The Crab Nebula Spectrum and its Physical Interpretation

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Abstract

A new sequence of observations of the remnant of a tremendous stellar explosion is giving astronomers a remarkable look at the dynamic relationship between the tiny Crab Pulsar and the vast nebula that it powers. Both the nebula and the pulsar are bright sources of radiation at all wavelengths from radio to high energies in the TeV range. The radiation is produced mainly by high–energy particles accelerated by the energy of the rotating neutron star. These energetic particles spiral around magnetic field lines in the nebula and give off radiation by the synchrotron and inverse Compton processes. By studying the global spectrum of the Crab nebula, we learn a lot about processes responsible for the radiation in quasars e.g.

1 Introduction

The Crab Nebula is the most famous and conspicuous known supernova remnant, the expanding cloud of gas created in the explosion of a star as supernova which was observed in the year 1054 AD. The supernova was noted on July 4, 1054 A.D. by Chinese astronomers as a new or *guest star*, and was about four times brighter than Venus, or about mag -6. According to the records, it was visible in daylight for 23 days, and 653 days to the naked eye in the night sky. It was probably also recorded by Anasazi Indian artists (in present-day Arizona and New Mexico), as findings in Navaho Canyon and White Mesa (both Arizona) as well as in the Chaco Canyon National Park (New Mexico) indicate. In addition, Ralph R. Robbins of the University of Texas has found Mimbres Indian art from New Mexico, possibly depicting the supernova.

The nebula consists of the material ejected in the supernova explosion, which has been spread over a volume approximately 10 light years in diameter, and is still expanding at the very high velocity of about 1,800 km/sec. The notion of gaseous filaments and a continuum background was photographically confirmed by Walter Baade and Rudolph Minkowski in 1930: The filaments are apparently the remnants from the former outer layers of the former star (the *pre-supernova* or supernova *progenitor*), while the inner, blueish nebula emits continuous light consisting of highly polarised synchrotron radiation, which is emitted by high-energy (fast moving) electrons in a strong magnetic field (see e.g. the VLT image). This explanation was first proposed by the Soviet astronomer J. Shklovsky (1953) and supported by observations of Jan H. Oort and T. Walraven (1956).

At the center of the nebula is the Crab Pulsar, a neutron star remnant of the supernova which is roughly 11 km in radius. It was discovered in 1968. The Crab Pulsar rotates once every 33 milliseconds, or 30 times each second. The most dynamic feature in the inner part of the nebula is the point where the relativistic pulsar wind slams into the surrounding material forming a shock front (for this, see HST and Chandra images). The shape and position of this feature shifts rapidly, with the equatorial wind appearing as a series of wisp-like features that steepen, brighten, then fade as they move away from the pulsar to well out into the main body of the nebula.

The Crab Nebula is often used as a calibration source in X–ray and gamma–ray astronomy. It is very bright in X- and gamma-rays, and the flux density and spectrum are known to be constant, with the exception of the pulsar. The pulsar provides a strong periodic signal that is used to check the timing of the X-ray detectors. In X-ray astronomy, 'Crab' and 'milliCrab' are sometimes used as units of flux density. Very few X-ray sources ever exceed one Crab in brightness.

2 Expansion and Distance of the Crab Nebula

For this, solve lab 7.1.

3 Spectrum and Energy–Loss of the Crab Nebula

The emission of the Crab Nebula is of non–thermal origin, and the spectrum covers 20 orders of magnitude in energy space. The overall spectrum is typical for synchrotron emitters (supernovae remnants, radio galaxies, quasars and BL Lac objects). For the last 50 years, every new instrument in Astronomy has been calibrated with this object. In the last years, new instruments have been built to measure the high–energy gamma–ray flux from the Crab Nebula (Compton Observatory, HEGRA, HESS, and many others).

3.1 Data Sets

A selection of spectral data is collected in the Appendix. We divide these data sets into low energy regime from radio frequencies to X–ray energies, and in high energy data covering the range from MeV to TeV regime.

For this you have to compile **two spectral data sets** in ascii format out of the 4 data sets listed at the appendix and given to you on CD (*note that you have to convert photon fluxes to Janskys first before compiling*):

Lab–Tasks 1:

- What is the meaning of the flux unit **Jansky** ?
- Generate by yourself 5 types of spectra in double–logarithmic form: a low–frequency flux spectrum S_ν and energy distribution νS_ν, a high–energy photon spectrum and high-energy energy distribution, and the global energy distribution νS_ν for all data loaded from your data sets. Comment these different spectra.
- Some portions of the spectrum can be represented in terms of **power–laws** of the following form

$$S_\nu = S_0 \, (\nu/\nu_0)^{-\alpha} \quad , \quad \nu_0 \leq \nu \leq \nu_1 \, . \tag{1}$$

  α is called **spectral index**, and S_0 is the corresponding flux normalisation. How many spectral ranges are necessary to cover the entire spectrum ? Calculate the corresponding spectral indices by using a plot of the flux distribution.
- Consider in particular the gamma–spectrum and try to find an interpretation for the rather complicated behaviour of the spectrum.
- Find out the meaning of COMPTEL, EGRET, WHIPPLE, HEGRA and HESS.

3.2 The Energy–Loss of the Crab Nebula

You can use the plot for the energy distribution to estimate the total energy lost by the Crab Nebula. Remember that $\nu \, S_\nu$ is in units of Watt/m^2. We can therefore integrate the spectrum

$$S = \int_0^\infty S_\nu \, d\nu = \int_0^\infty (\nu \, S_\nu) \, d\log\nu \simeq \sum_{n=0}^{N} (\nu S_\nu)_n \, (\Delta \log\nu)_n \, . \tag{2}$$

Lab–Tasks 2:

- Print out a plot of the energy distribution νS_ν.
- Interprete the power spectrum. In which spectral ranges does the Crab Nebula emit most of the energy ?
- Using equation (2), you can easily estimate the total power S emitted by the Crab Nebula. Compare this value with the solar constant $S_\odot$.
- Use the distance d of the Crab Nebula to determine its total luminosity L. Give this value in solar units.
- Give an interpretation of the infrared bump in the spectrum.

3.3 Properties of Synchrotron Emission

Monoenergetic relativistic electrons (and positrons) spiraling in a magnetic field structure cool by synchrotron emission with a characteristic frequency ν_c, which is related to the **cyclotron frequency** ν_B

$$\nu_c \simeq 0.29\,\nu_0 \quad , \quad \nu_0 = \gamma_e^2\,\nu_B\,\sin\chi\,. \tag{3}$$

γ_e is the Lorentz factor of the electron with energy $E_e = \gamma_e m_e c^2$, and χ is the pitch angle of the electron's helical motion in the magnetic field with field strength B. One finds then the relation

$$\nu_0 = 42\,\mathrm{GHz}\,B_\perp\,\gamma_e^2\,, \tag{4}$$

where $B_\perp$ is the magnetic field perpendicular to the line of sight ($B_\perp$ is given in units of Tesla). The magnetic field in the inner part of the Crab nebula has been found to be of order 30 to 50 nano-Tesla.

Due to this synchrotron cooling the electrons loose energy with the rate

$$\dot{E}_e = \frac{4}{3}\,\sigma_T c\,\frac{B^2}{2\mu_0}\,\gamma_e^2\,. \tag{5}$$

The energy loss is proportional to the energy density in the magnetic field, and it scales with the square of the Lorentz factor γ_e. σ_T is the Thomson scattering cross–section for electrons. From this formula we get a characteristic **cooling time** for electrons, $t_S = \gamma_e/\dot{\gamma}_e$.

It is often assumed in Astronomy that the pitch angles of synchrotron electrons are isotropically distributed. Under this assumption, the electron distribution in the Crab nebula depends on the location $\vec{x}$ in the nebula and on the Lorentz factor

$$n_e(\vec{x},\gamma_e)\,d\gamma_e = n_0(\vec{x})\,N_e(\gamma_e)\,d\gamma_e\,. \tag{6}$$

The energy distribution of the electrons $N_e(\gamma_e)$ is typically a power–law, i.e. $N_e(\gamma_e) \propto \gamma_e^{-p}$ with some energy index p. The radiation flux observed in a telescope is therefore given by the line of sight integration of the local spectral emissivity $j_\nu(B_\perp,\gamma_e)$ over the entire volume of the nebula

$$S_\nu = \frac{1}{4\pi d^2}\int_{\mathrm{nebula}} n_0(\vec{x})\left[\int_0^\infty N_e(\gamma_e) j_\nu(B_\perp,\gamma_e)\,d\gamma_e\right] dV\,. \tag{7}$$

The integration of the synchrotron emissivity j_ν over the energy distribution N_e leads to a power–law in the observed spectrum, $S_\nu \propto \nu^{-\alpha}$, where the spectral index is related to the energy spectral index p over the relation

$$\alpha = \frac{p-1}{2}\,. \tag{8}$$

This explains, why the observed spectrum is a sequence of power–laws.

Lab–Tasks 3:

- Why is the emission from the Crab nebula synchrotron emission (give two arguments).
- Determine the typical Lorentz factors γ_e for the electrons which emit radio, optical, X– and gamma–rays, respectively.
- Determine the typical cooling times t_S for these electrons (radio, optical, X- and gamma-rays). What do we learn from these numbers ?
- Give arguments for the formula (7) by considering the radiation transport equation.
- Find an interpretation for the TeV–spectrum of the Crab nebula.
- What is **inverse Compton emission** ? Which parameters determine the typical energy E_c emitted by inverse Compton processes (search in the literature) ? Look for other sources, where inverse Compton emission is important.

3.4 The Energy Source of the Crab Nebula

So far we have found that the continuum emission of the Crab nebula is due to synchrotron losses of relativistic electrons (and positrons), which have to be refuelled by the central Pulsar. The only energy reservoir of a rotating neutron star is its rotational energy, which can be derived from the observed rotational period $P = 33.3$ ms and the moment of inertia of the neutron star, $I_* \simeq 0.4 M_* R_*^2$. In addition, we know the braking of the rotation from observations, $\dot{P} = 4.22 \times 10^{-13}$ s/s, from pulsar timing.

Lab–Tasks 4:

- Calculate the rotational energy $E_{\rm rot}$ of the Crab Pulsar and from this its energy–loss $\dot{E}_{\rm rot}$. Compare with your number for the total luminosity L of the Crab nebula.
- The observed total luminosity L must be smaller than the rotational energy–loss $\dot{E}_{\rm rot}$. Where is the rest of the energy ?

3.5 The Inner Structure of the Crab Nebula

The Crab Nebula is the archetypal filled–center supernova remnant, or **plerion**. A central pulsar powers each filled-center supernova remnant. Thus, the inner nebula of a plerion is particularly interesting, since it is the site of conversion of pulsar-supplied energy into synchrotron-emitting electrons. The Crab Pulsar generates a relativistic wind which creates a cavity of 10 arcsec around the pulsar (see HST and Chandra images, Scargle's optical hole). At this distance, the pulsar wind is shocked and particles get accelerated to even higher energies. This structure is seen as a torus-like ring around the Pulsar, since the wind is not spherically symmetric, but more concentrated towards the equatorial plane. The inner Crab ring is one light year in diameter; in Vela it is 0.1 light year. Chandra images reveal, for the first time, an X-ray inner ring within the X-ray torus, the suggestion of a hollow-tube structure for the torus, and X-ray knots along the inner ring and perhaps along the inward extension of the X-ray jet.

References

[1] F. Aharonian et al. (HEGRA Coll.) 2004: *The Crab Nebula and Pulsar between 500 GeV and 80 TeV: Observations with the HEGRA stereoscopic air Cherenkov telescopes*, ApJ 614, 897

[2] J. Kuiper et al. 2001: *The Crab pulsar in the 0.75-30 MeV range as seen by CGRO COMPTEL*, A&A 378, 918; astro-ph/0109200

[3] http://cossc.gsfc.nasa.gov/docs/cgro/index.html

[4] http://www.mpi-hd.mpg.de/hfm/HESS/HESS.html

[5] http://heasarc.gsfc.nasa.gov/docs/xte/XTE.html

4 Appendix: Data Sets

Frequency/Wavelength	**Spectral Flux**
100 MHz	2080 Jy
1 GHz	1040 Jy
22 GHz	411 Jy
250 GHz	204 Jy
0.3 mm	120 Jy
100 μm	184 Jy
60 μm	210 Jy
25 μm	67 Jy
12 μm	37 Jy
2.2 μm	9.12 Jy
1.6 μm	9.33 Jy
740 nm	5.25 Jy
470 nm	4.26 Jy
330 nm	1.91 Jy
155 nm	1.37 Jy
2.463 keV	1.351 ph/cm^2 s keV
2.866 keV	0.9778 ”
4.210 keV	0.4412 ”
6.069 keV	0.2052 ”
8.124 keV	0.1135 ”
10.43 keV	0.0677 ”
15.07 keV	0.0317 ”
22.37 keV	0.0143 ”
44.62 keV	0.0041 ”
67.75 keV	0.0011 ”

Table 1: Spectral fluxes in the low–frequency regime. X–ray data from RXTE [5].

Energy window [MeV]	**Nebula Photon Flux [ph/cm^2 s MeV]**
0.75 - 1.00	(2.585 ± 0.089)E-03
1.00 - 1.25	(1.563 ± 0.054)E-03
1.25 - 1.50	(1.127 ± 0.043)E-03
1.50 - 2.00	(0.617 ± 0.020)E-03
2.00 - 2.50	(0.306 ± 0.014)E-03
2.50 - 3.00	(0.217 ± 0.010)E-03
3.0 - 4.00	(1.312 ± 0.055)E-04
4.00 - 6.00	(0.613 ± 0.022)E-04
6.00 - 8.00	(0.284 ± 0.014)E-04
8.00 - 10.0	(1.637 ± 0.082)E-05
10.0 - 15.0	(0.734 ± 0.033)E-05
15.0 - 30.0	(0.201 ± 0.013)E-05

Table 2: COMPTEL spectra of the Crab nebula [2].

Energy	**Photon Flux [ph/cm^2 s MeV]**
50 MeV	1.5E-07
120 MeV	9.0E-09
200 MeV	1.5E-09
400 MeV	2.0E-10
800 MeV	3.0E-11
2.0 GeV	1.2E-11
8.0 GeV	3.0E-12

Table 3: EGRET data [3].

Energy [TeV]	**Photon Flux [ph/cm^2 s TeV]**
0.365	(1.97 ± 1.17)E-10
0.487	(1.76 ± 0.24)E-10
0.649	(8.78 ± 0.53)E-11
0.866	(4.02 ± 0.13)E-11
1.155	(1.87 ± 0.09)E-11
1.540	(9.05 ± 0.26)E-12
2.054	(4.51 ± 0.12)E-12
2.738	(2.16 ± 0.07)E-12
3.652	(9.33 ± 0.36)E-13
4.870	(4.18 ± 0.20)E-13
6.494	(1.93 ± 0.12)E-13
8.660	(1.02 ± 0.07)E-13
13.335	(3.28 ± 0.31)E-14
23.714	(5.28 ± 0.70)E-15
42.170	(1.10 ± 0.25)E-16
74.989	(2.05 ± 1.01)E-16

Table 4: HEGRA data for Crab nebula [1].