$

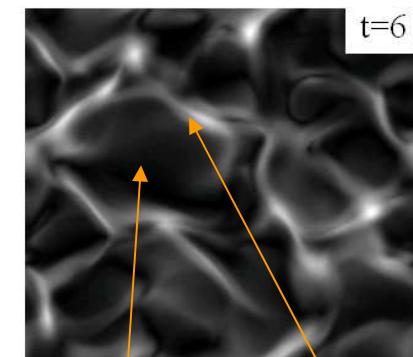
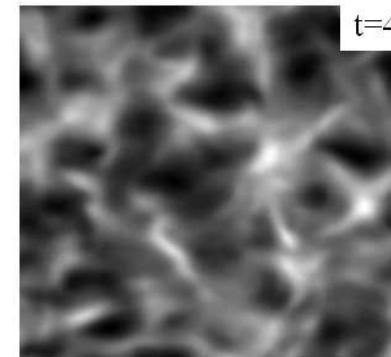
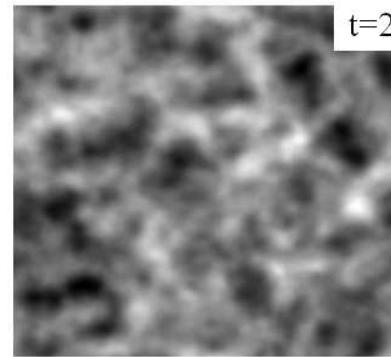
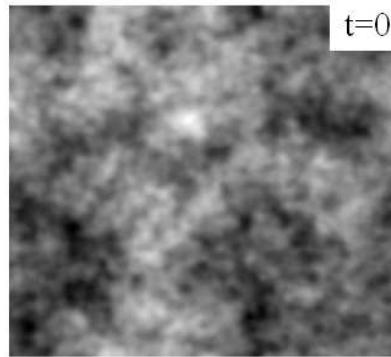
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})$$

$$\gamma = \left(\frac{k B_0 j_{CR}}{\rho} \right)^{1/2}$$

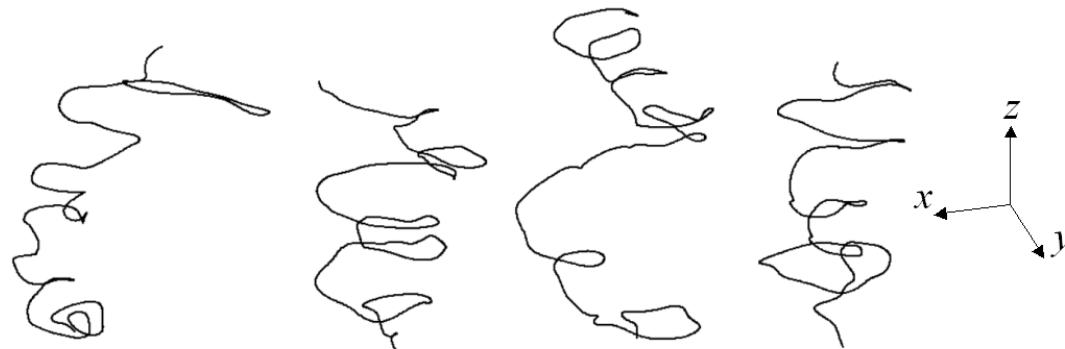
Magnetic tension inhibits instability

Non-linear growth – expanding loops

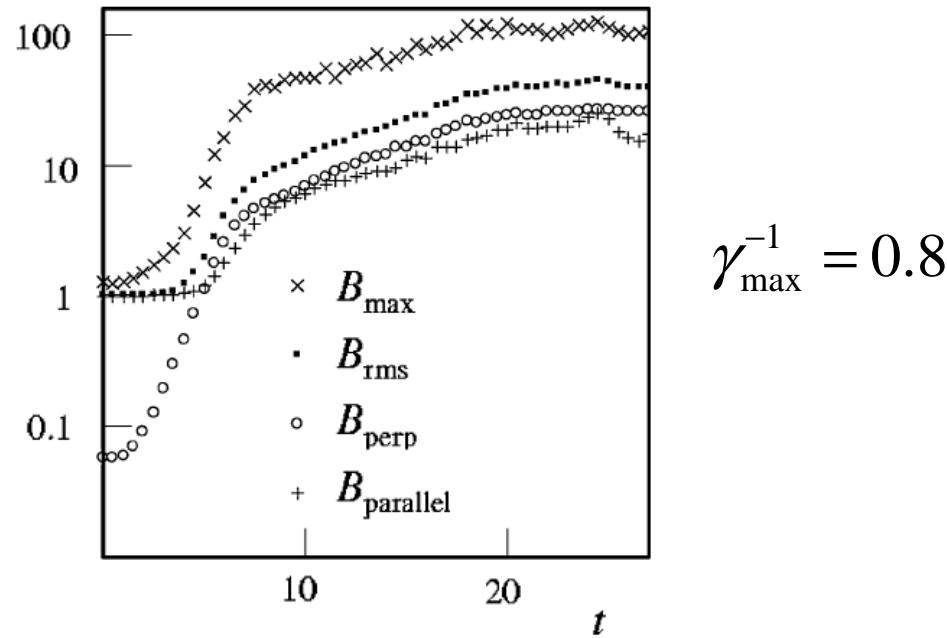
Slices through $|B|$ - time sequence (fixed CR current)



Field lines: wandering spirals



Non-linear growth



Strong fields for $t > 10\gamma_{\max}^{-1}$

$$\gamma_{\max} = j_{CR} \left(\frac{\mu_0}{2\rho} \right)^{1/2} \implies j_{CR} > 10 \left(\frac{2\rho}{\mu_0} \right)^{1/2} t$$

$$\implies E_{\max} = 4 \times 10^{15} \left(\frac{Q_{CR}}{\rho u_{\text{shock}}^3} \right) \left(\frac{u_{\text{shock}}}{5000 \text{ km s}^{-1}} \right)^3 \left(\frac{t}{300 \text{ yr}} \right) \left(\frac{n_e}{\text{cm}^{-3}} \right)^{1/2}$$

Limits on magnetic field

Limits on magnetic field growth

1) Magnetic tension

$$\frac{B^2}{\mu_0} \approx \left(\frac{u_{shock}}{c} \right) \left(\frac{Q_{CR}}{\rho u_{shock}^3} \right) \rho u_{shock}^2$$

2) Low instability growth rate, but normally compensated by drop in CR escape energy

$$\text{CR current} \propto \frac{\rho u_{shock}^3}{\text{CR escape energy}}$$

3) Confines CR to shock, inhibits CR current, reduces instability growth

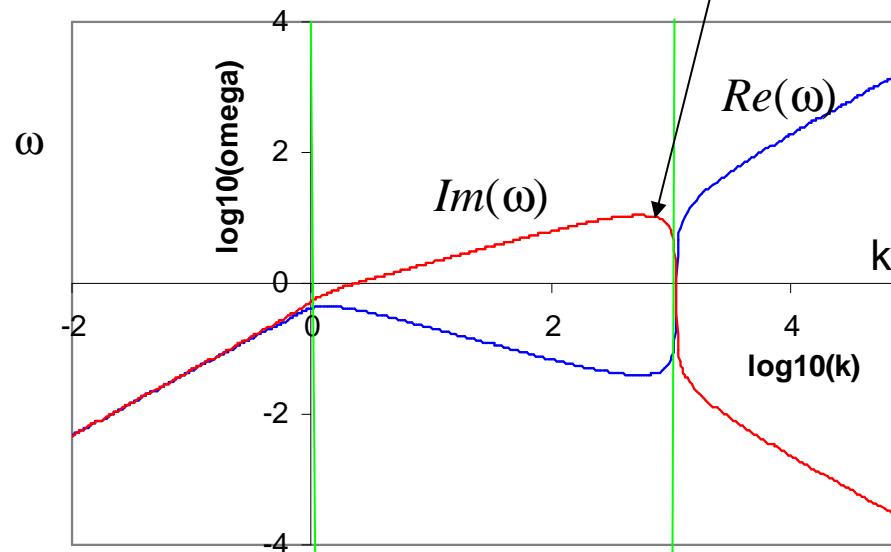
$$\frac{B^2}{\mu_0} \approx \frac{1}{20} \left(\frac{Q_{CR}}{\rho u_{shock}^3} \right)^2 \rho u_{shock}^2$$

Dispersion relation

Max growth rate

$$\gamma_{\max} = j_{CR} \left(\frac{\mu_0}{2\rho} \right)^{1/2}$$

Red line is growth rate



k in units of r_g^{-1}
 ω in units of $v_s^2/c r_g$

Wavelength longer than Larmor radius
 CR follow field lines.
 $j \times B$ drives weak instability

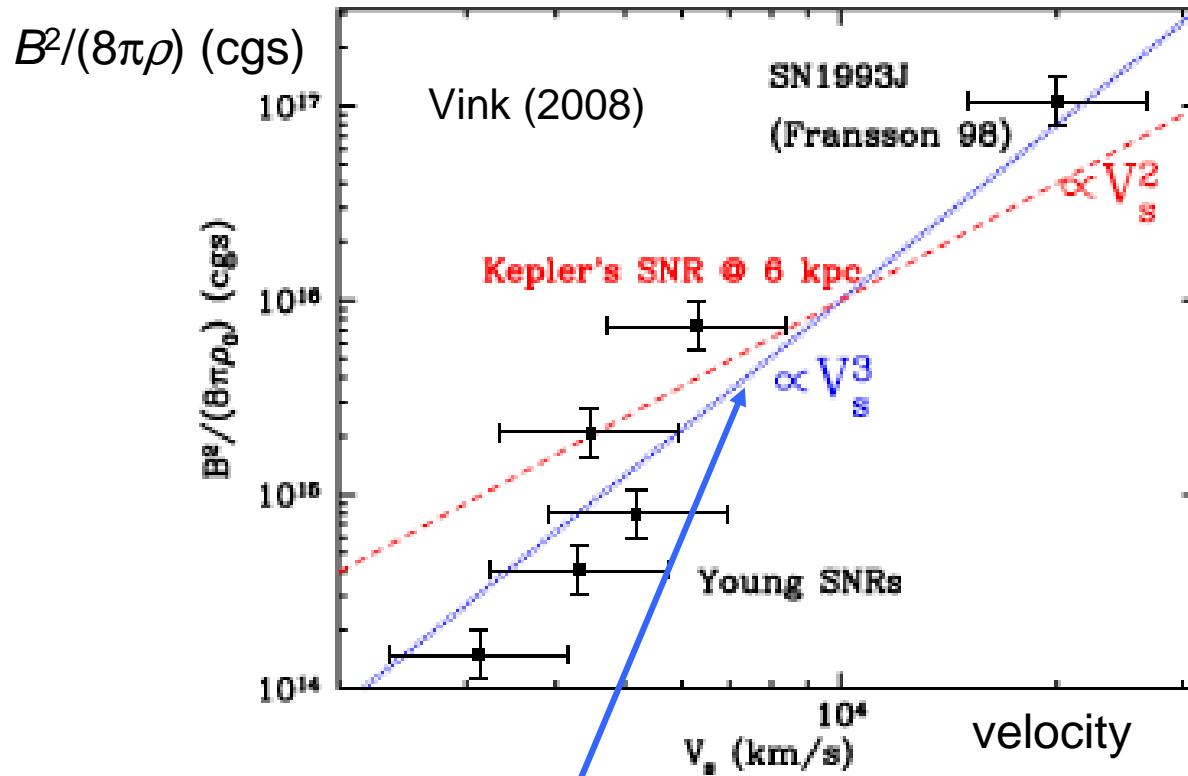
$$\rho \frac{d\mathbf{u}}{dt} = -\mathbf{j}_{CR} \times \mathbf{B}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})$$

$$\gamma = \left(\frac{k B_0 j_{CR}}{\rho} \right)^{1/2}$$

Magnetic tension inhibits instability

Magnetic field limited by magnetic tension



Data for
RCW86, SN1006, Tycho,
Kepler, Cas A, SN1993J

Fit to obs (Vink):

$$B \approx 700 \left(\frac{u}{10^4 \text{ km s}^{-1}} \right)^{3/2} \left(\frac{n_e}{\text{cm}^{-3}} \right)^{1/2} \mu\text{G}$$

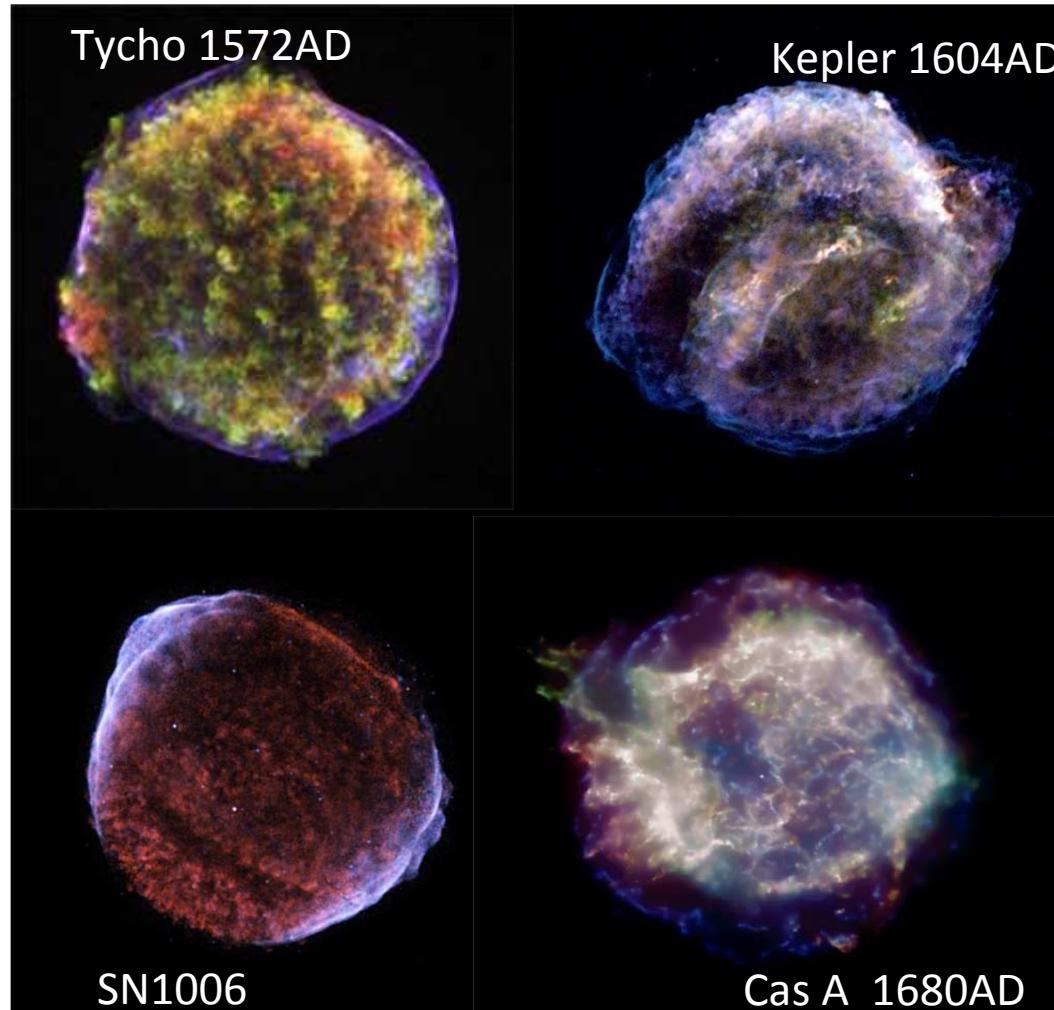
Theory:

$$B \approx 400 \left(\frac{u}{10^4 \text{ km s}^{-1}} \right)^{3/2} \left(\frac{n_e}{\text{cm}^{-3}} \right)^{1/2} \left(\frac{\eta}{0.1} \right)^{1/2} \mu\text{G}$$

Can we observe the turbulence?

Historical shell supernova remnants

(Vink & Laming, 2003; Völk, Berezhko, Ksenofontov, 2005)



Chandra observations

NASA/CXC/Rutgers/
J.Hughes et al.

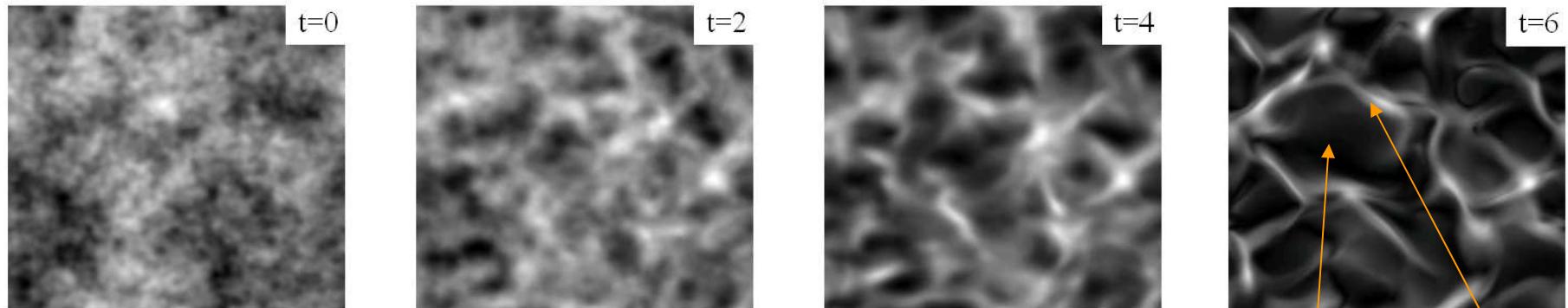
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NASA/CXC/NCSU/
S.Reynolds et al.

NASA/CXC/MIT/UMass Amherst/
M.D.Stage et al.

Characteristic distances and times

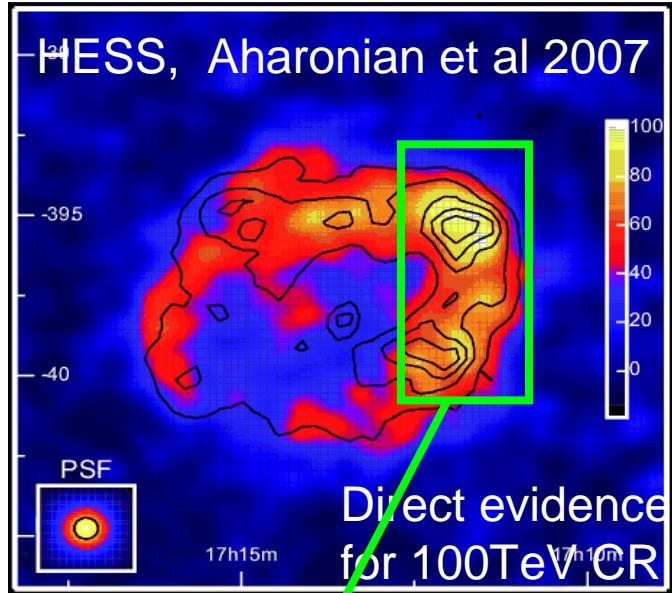
Slices through $|B|$ - time sequence (fixed CR current)



Cavities and walls
in $|B|$ & ρ

Larmor radius of CR at 10^{15} eV in $100\mu\text{G}$ field = $3 \times 10^{14}\text{m} = 0.5$ arc sec at 3kpc

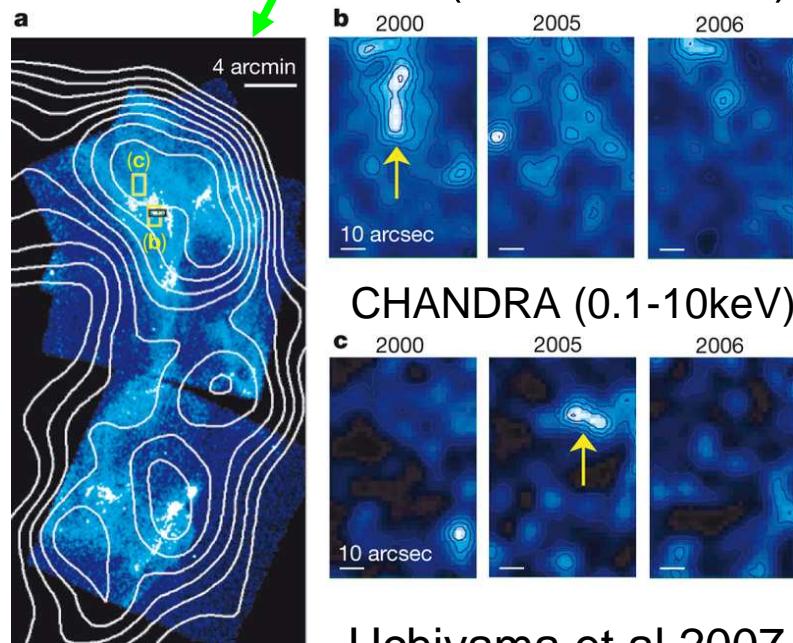
Shock at 5000 km s^{-1} traverses Larmor radius in 2 years



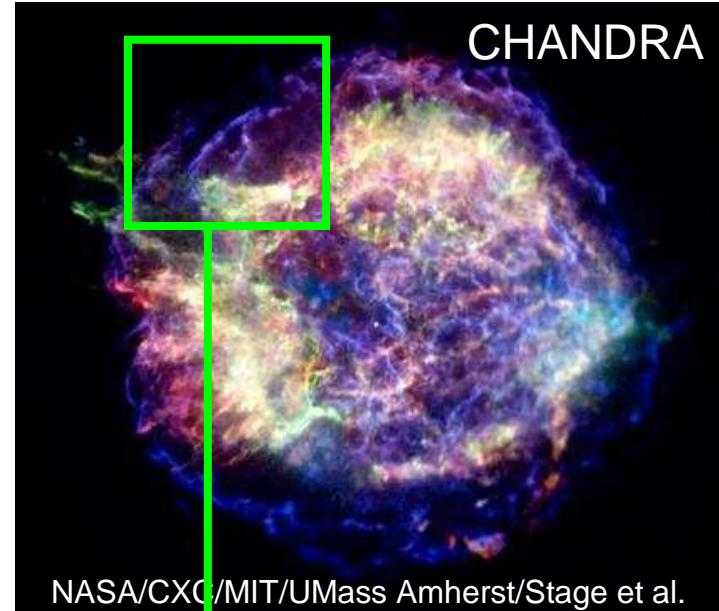
Two SN remnants with varying shock structure

Shock sweeps out pre-shock medium

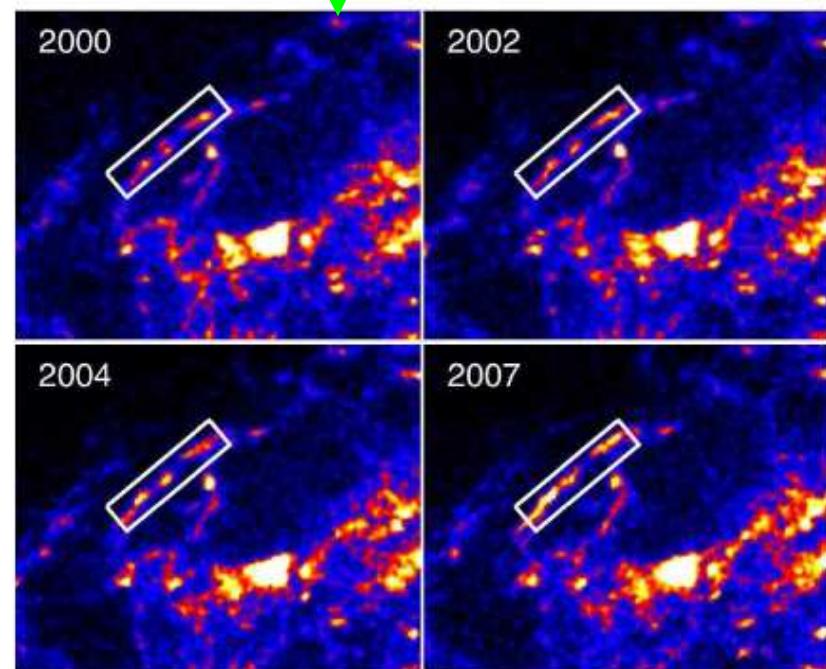
RX J1713.7-3946 (SN of 393AD)



Uchiyama et al 2007



Cas A (1680AD)
CHANDRA (Patnaude et al 2008)



The situation as I see it...

We probably understand the basic microphysics of shock acceleration

- 1st order Fermi produces $\sim E^{-2}$
- CR are scattered by self-excited turbulence

But the global picture is incomplete...

- Lack of clinching observational evidence that SNR accelerate protons to PeV
- Acceleration to a few PeV stretches theory to limit
- Are protons accelerated to 10-100PeV and how?
- How do CR escape into the interstellar medium?
- Extra-Galactic accelerators