Task 7.1

Distance of the pulsar in the Crab Nebula and interstellar electron density

Material

1. Reprint of red filter images of the Crab Nebula
2. Reprint of the spectrum of the Crab Nebula (slit along the semi-major axis).
3. Recordings of radio signals of three pulsars at three different frequencies.

Tasks

1. Determine the "kinematic" age of the Crab Nebula.
2. Determine the distance of the Crab Nebula from the proper motion of knots of emission and the expansion velocity of the nebula along the line of sight.
3. Determine the periods and
4. the dispersion of PSR 0809+74, PSR 0950+08 and PSR 0329+54
5. Determine the interstellar electron density from distance and dispersion of the pulsar in the Crab Nebula. The dispersion of the pulsar in the Crab Nebula is $nd = 56 cm^{-3}$ pc.
6. Estimate the distance to PSR 0809+74, PSR 0950+08 and PSR 0329+54 with the dispersion.

Hints

to 1. Age of the Crab Nebula
The Crab Nebula is the expanding supernova remnant (SNR) of the "Chinese supernova" of the year 1054. You shall determine the proper motion $\mu = \frac{\Delta x}{\Delta t}$ for a number of knots of emission from the red filter images (time interval $\Delta t = 34$a). The linear shift will be measured with respect to the pulsar in the Crab Nebula. The scale on both images can be determined from the two marked stars, which are separated by $576''$. The pulsar in the Crab Nebula is the southern of the two stars in the center of the nebula. We assume that matter ejected by the supernova moves with a constant velocity; the gas ejected with the highest velocity has the largest distance to the central star. Using the measured angular velocity of each knot and the distance to the pulsar the "kinematic" age of the Crab Nebula and hence the year of the supernova explosion can be estimated. Check if your estimate corresponds (within the errors) to the dates found in Chinese sources.

to 2. Distance of the Crab Nebula
If one measures the expansion velocity of the nebula along the line of sight (radial velocity in km s^{-1}), the distance of the pulsar in the Crab Nebula can be determined from the angular velocity and the linear velocity. The expansion velocity can be determined from the splitting of the [O II] $\lambda 3727$ line in the enlarged spectrum (slit along the semi-major axis) of the Crab Nebula.

to 3. Pulsar periods
From the recordings you can determine the pulsar periods of PSR 0809+74, PSR 0950+08 and PSR 0329+54. The highest accuracy can be achieved by using the widest separated pulses divided by the number of periods. Since the period is a characteristic quantity of a pulsar, it can be measured separately at three different frequencies (see Scheffler & Elsässer, Physik der Sterne und der Sonne).

to 4. Dispersion
Radio pulses emitted at different frequencies by the pulsar move with different velocities through the interstellar medium due to interaction with free electrons. Pulses with a lower frequencies are more delayed by the free electrons and hence later detected (see also frequency dependence in the recordings). This effect is called "pulse dispersion". The delay is proportional to the number of electrons n [cm^{-3}] and the distance d [pc] of the pulsar. The quantity nd is called dispersion. The equation to calculate differences of travel time Δt for two frequencies f_1 and f_2 is

$$\Delta t = 4150 * nd * \left(\frac{1}{f_1^2} - \frac{1}{f_2^2} \right) \tag{1}$$

with Δt in s, n in cm^{-3}, d in pc and f_1,f_2 in MHz (see for the derivation of the equation e.g. Scheffler & Elsässer, Physik der Sterne und der Sonne). Determine the dispersion of the three pulsars using the recordings.

to 5. Electron density in the interstellar medium
Equation (1) can be used to determine the electron density in the interstellar medium with known distance of the pulsar. The dispersion of the pulsar in the Crab Nebula is $nd = 56 cm^{-3}$ pc. Determine the mean electron density using the distance derived in task 2 along the line of sight to the Crab Nebula.

to 6. Pulsar distances
Equation (1) is often used for distance determination of pulsars. Assuming a mean electron density $n \approx 0.03 cm^{-3}$ the distance of PSR 0809+74, PSR 0950+08 and PSR 0329+54 shall be estimated.

References

[1] Scheffler/Elsässer "Physik der Sterne und der Sonne"