Loop Quantum Gravity

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IQG - FAU Erlangen

I. Brief introduction to LQG

How do we get LQG from classical GR?

Dynamics of LQG: Quantum Einstein Equations

Geometrical Operators in LQG

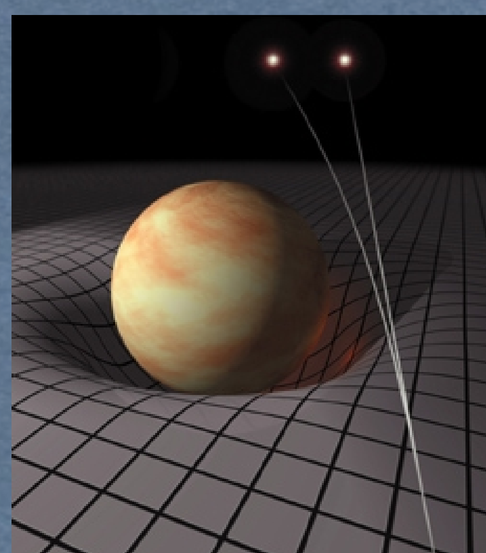
II. Current Research in LQG

Loop Quantum Cosmology

Semiclassical Sector of LQG

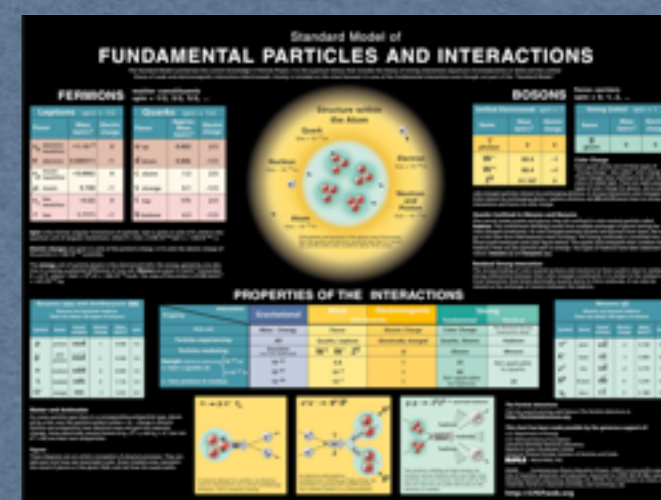
Modern Theoretical Physics

General Relativity



macroscopic
classical physics

Quantum Field Theory



microscopic
quantum physics

Search for a Theory of Quantum Gravity

General Relativity

Quantum Field Theory

Quantum Gravity

Candidates:

String theory

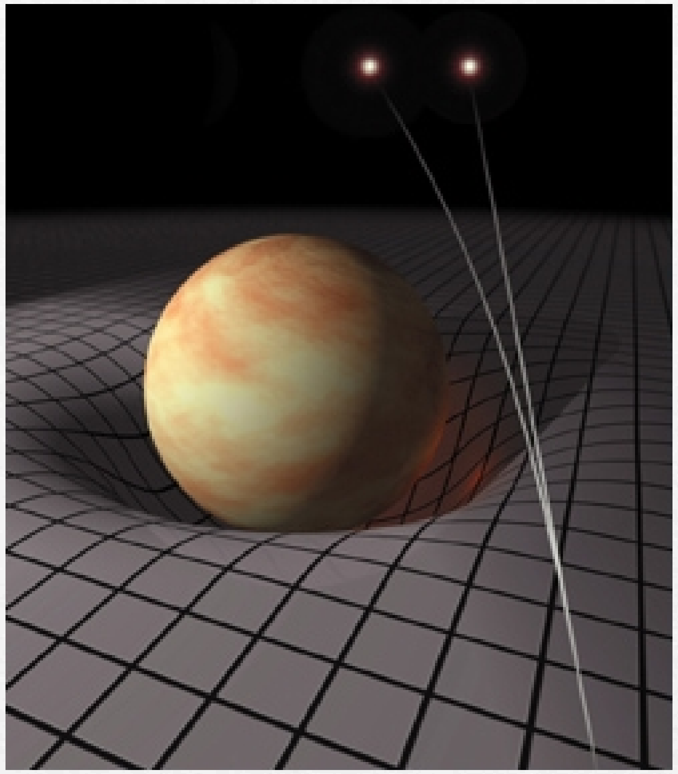
Loop Quantum Gravity, ...

Part I.

Brief introduction to LQG

General Relativity

Covariance: Diffeomorphism symmetry



Einstein Equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu}$$

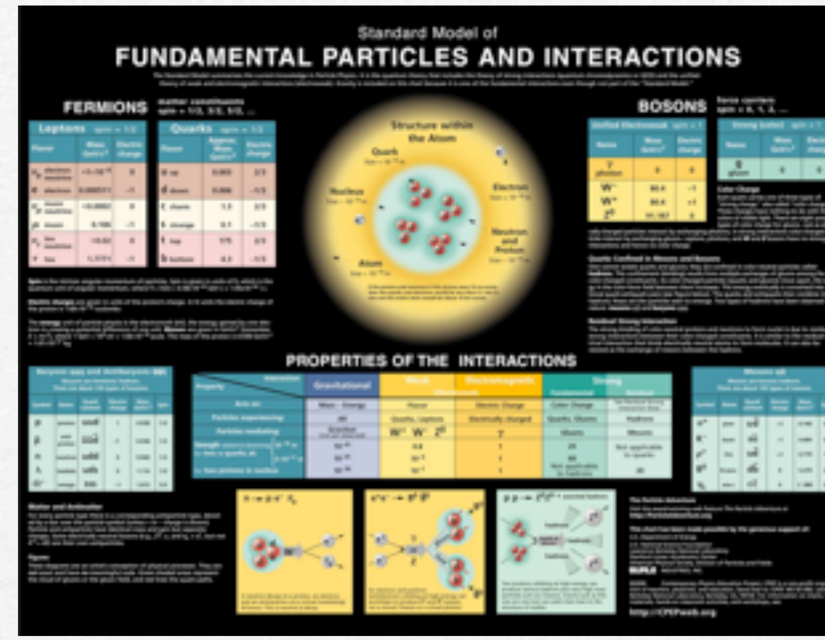
$g_{\mu\nu}$ Metric encodes spacetime geometry

Metric becomes dynamical object in GR

background independent and non-perturbative

Quantum Field Theory

QFT combines classical field theory
& quantum mechanics



Canonical quantization

Classical: $\{\varphi(x), \pi(y)\} = \lambda\delta^3(x, y)$

QFT: $[\hat{\varphi}(x), \hat{\pi}(y)] = i\lambda\hbar\delta^3(x, y)$

preferred background metric: Minkowski

Wightman axioms rely on high symmetry of $\eta_{\mu\nu}$

background dependent, perturbative quantum theory

Interaction spacetime and matter

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu}$$

↓ ↓

Geometry = Matter

↓ ↓

Classical ? Quantum

From GR to LQG

Step 1: Take GR as a classical starting point and apply canonical quantization

Back to QM: Phase space formulation (q,p): $\{q,p\} = 1$

Dynamics is described by Hamiltonian H(q,p)

Quantum algebra: $[\hat{q},\hat{p}] = i\hbar$

Stone-von Neumann Theorem for QM

Representation: $\mathcal{H}_{\rm QM} = L_2(\mathbb{R}, dx)$ unique!

$$\hat{q}\Psi(x) = x\Psi(x) \quad \hat{p}\Psi(x) = -i\hbar\frac{d}{dx}\Psi(x)$$

SU(2)-Gauge Theory

Minkowski spacetime, canonical formulation

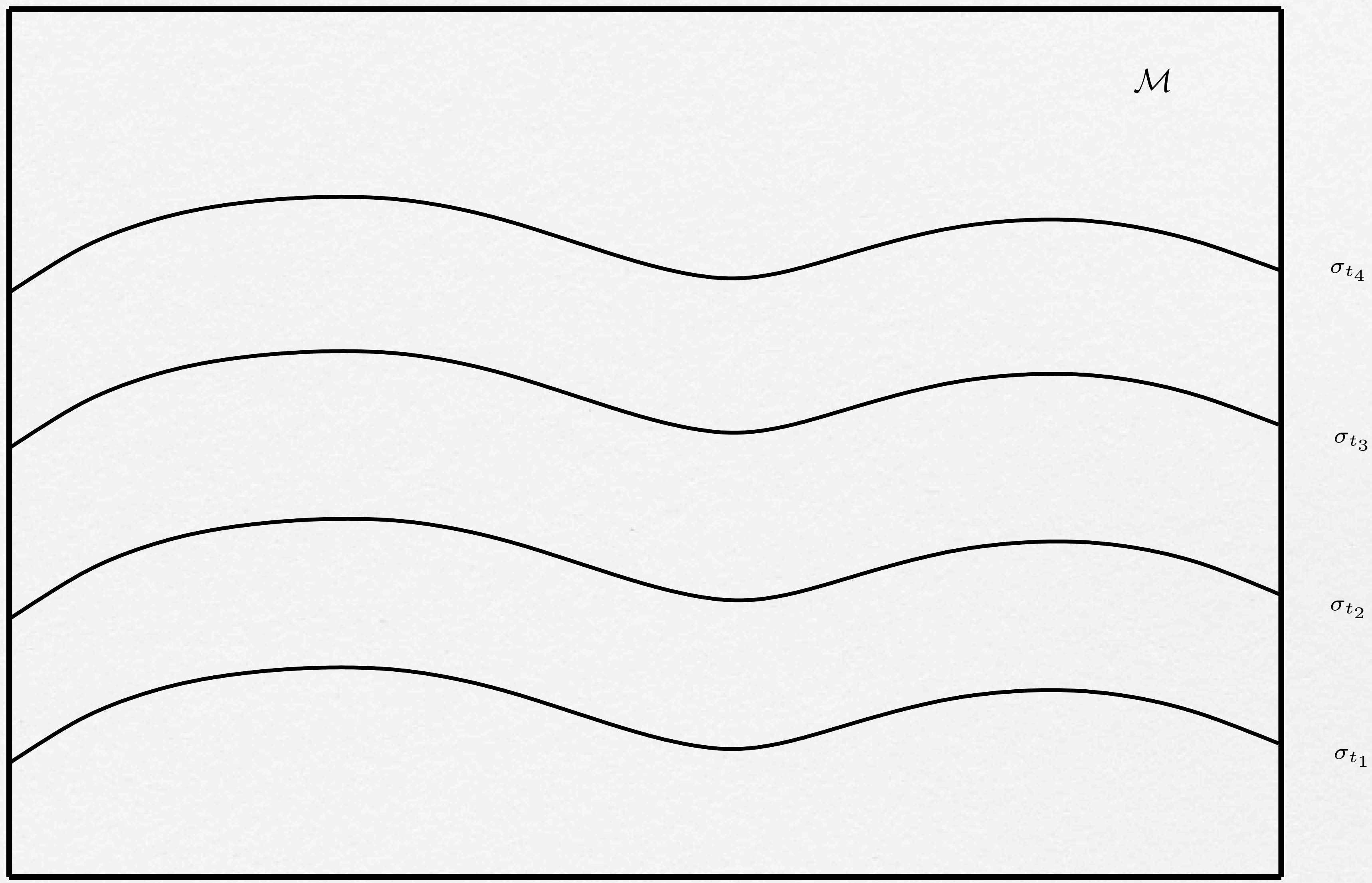
Elementary variables

$$A^j_a, E^a_j \quad \{A^j_a(x), E^b_k(y)\} = g^2 \delta^j_k \delta^b_a \delta^3(x,y)$$

Constraint: $G_j(A,E) = 0$

Dynamics: $\dot{A}^j_a = \{A^j_a(x), H\}, \quad \dot{E}^a_j = \{E^a_j(y), H\}$

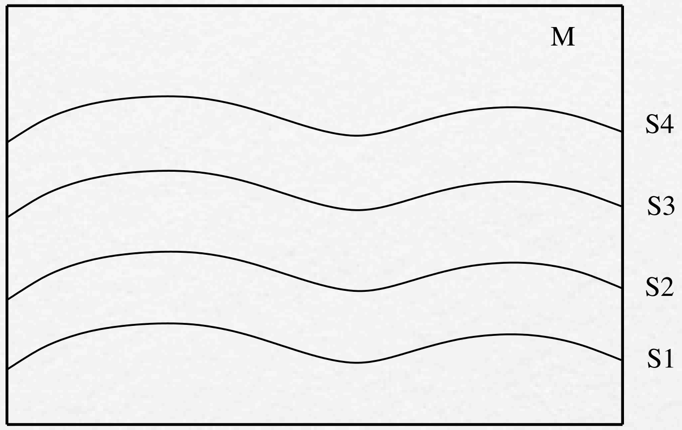
Hamiltonian: $$H = \frac{1}{2g^2} \int d^3x Tr(E^2 + B^2)$$



3+1-split of spacetime

Ashtekar Variables in GR

(3+1) split into space and time [ADM 1959]



Elementary variables [Ashtekar 1986]

$$A^j_a, E^a_j \quad \{A^j_a(x), E^b_k(y)\} = \kappa \delta^j_k \delta^b_a \delta^3(x,y)$$

Constraints:

$$C(A,E) = 0, C_a(A,E) = 0, G_j(A,E) = 0$$

$$\dot{A}^j_a = \{A^j_a, H_{\text{can}}\} \quad \dot{E}^a_j = \{E^a_j, H_{\text{can}}\}$$

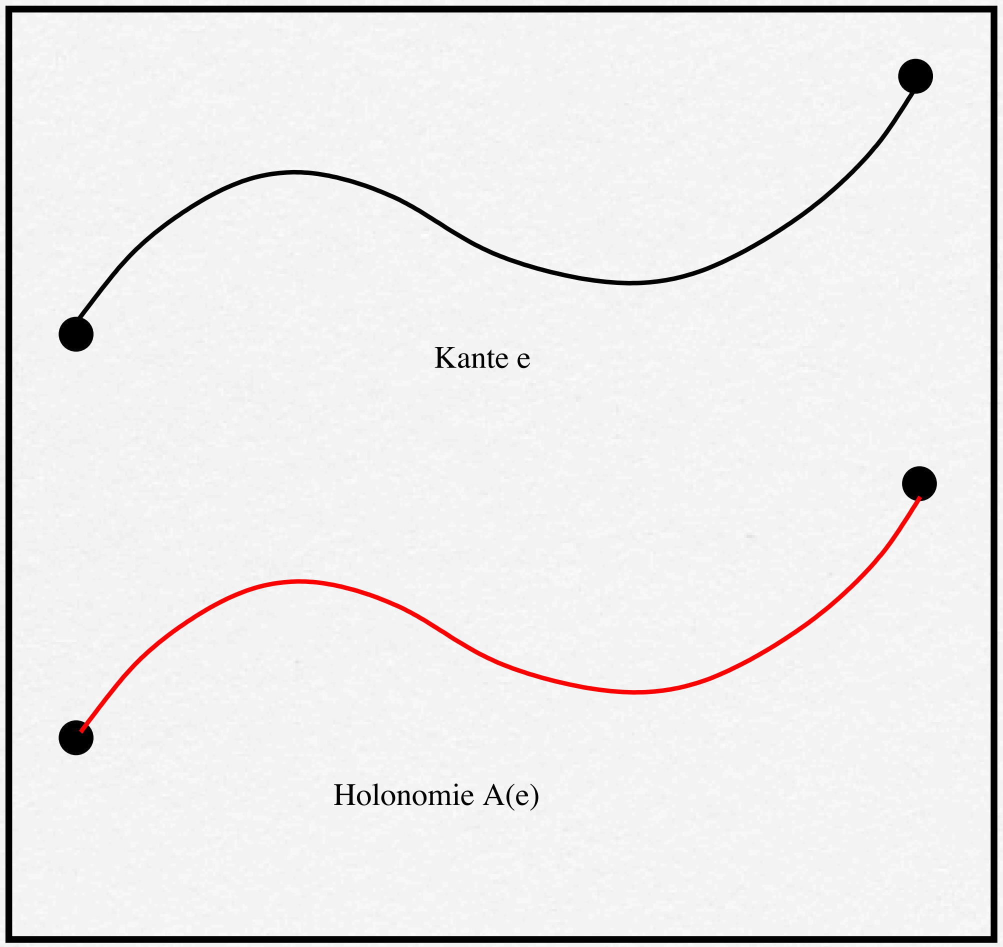
Hamiltonian:

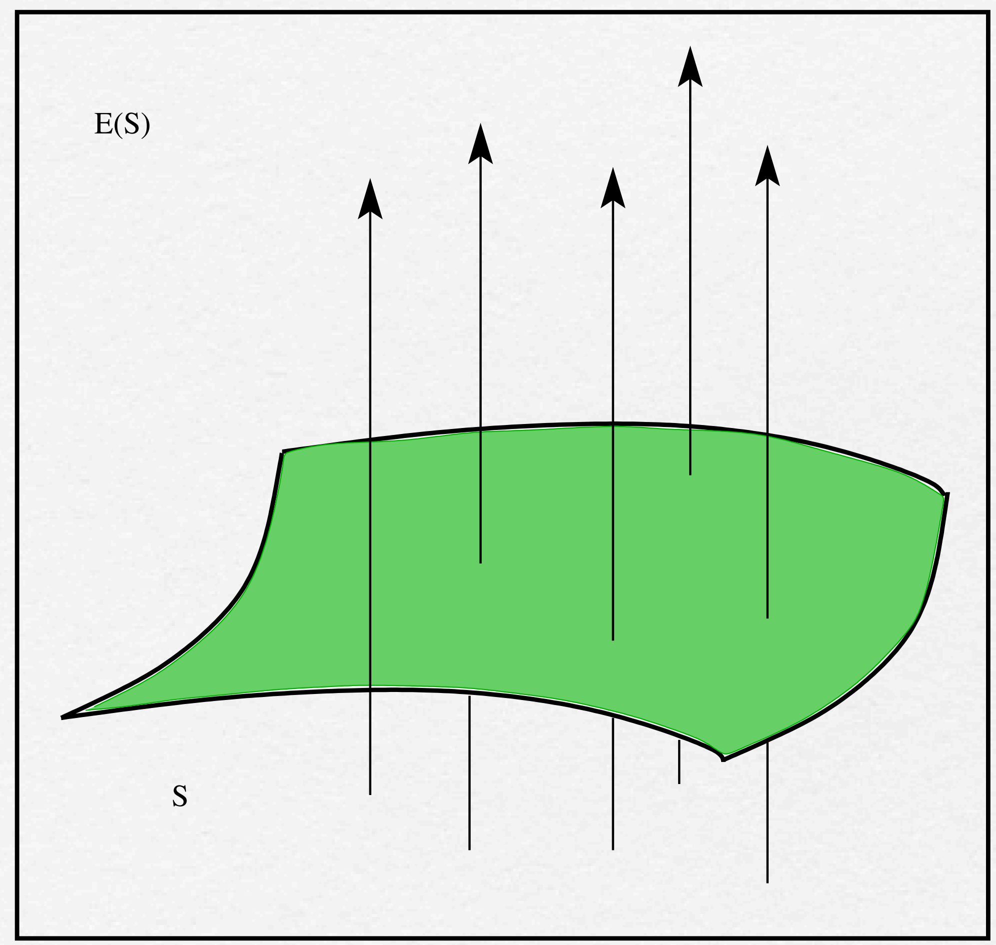
$$H_{\text{can}} = \frac{1}{\kappa} \int_\sigma d^3x (NC + N^a C_a + \Lambda^j G_j)$$

Holonomies and Fluxes, starting point for quantization in LQG:

$$h_e(A), \; E(S)$$

Holonomies and Fluxes





Constraint Quantization

Dirac Quantization

Constraints are solved in quantum theory

as an intermediate step kinematical $\mathcal{H}_{\rm kin}$

Constraints become operators

Solutions: $\hat{C}\psi_{\rm phys} = 0$

Reduced Quantization

Constraints are reduced in classical theory

direct access to $\mathcal{H}_{\rm phys}$ physical Hilbert space

physical Hamilton in QT

algebra of observables might be complicated

Dirac Quantization: LQG

Starts with holonomy-flux algebra $\{h_e(A), E(S)\}$

One gets kinematical Hilbert space $\mathcal{H}_{\text{kin}} = L_2(\bar{A}, d\mu_{\text{AL}})$

[Ashtekar, Isham, Lewandowski, Rovelli, Smolin `90]

LOST-Theorem: uniqueness of quantum theory

[Lewandowski, Okolow, Sahlmann, Thiemann `05, Fleischhack `05]

$\hat{h}_e(A),\ \hat{E}(S)$ multiplication and derivation operators

In $\mathcal{H}_{\text{kin}}$ constraints are implemented $\hat{C}, \hat{\vec{C}}, \hat{G}$

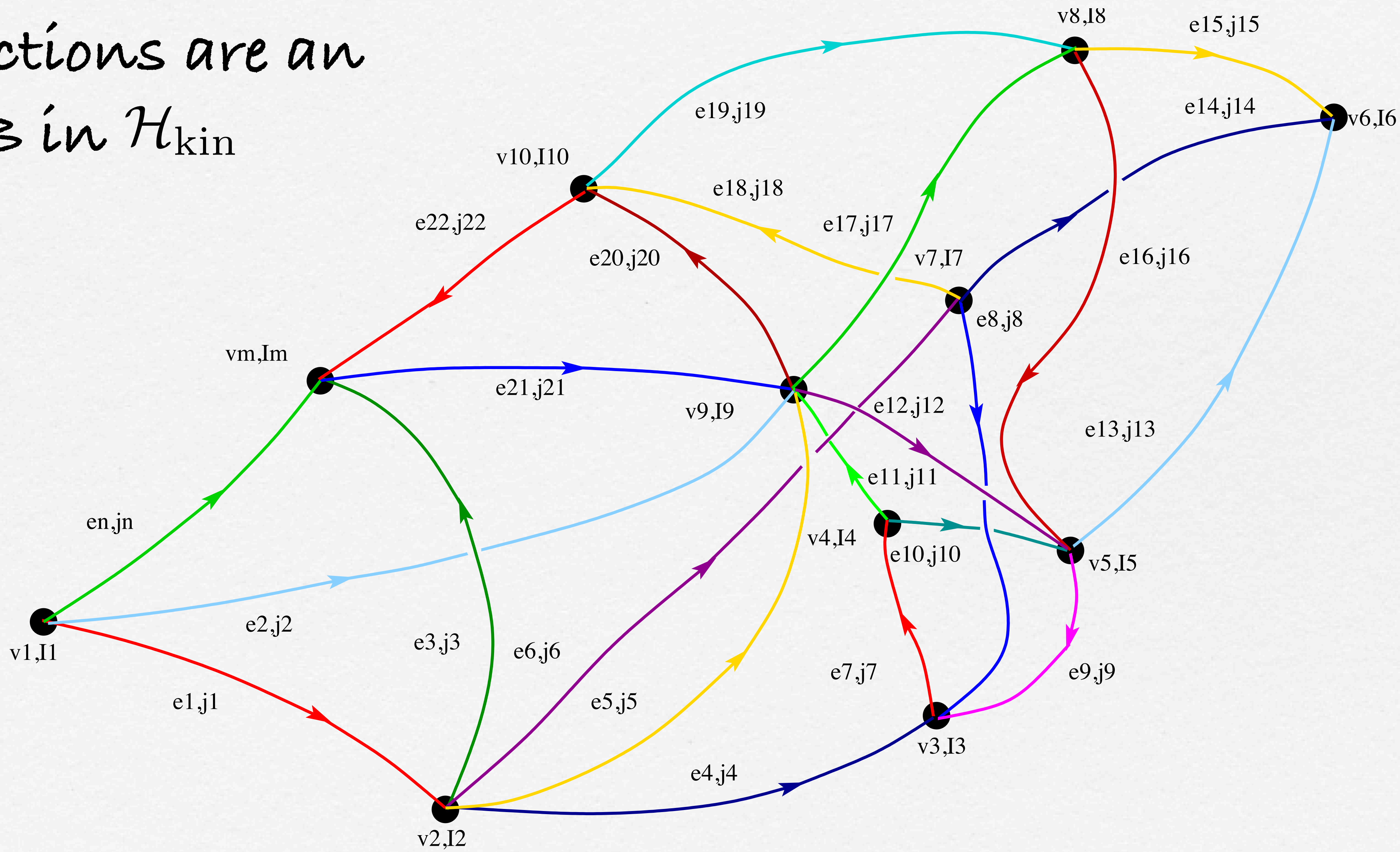
Quantum Einstein Equations:

$$\hat{C}\psi_{\text{phys}}(A) = 0 \quad \hat{\vec{C}}\psi_{\text{phys}}(A) = 0 \quad \hat{G}\psi_{\text{phys}}(A) = 0$$

Spin network functions

[Ashtekar, Isham, Lewandowski, Rovelli, Smolin `90]

Spin network functions are an ONB in $\mathcal{H}_{\rm kin}$



Geometrical Operators: Area

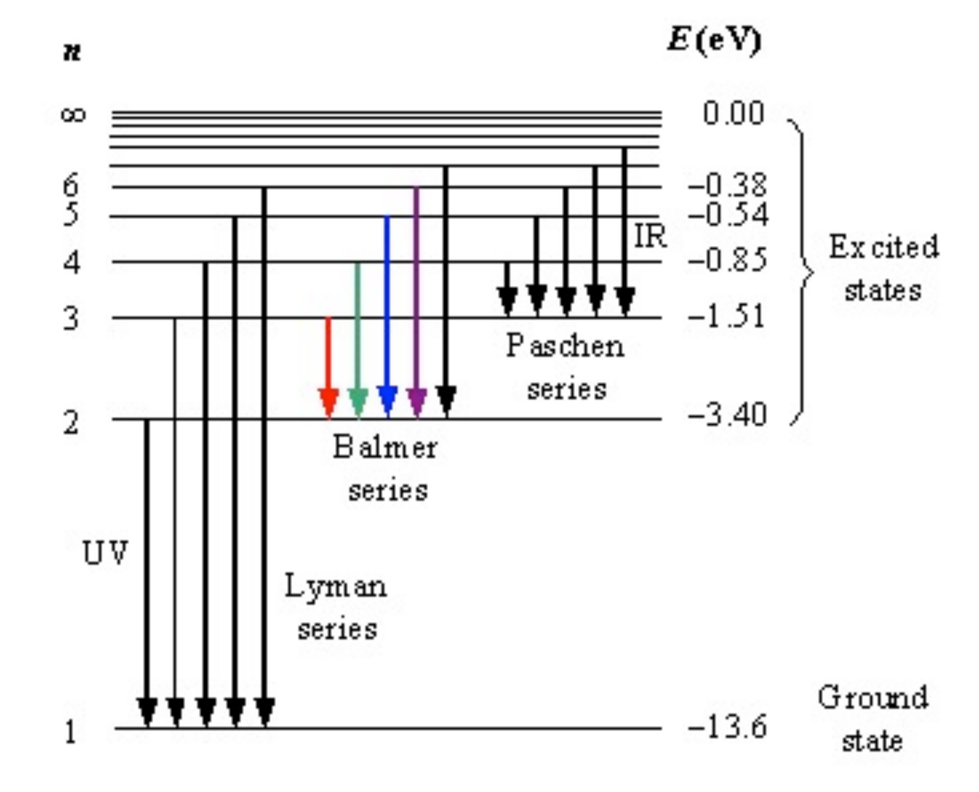
LQG allows to define operators for area, volume and length

$\hat{Ar}_S(E)$ function of fluxes

Area operator: $\hat{Ar}_S = Ar_S(\hat{E})$

Area operator has discrete spectrum

Area gap, minimal non-zero eigenvalue



Energy levels of the hydrogen atom with some of the transitions between them that give rise to the spectral lines indicated.

Volume operator

[Rovelli, Smolin `92]
[Ashtekar, Lewandowski `93]

Analogous to area also classical volume functional

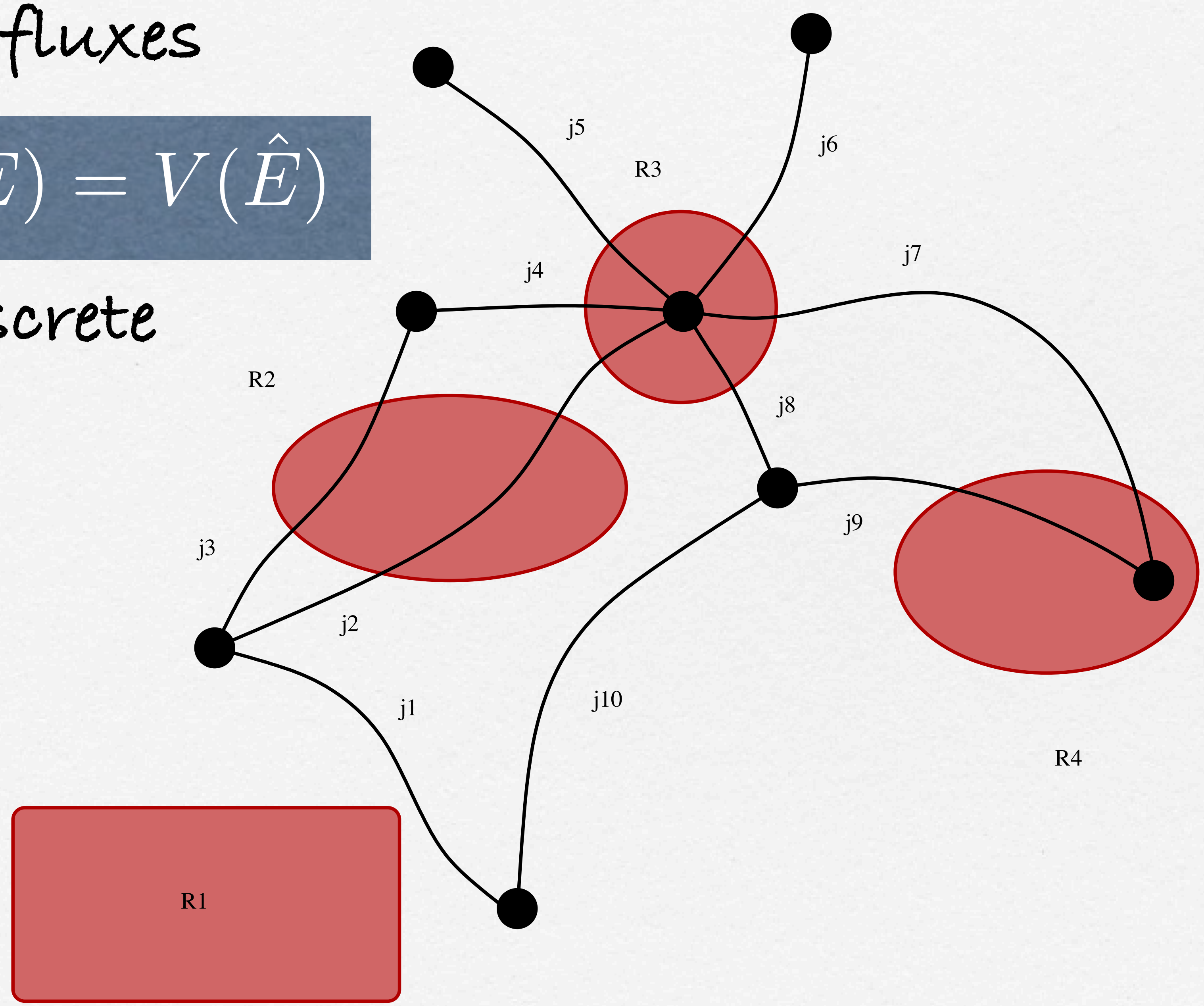
$V_R(E)$ is function of fluxes

Volume operator: $\hat{V}_R(E) = V(\hat{E})$

Volume operator has discrete spectrum, volume gap?

Also important for dynamics

$$\hat{V}_{R_1} = \hat{V}_{R_2} = 0$$
$$\hat{V}_{R_3} \neq 0, \hat{V}_{R_4} = 0$$



Part II.

Current research topics in LQG

Quantum Einstein Equations

$$\hat{C}\psi_{\text{phys}} = 0 \quad \hat{\vec{C}}\psi_{\text{phys}} = 0 \quad \hat{G}\psi_{\text{phys}} = 0$$

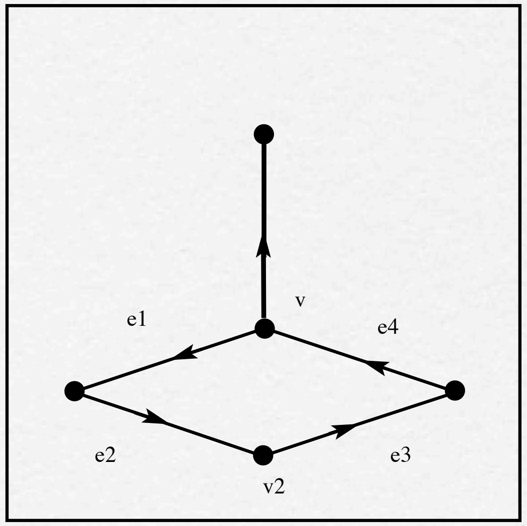
Gauss & Diffeo can be solved: Solutions in $\mathcal{H}^{G}_{\text{diff}}$

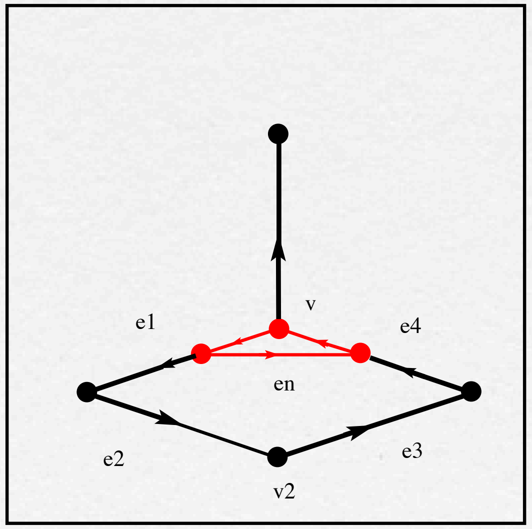
More complicated $\hat{C}\psi_{\text{phys}} = 0$ yields to $\mathcal{H}_{\text{phys}}$

$\hat{C}$ modifies (dual) graph

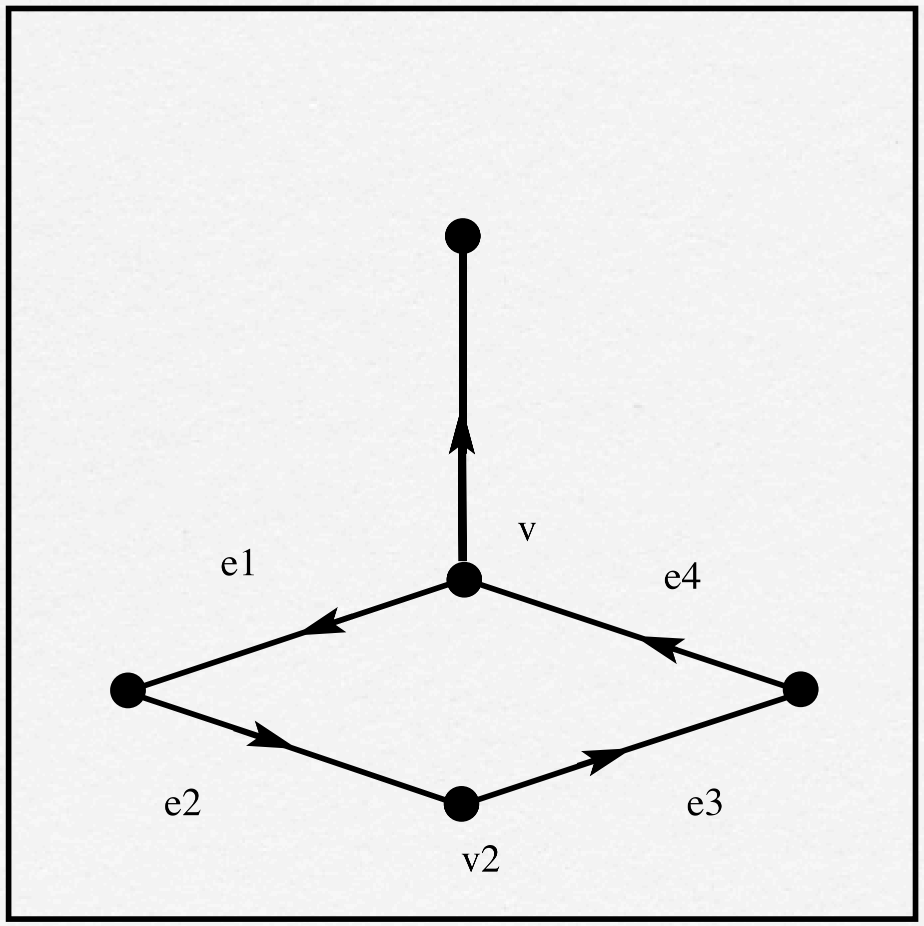
General solutions so far unknown

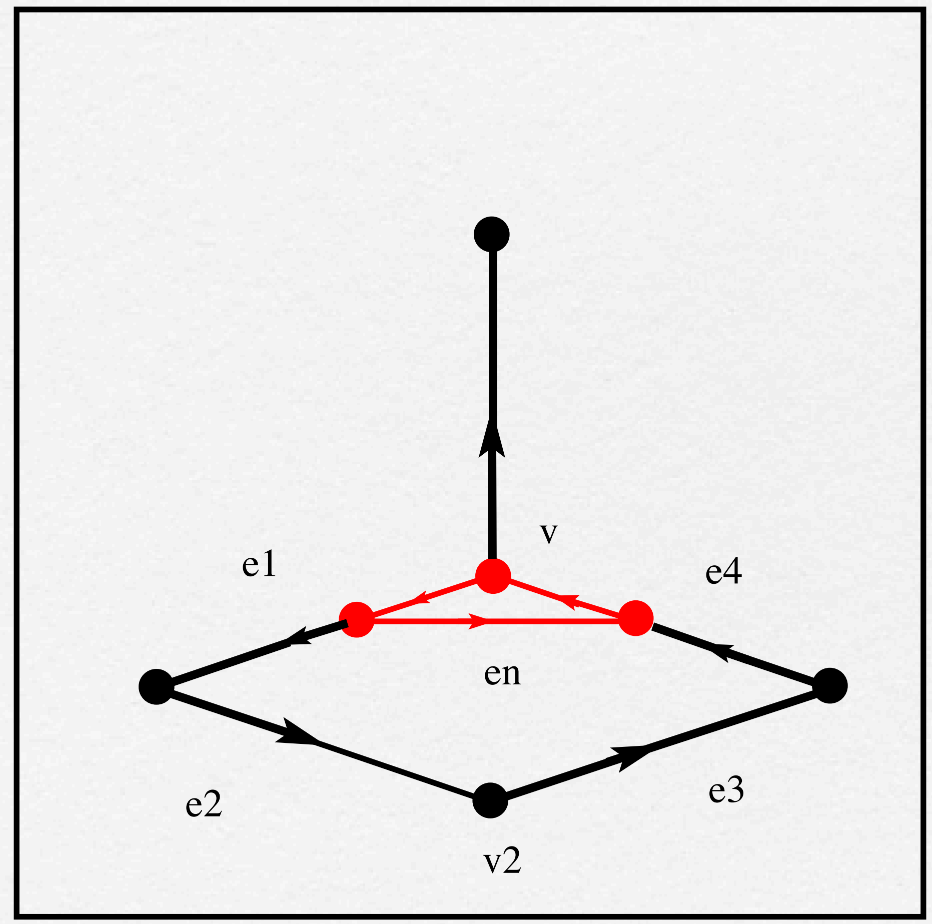
Also classical Einstein eqns cannot be solved analytically





Graph modifying Operator $\hat{C}$





Quantum Einstein Equations

Loop Quantum Cosmology (LQC) [Kastrup, Bojowald `98]

Idea: Reduce classical to cosmological sector then quantize

Ansatz for variables $A^{\rm FRW}, E^{\rm FRW}$

Dirac Quantization analog to full LQG

Simplified version of QEE: Can be solved

Resolution of big bang singularity $\longrightarrow$ bounce

Caution: Generalization to full LQG [Brunnemann, Thiemann`05]

Loop Quantum Cosmology

Classical FRW: gravity plus scalar field

$$H^2 = \frac{8\pi G}{3}\rho$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

Loop Quantum Cosmology

LQC: gravity plus scalar field [Ashtekar, Pawlowski, Singh `06]

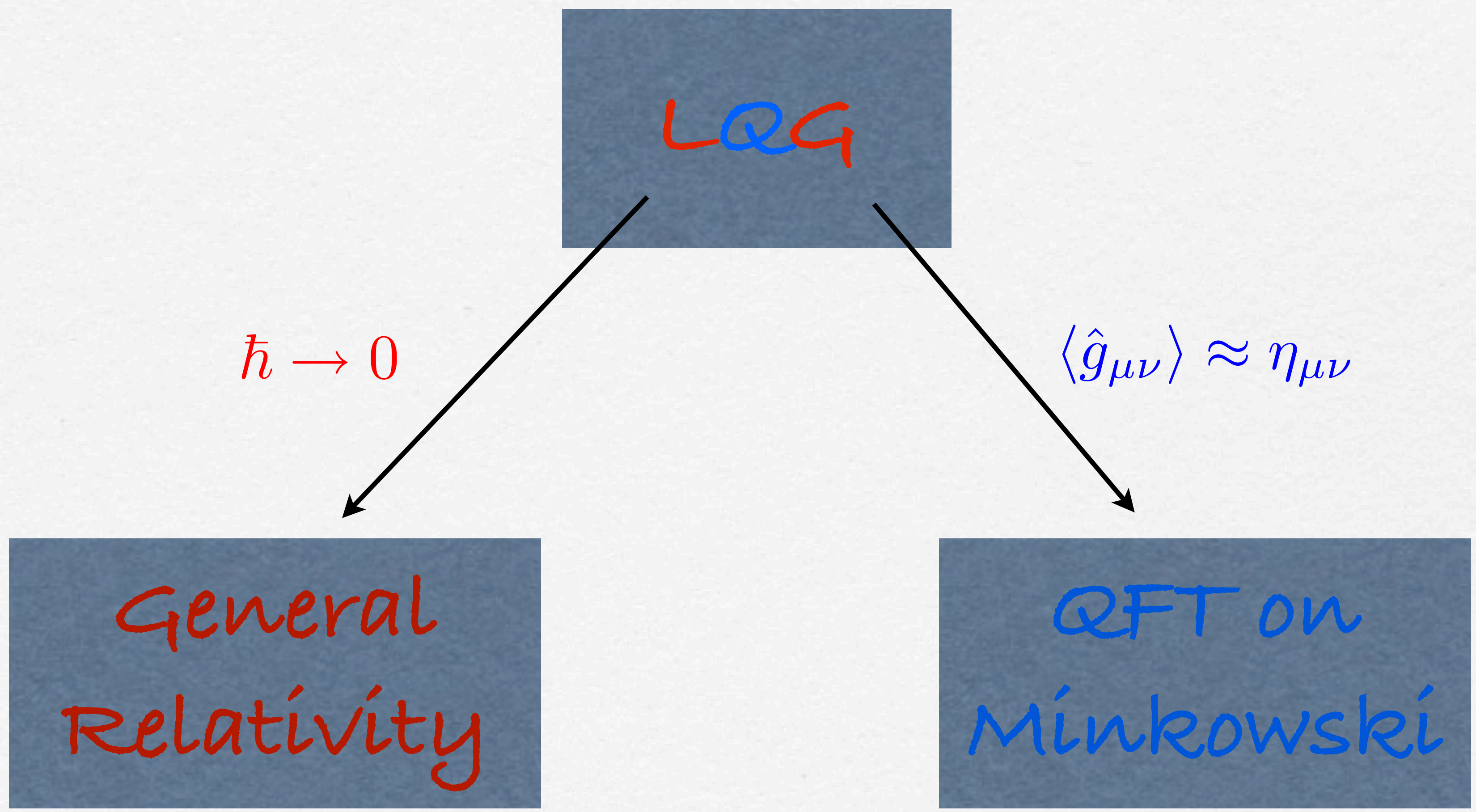
$$H^2 = \frac{8\pi G}{3}\rho(1 - \frac{\rho}{\rho_{\max}})$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho(1 - 4\frac{\rho}{\rho_{\max}}) - 4\pi G P(1 - 2\frac{\rho}{\rho_{\max}})$$

$\rho_{max} = \frac{3}{8\pi G\beta^2\lambda^2} \approx 0.41\rho_{\text{planck}}$ $\lambda \to 0$ for $\hbar \to 0$

Semiclassical LQG

<u>Minimal requirement for LQG:</u>



Semiclassical Dynamics

Only non-perturbative techniques are allowed

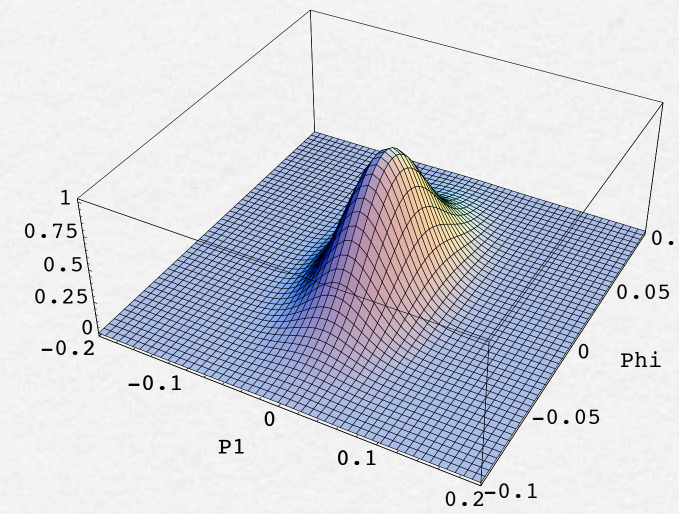
Coherent states for LQG: $\psi_{A_0,E_0} \in \mathcal{H}_{\text{kin}}$ [Sahlmann, Thiemann, Winkeler `03, Bahr `06]

Aim:

$$\langle \psi_{A_0,E_0} \hat{C} \psi_{A_0,E_0} \rangle = C(A_0, E_0) + o(\hbar)$$

$$\langle \psi_{A_0,E_0} \hat{G} \psi_{A_0,E_0} \rangle = G(A_0, E_0) + o(\hbar)$$

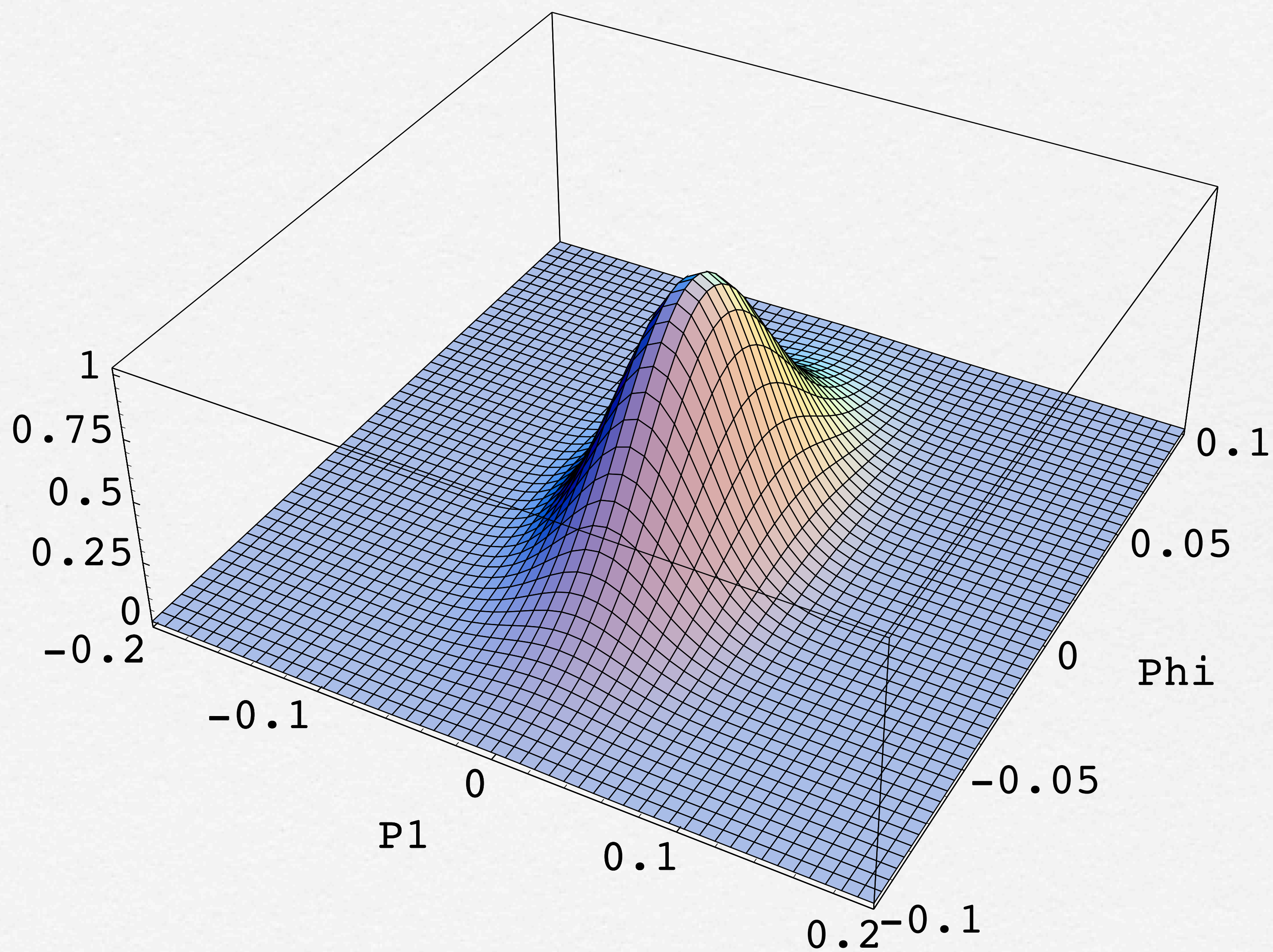
$$\langle \psi_{A_0,E_0} \hat{\vec{C}} \psi_{A_0,E_0} \rangle = \vec{C}(A_0, E_0) + o(\hbar)$$



Problem: Hamiltonian constraint

Idea: Reformulate dynamics in a technically simpler and physically equivalent way [Thiemann `03]

Coherent States for LQG



Master Constraint Program

Master Constraint

$$M = \int_\sigma d^3x \frac{C^2 + q^{ab}C_aC_b + \delta^{jk}G_jG_k}{\sqrt{\det(q)}}$$

Class. equiv.

$$M = 0 \quad \leftrightarrow \quad C = 0 \quad \wedge \quad \vec{C} = 0 \quad \wedge \quad G = 0$$

Quantum Einstein Equations

$$\hat{M}\psi_{\text{phys}} = 0$$

Semiclassical dynamics

[K.G., Thiemann `06]

$$\langle \psi_{A_0,E_0}, \hat{M}\psi_{A_0,E_0}\rangle = M(A_0,E_0) + o(\hbar)$$

Reduced Quantization of LQG

[K.G., Thiemann `10]

(Mathem.) Observables for basic variables O_A, O_E

Quantum Einstein Equations $\frac{d}{d\tau}\hat{O}_A(\tau) = [\hat{O}_A(\tau), \hat{H}_{phys}]$

Semiclassical Analysis $\langle \psi_{A_0,E_0}, \hat{H}_{\mathrm{phys}} \psi_{A_0,E_0} \rangle = H_{\mathrm{phys}}(A_0, E_0) + o(\hbar)$

Master Constraint and phys. Hamiltonian Analysis:

First step towards showing that GR is semiclassical limit

To go further: Improvement of coherent states needed

Summary

LQG: GR formulated in terms of holonomies and fluxes

Can. Quantization yields Quantum Einstein Equations

General solutions to Hamiltonian constraint unknown

What can we still learn about QEE?

Symmetry reduced model LQC:
Corrections to classical FRW equations

Full LQG: Semiclassical Analysis of dynamics
(Master Constraint, physical Hamiltonian)

Outlook

Aim: Understand physical implications of LQG

Improve semiclassical techniques of LQG

Analyze choices made for Quantum Einstein Equations

Mathem.: Representation of the Quantum theory

Understand cosmological sector of full LQG

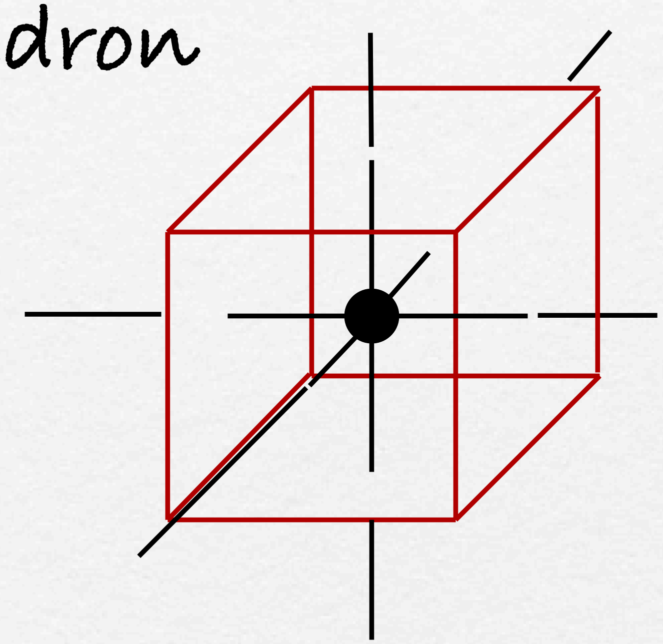
cosmological perturbation theory within LQG

⟶ manifestly gauge invariant perturbation theory [K.G., Hofmann, Thiemann, Winkler `10]

Scattering theory for QFT on curved spacetimes

Back to Quantum Dynamics

SNF on graph γ, consider dual polyhedron

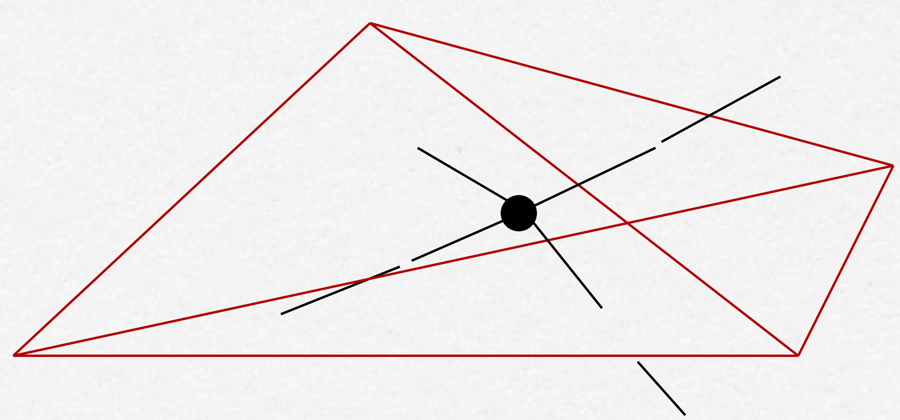


Example 1: 6-valent graph

→ dual polyhedron is a cube

Example 2: 4-valent graph

→ dual polyhedron is a tetrahedron



Quantum Einstein Equations

a little movie about quantum dynamics