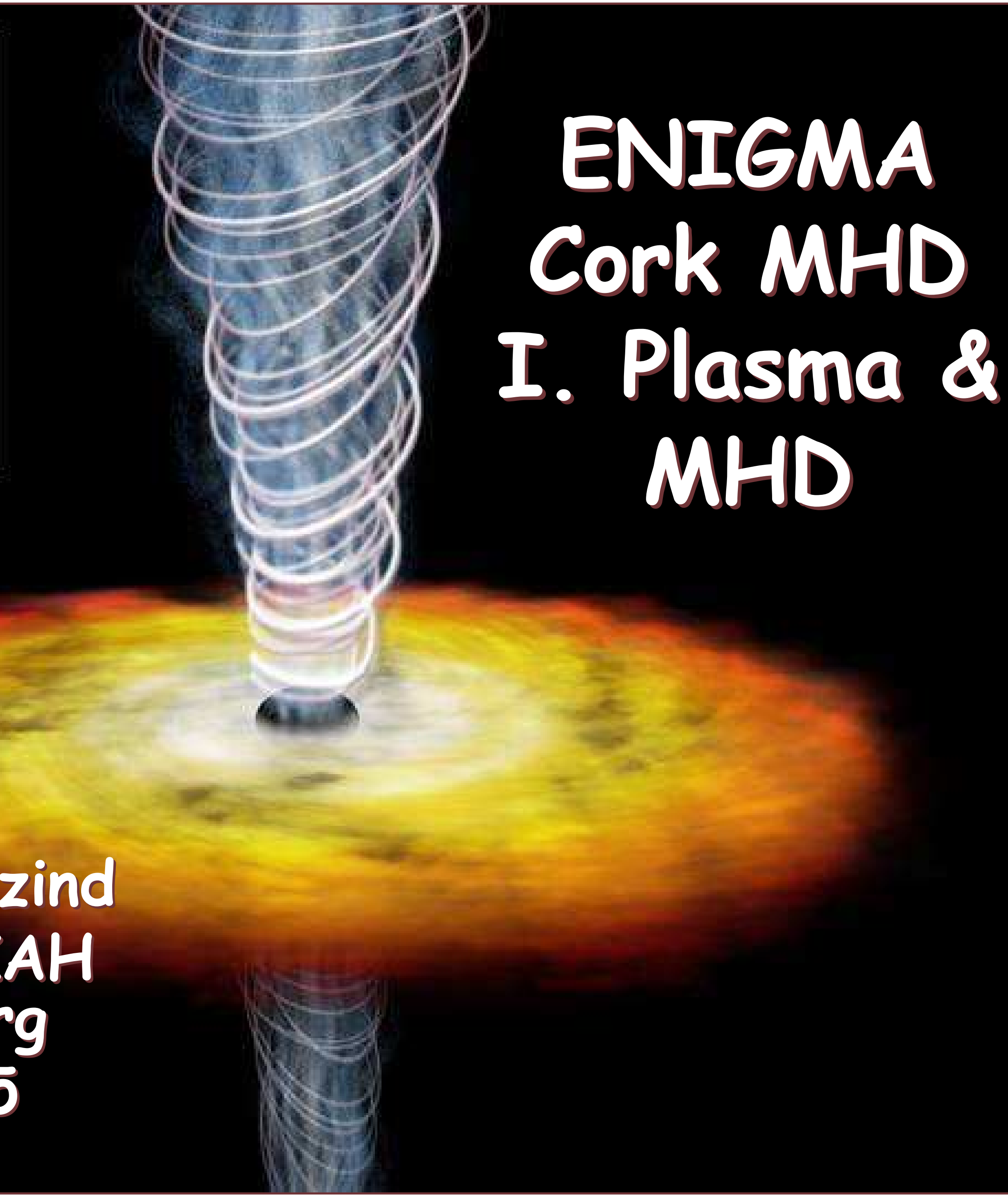


ENIGMA Cork MHD I. Plasma & MHD

Max Camenzind
@ LSW, ZAH
Heidelberg
11-2005

Pre-Remarks

- The motion of gases is dictated by hydrodynamics (Lab applications, aircraft wings, …, baryons in the cosmos). Numerical applications go under the name of **Astro-Hydrodynamics** (with many codes available, even relativistic ones).
- In many space plasmas, however, magnetic fields are involved (solar corona and solar wind e.g., everywhere with synchrotron emission) → field of **Magnetohydrodynamics (MHD).** A few codes are available for astrophysical applications – but this field is not yet maturated.
- Still problems with coordinate systems !

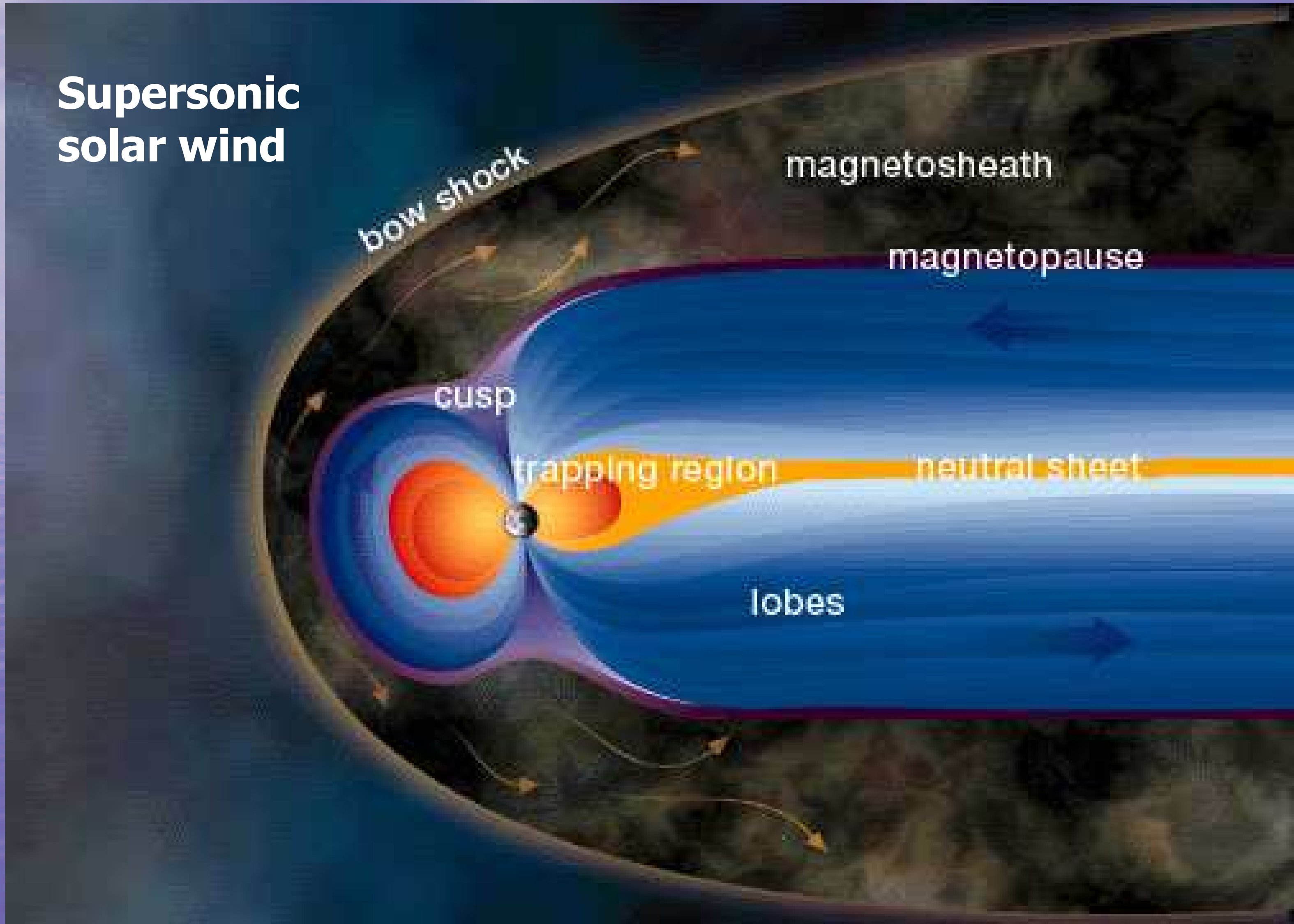
Plan for the Lectures

- I. What is MHD ? What can we learn by performing MHD simulations ?
  → Basics: Space Plasmas and MHD
- II. Newtonian Ideal MHD and Numerical Codes
  → Classical public-domain codes
- III. Applications and Visualisations

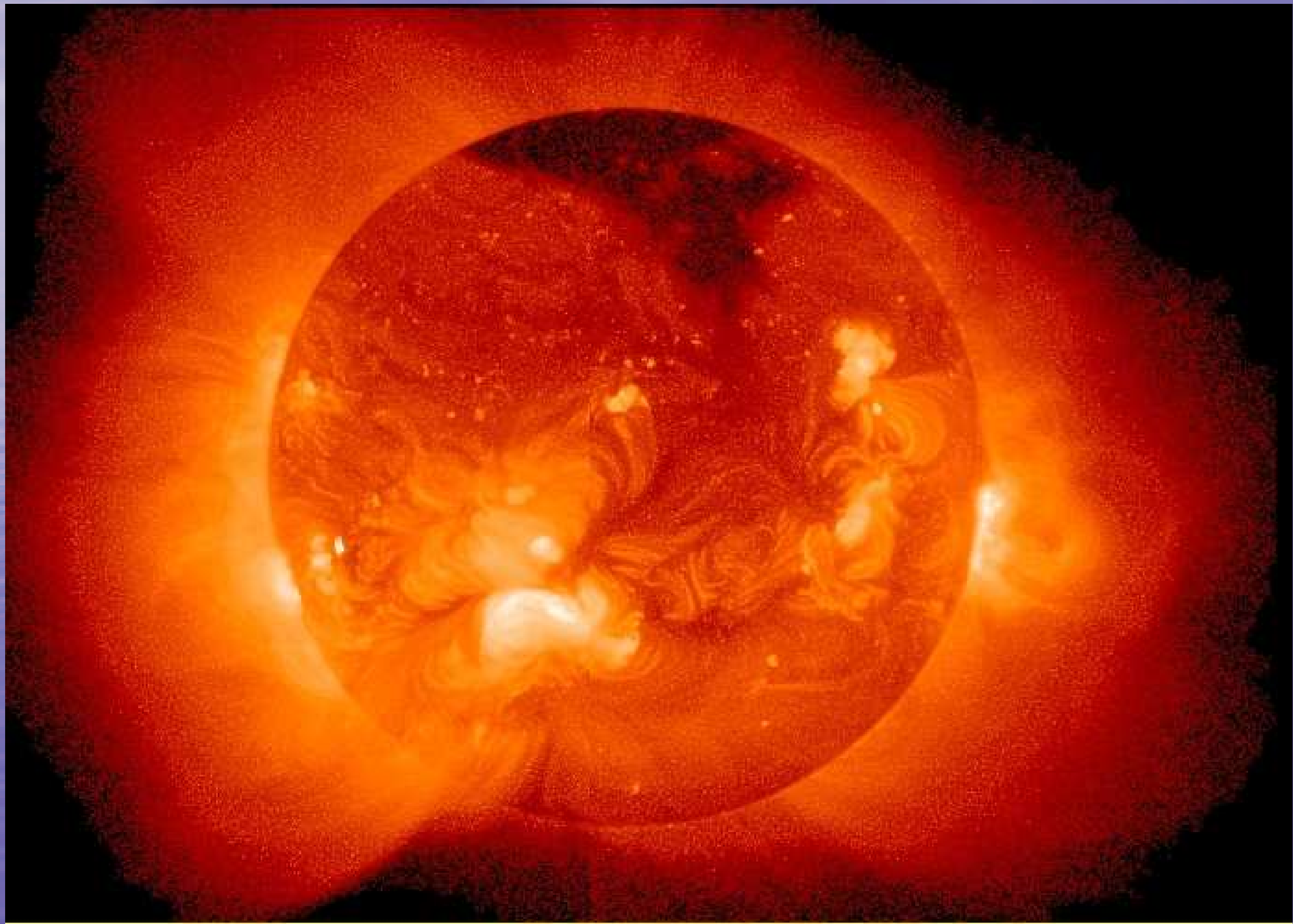
I. Plasma and MHD

- Solar Corona – best studied thermal plasma
- Structure of Extragalactic Jets reveal non-thermal plasmas – even relativistic !
- Jets fill Bubbles in Clusters with non-thermal plasma, magnetic fields and heavy elements
- Maxwell's Equations as Part of MHD
- Maxwell's Equations extended to Black Holes
  → Question of Jet Formation in Blazars
- → Application: The Blandford-Znajek Mechanism (1977) for the Launch of Relativistic Jets

Solar Wind & Earth`s Magnetopshere



Nearest Plasma - Solar Corona

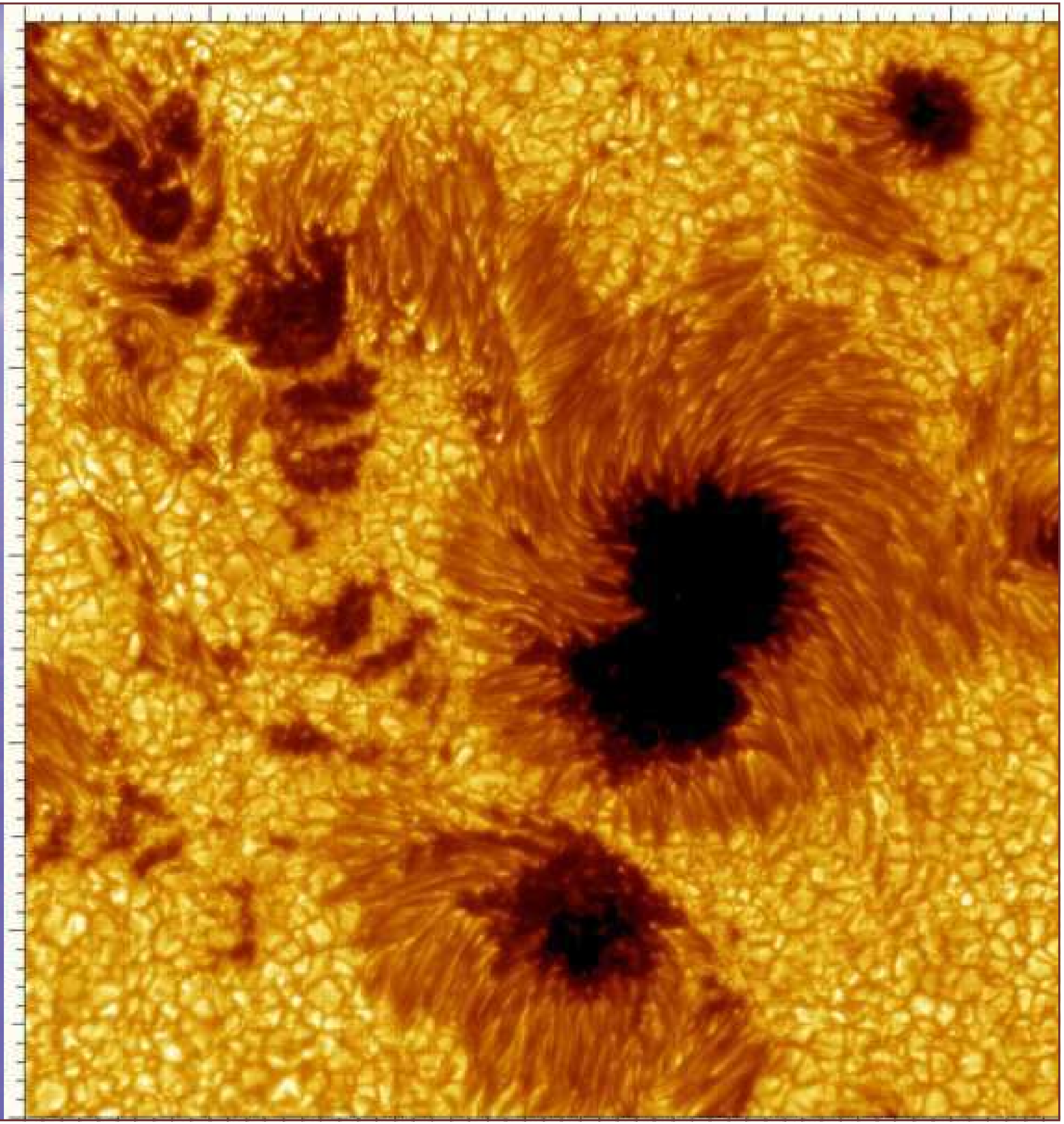


Stunning Image

(Swedish telescope)

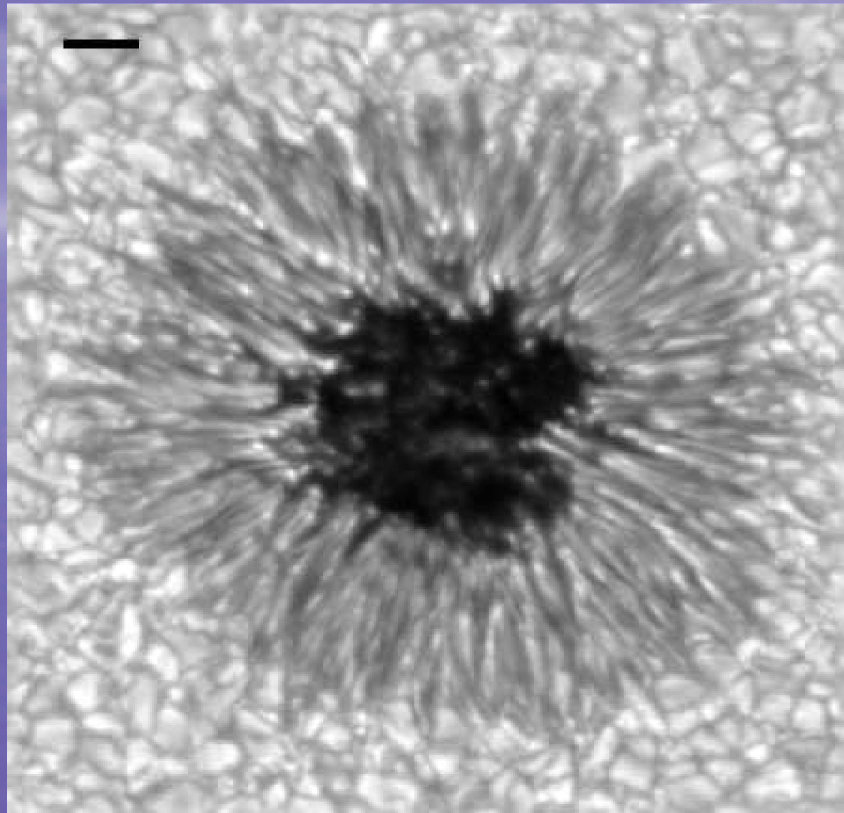
[Scharmer & van der Voort]

Close-up of **penumbral structure** created by magnetic fields



Magnetic Field Effects

A Sunspot e.g.



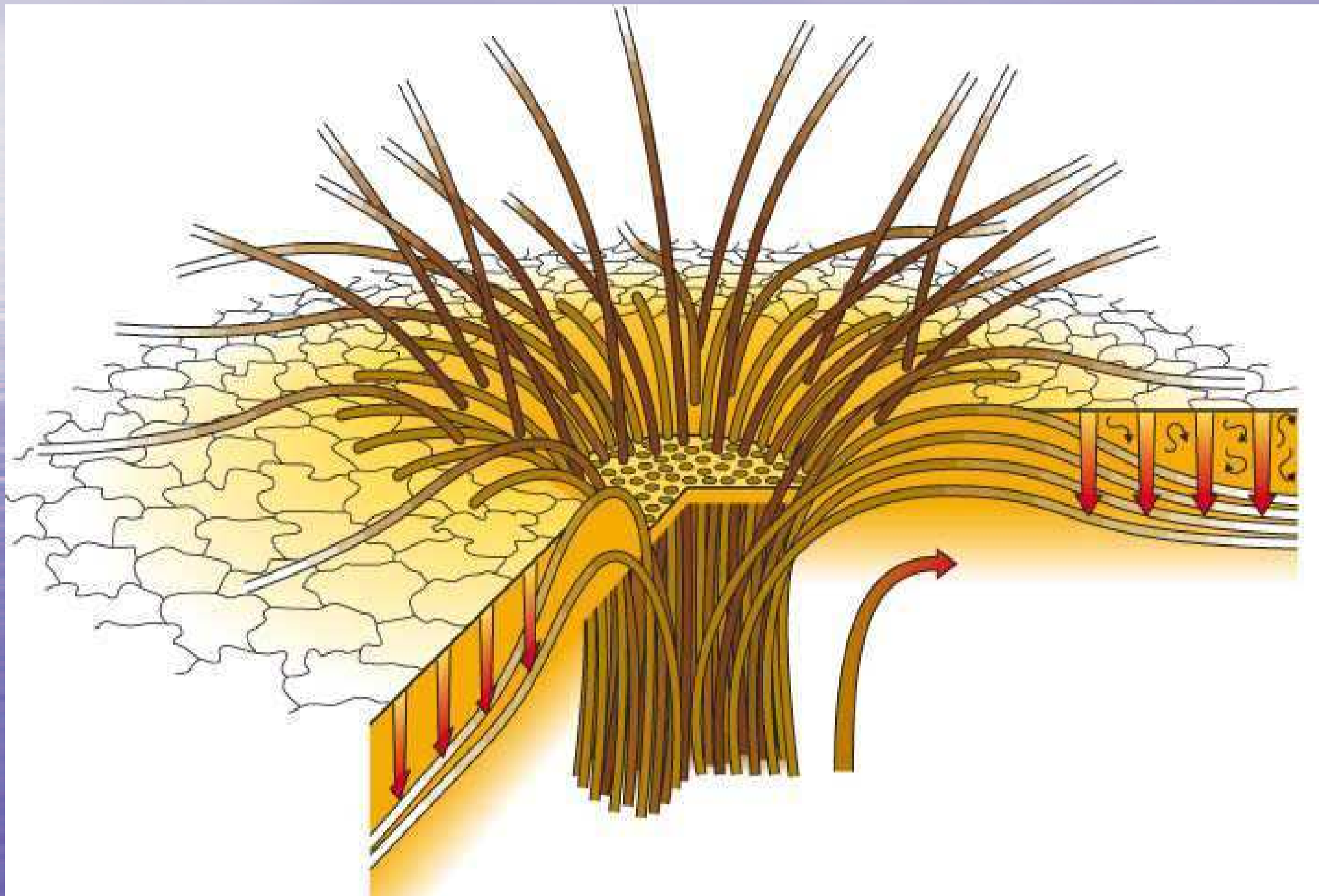
B exerts a force:

-- creates intricate structure

*What is equilibrium ? / nature of instabilities *

SunSpot Model

Penumbra - a mixture of interlocked field lines

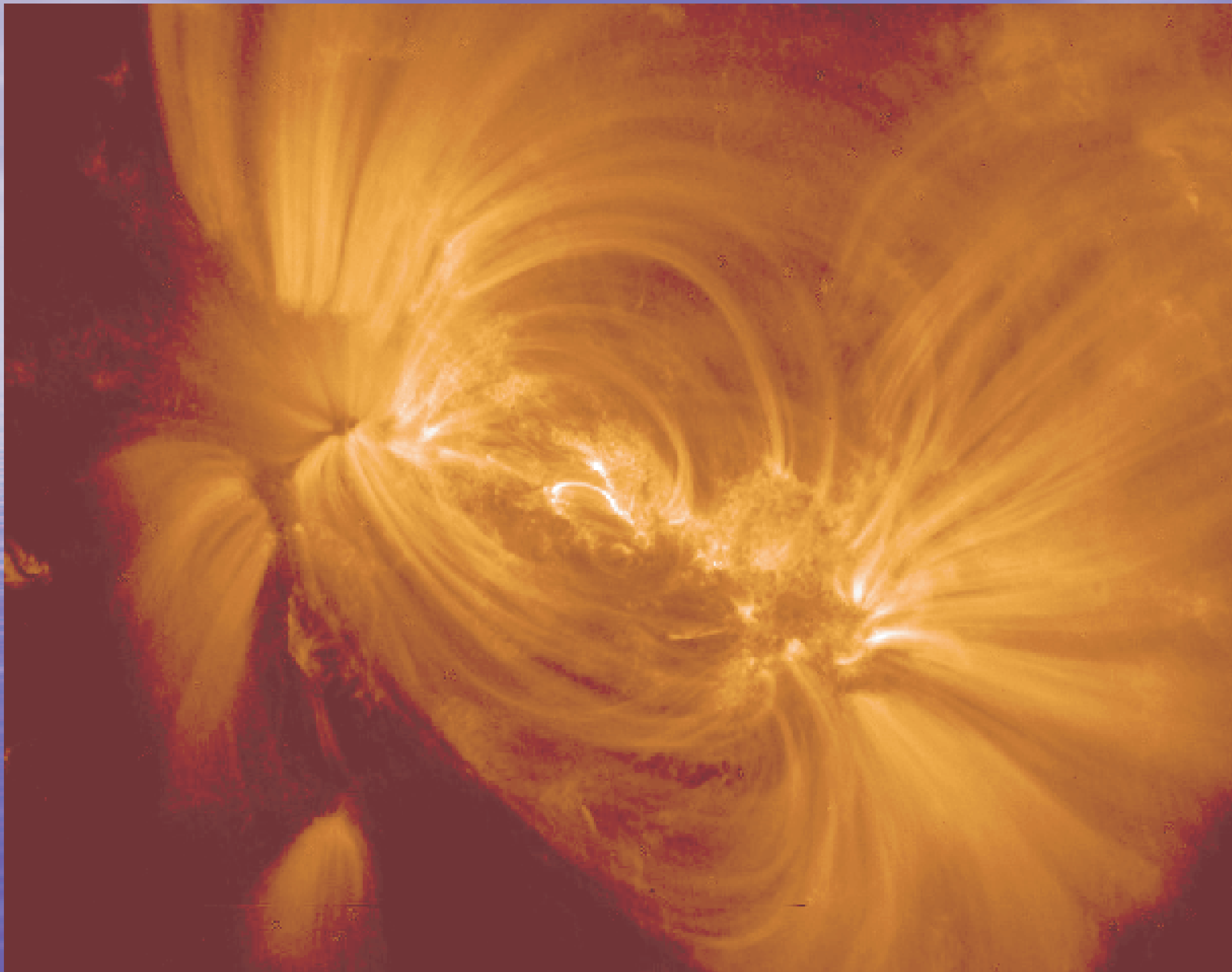


Dark filaments-
(low)

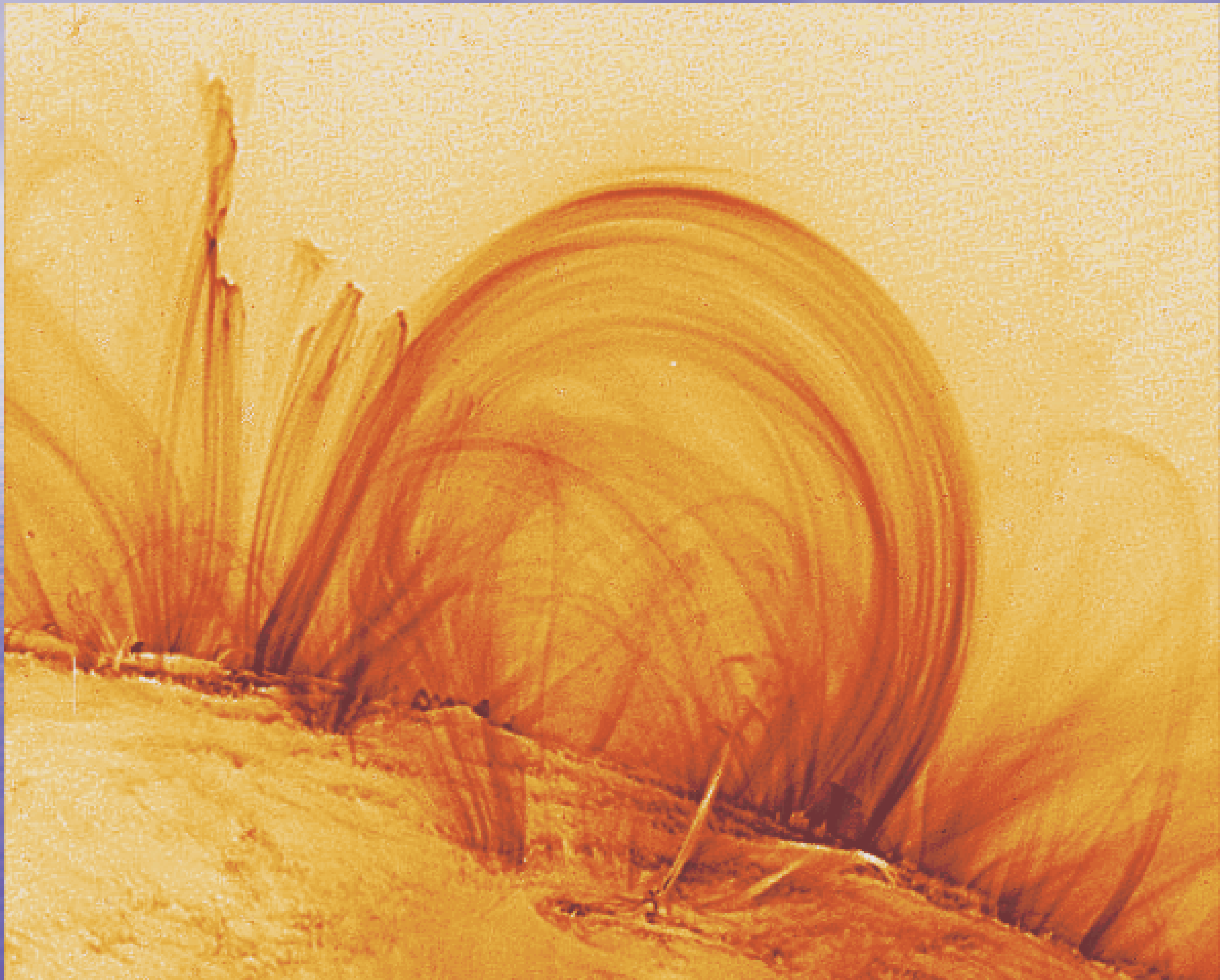
held down by granule flux pumping

Bright filaments-
(high)

TRACE (Active region) - from above



TRACE - from side - intricate structure



Not isolated coronal loops -

plasma that is at one temperature of 1.5 MK

Magnetohydrodynamics

- **MHD - the study of the interaction between a magnetic field and a plasma, treated as a continuous medium**

- **The assumption of a continuous medium is valid for length-scales (Debye Length), completely ionized**

$$L >> 300\left(\frac{T}{10^6 K}\right)^2\left(\frac{n}{10^{17} m^{-3}}\right)^{-1} km$$

  - **Chromosphere** $\left(T = 10^4, n = 10^{20}\right)\ L >> 3\,cm$
  - **Corona** $\left(T = 10^6, n = 10^{16}\right)\ L >> 30 km$

Plasma Beta & Alfven Speed

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j}\times\mathbf{B} + \rho\mathbf{g}$$

(1) **(2)** **(3)** **(4)**

(i) $$\frac{(2)}{(3)} = \beta = \frac{p}{B^2/(2\mu)}$$ * **Plasma beta** *

When $\beta << 1$, $\mathbf{j}\times\mathbf{B}$ dominates

(ii) $$(1) \approx (3) \rightarrow v \approx v_A = \frac{B}{\sqrt{\mu\rho}}$$ ***Alfvén speed***

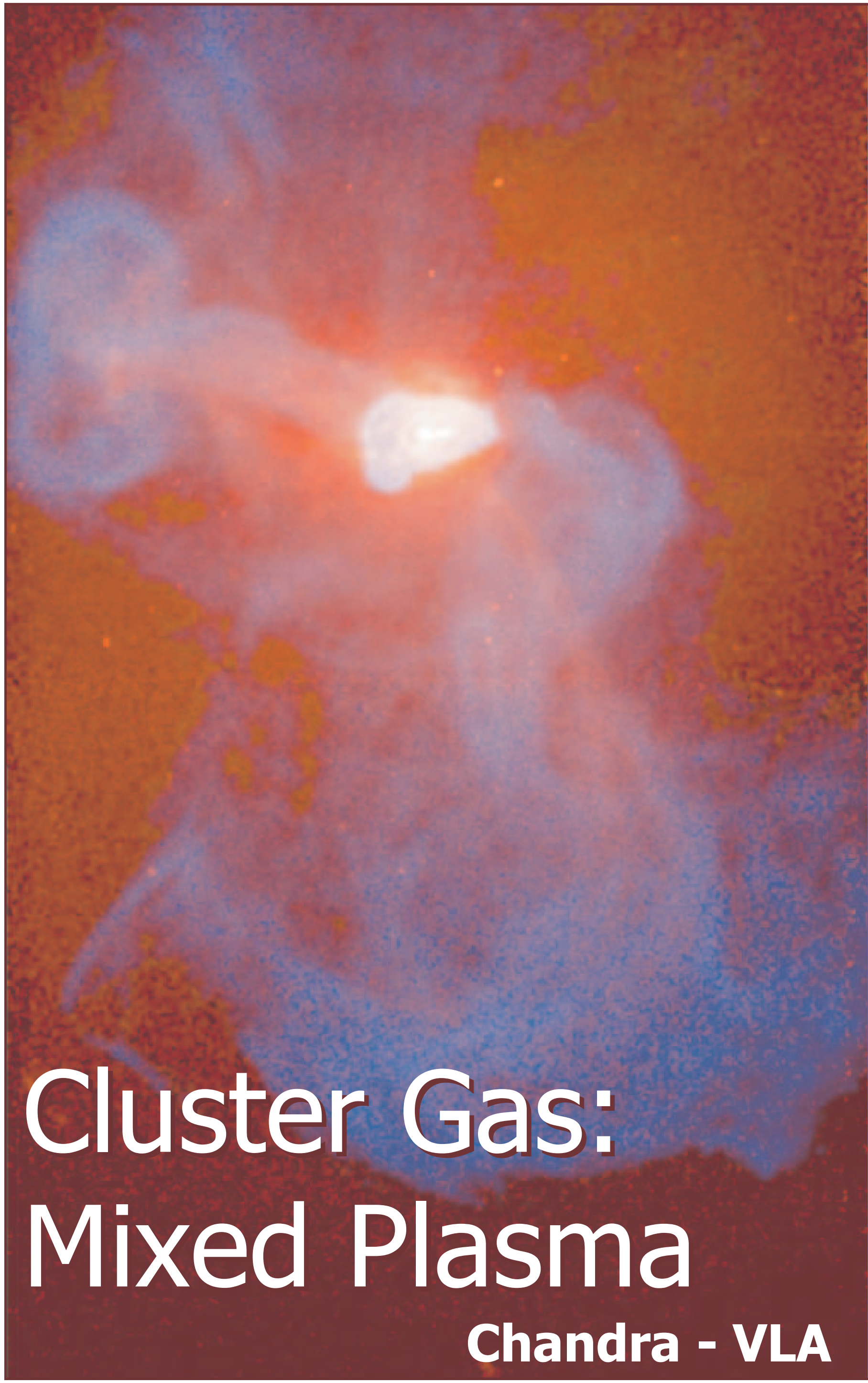
Typical Values Solar Plasma

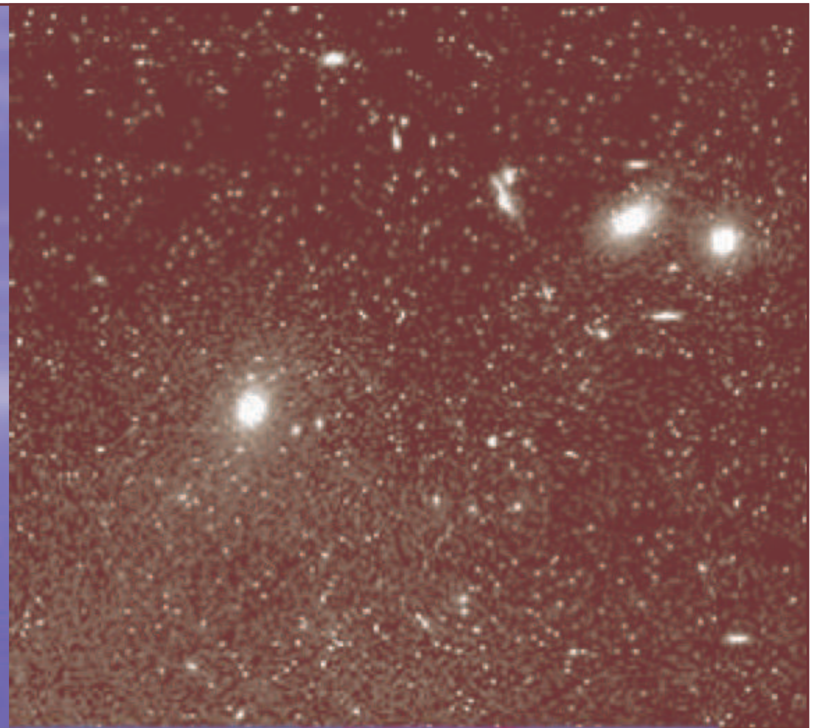
	Photosphere	Chromosphere	Corona
N (m^{-3})	10^{23}	10^{20}	10^{15}
T (K)	6000	10^{4}	10^{6}
B (G)	5 - 10^{3}	100	10
Plasma β	10^{6} - 1	10^{-1}	10^{-3}
v_A (km/s)	0.05 - 10	10	10^{3}

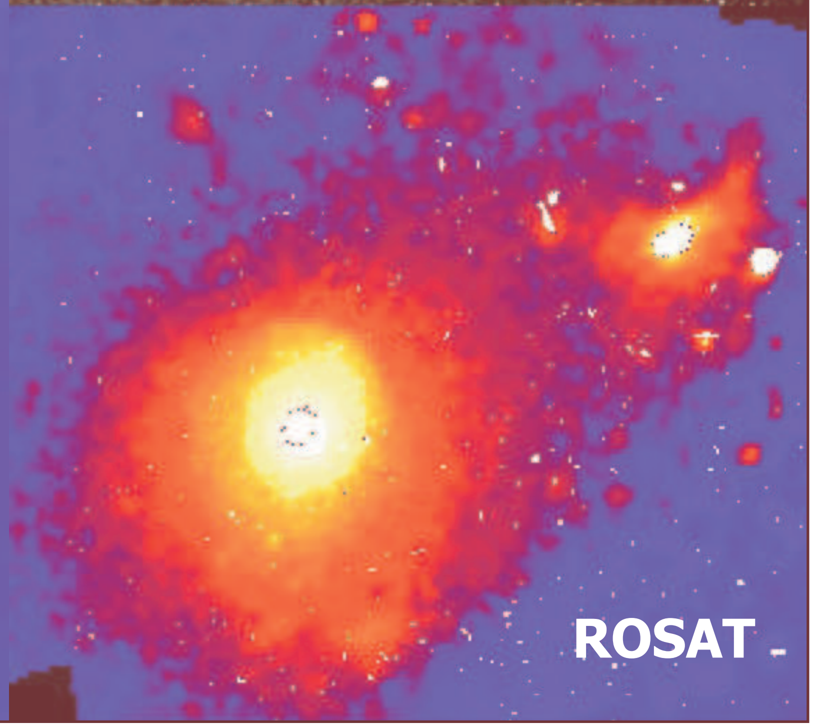
N (m^{-3}) = 10^6 N (cm^{-3}), B (G) = 10^4 B (tesla)

$\beta = 3.5 \times 10^{-21} N\, T/B^2$, $v_A = 2 \times 10^9 B/N^{1/2}$

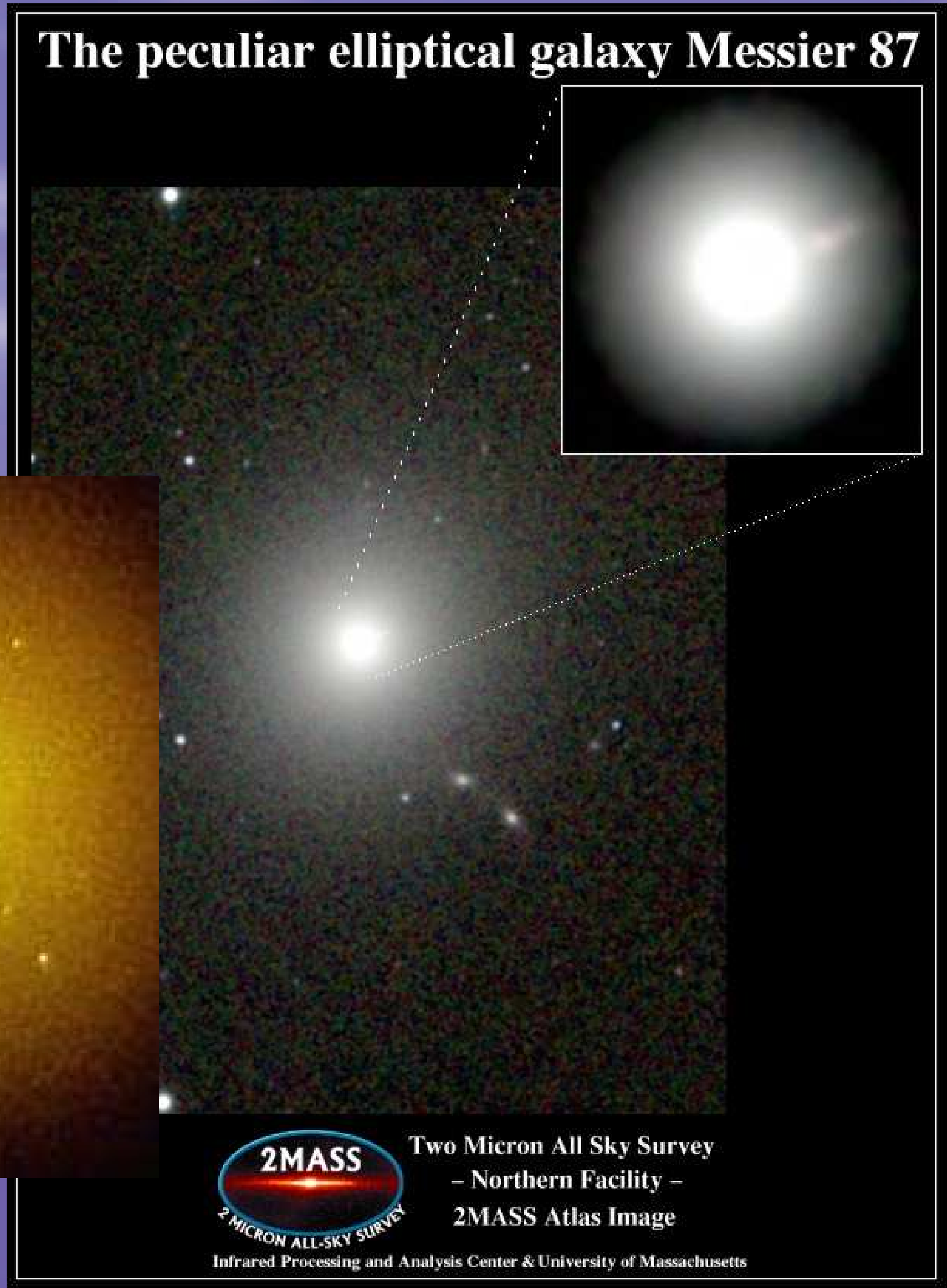
Cluster Gas: Mixed Plasma

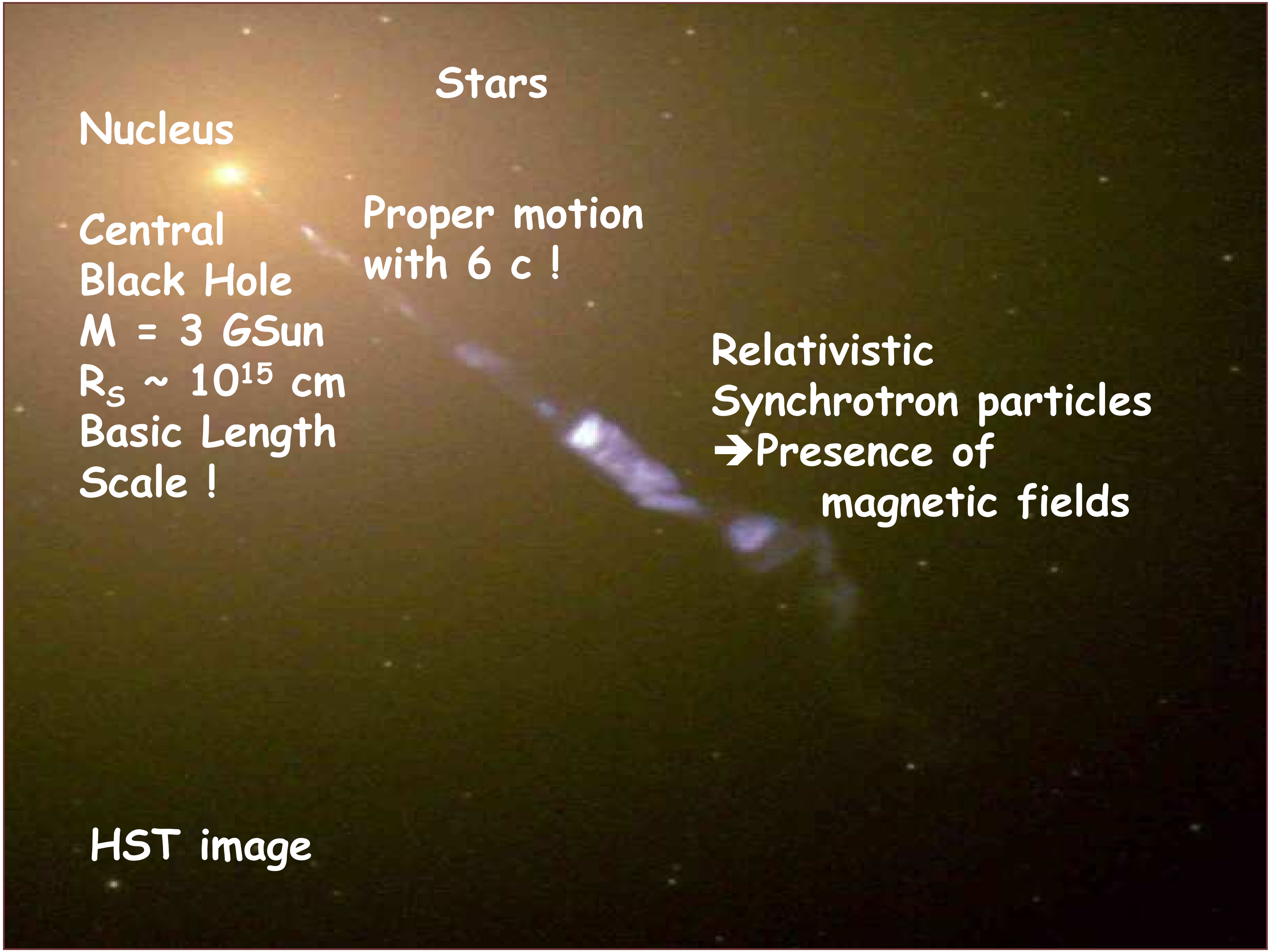


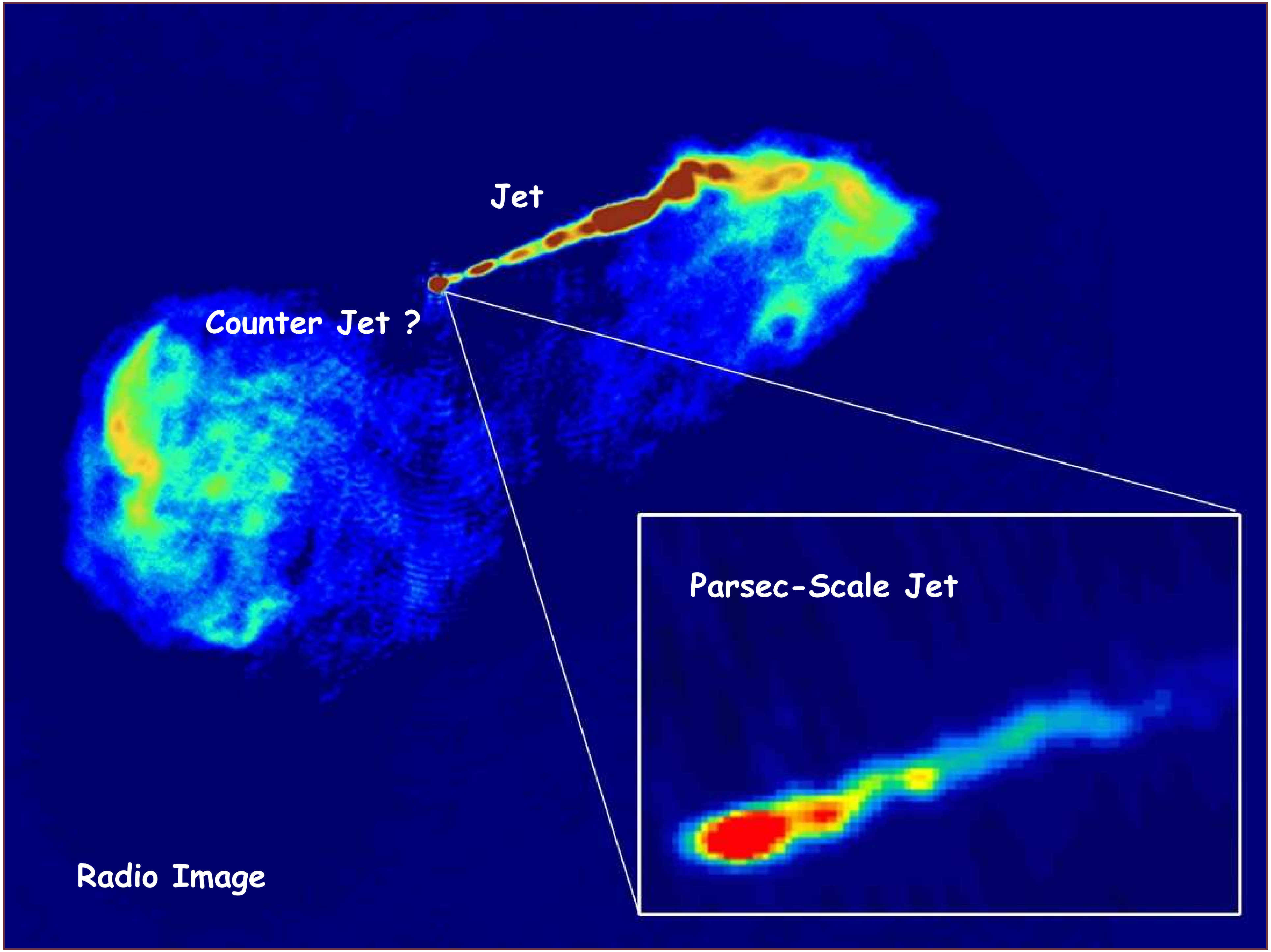




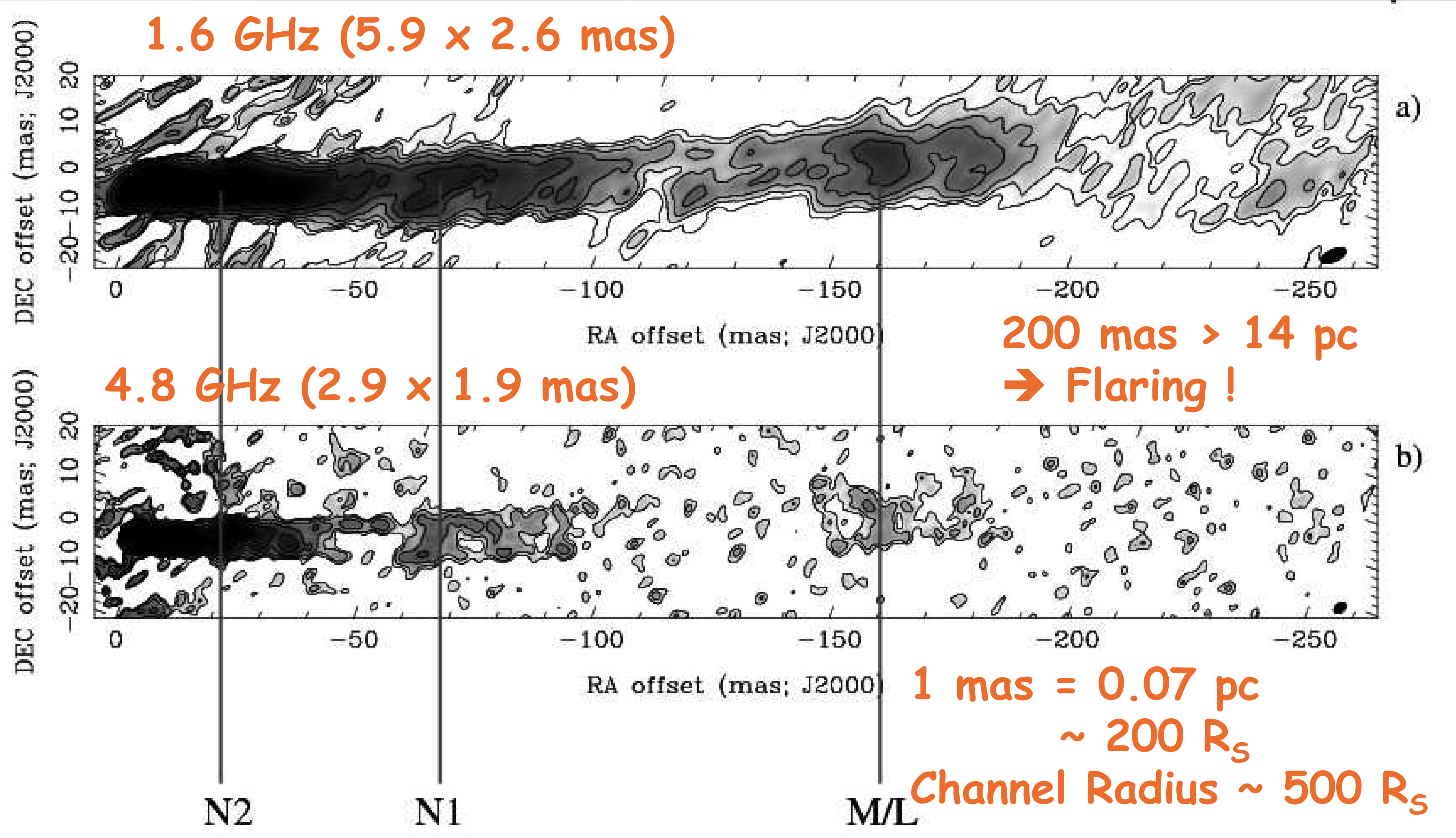
Messier 87 Central Galaxy in Virgo



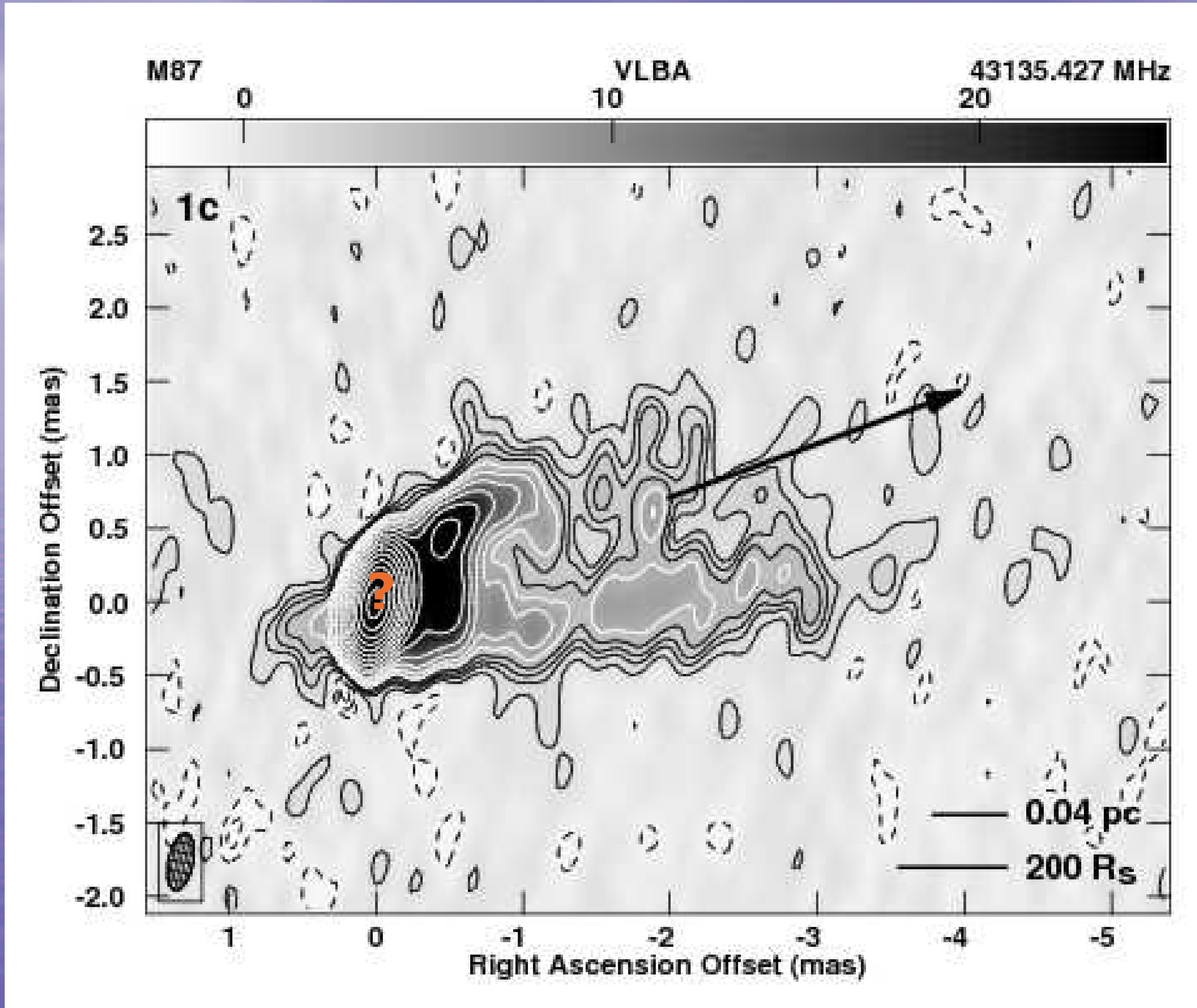

Stars
Nucleus
Central
Black Hole
M = 3 GSun
$R_S \sim 10^{15}$ cm
Basic Length
Scale !
Proper motion
with 6 c !
Relativistic
Synchrotron particles
→Presence of
magnetic fields
HST image

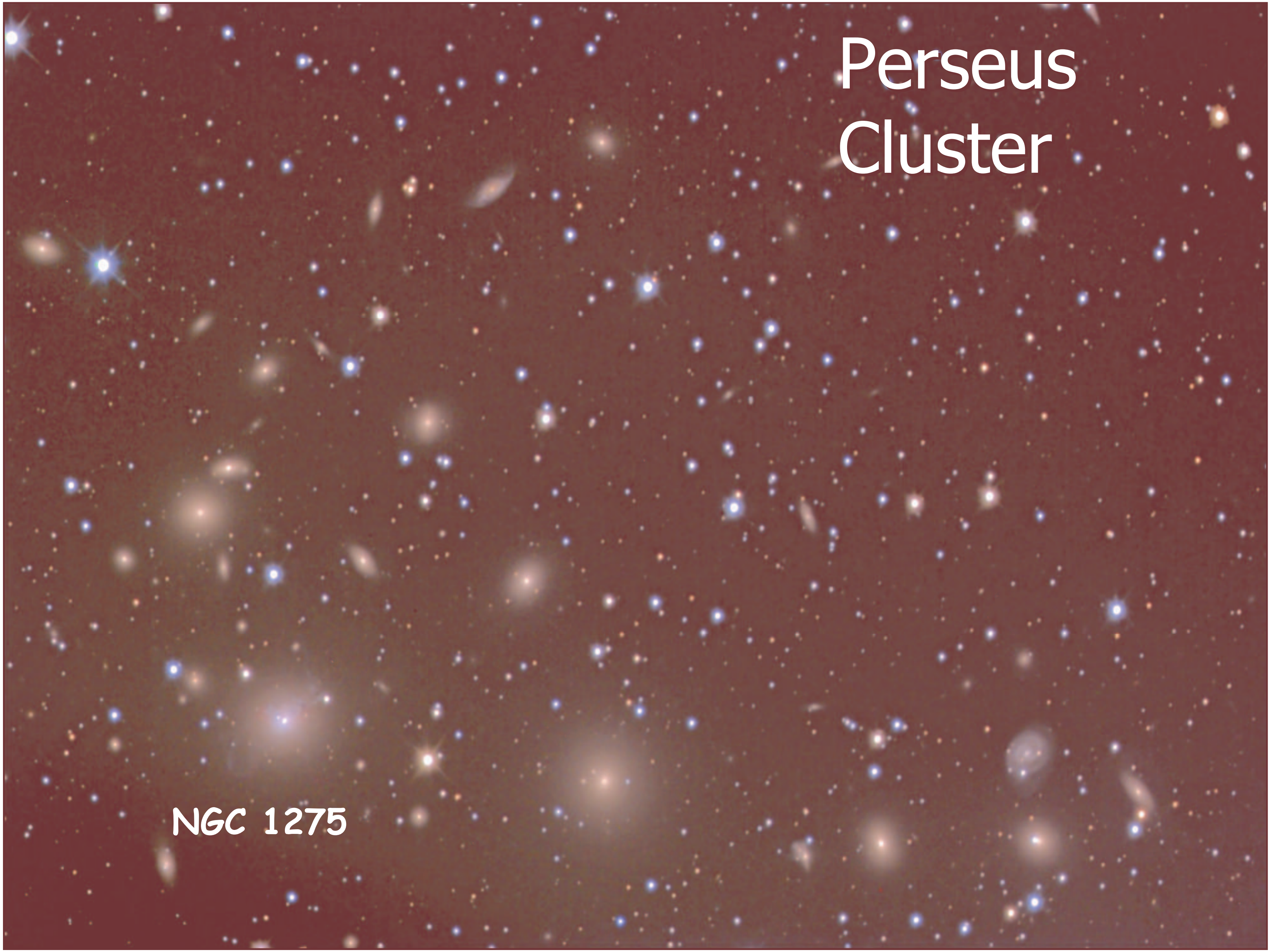

Jet
Counter Jet ?
Parsec-Scale Jet
Radio Image

M 87 – VLBA ➔ Collimated Plasma

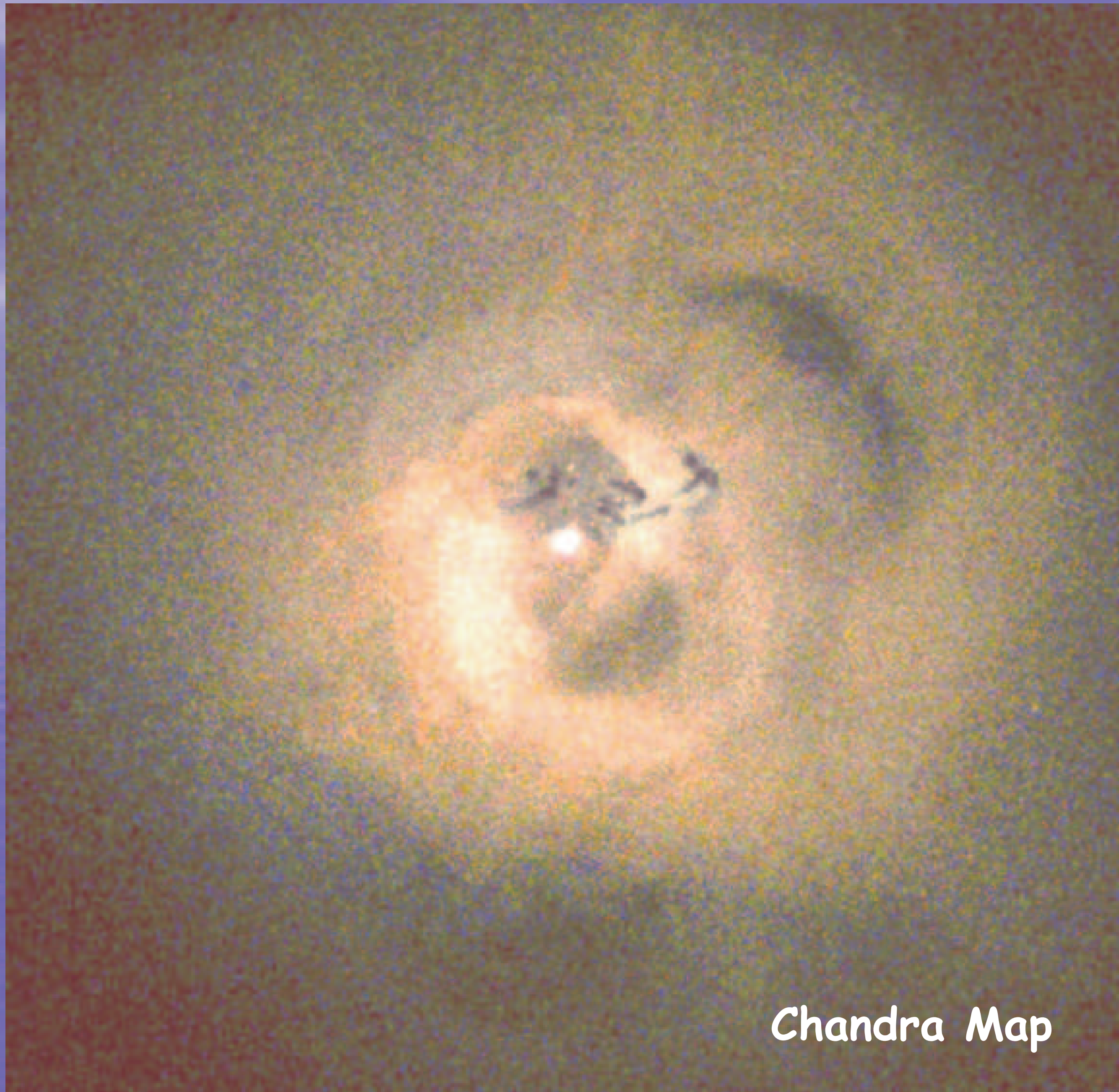


Dodson et al. 2005


M87
VLBA
43135.427 MHz
0
10
20
1c
?
0.04 pc
200 Rs
Declination Offset (mas)
Right Ascension Offset (mas)


Perseus
Cluster
NGC 1275

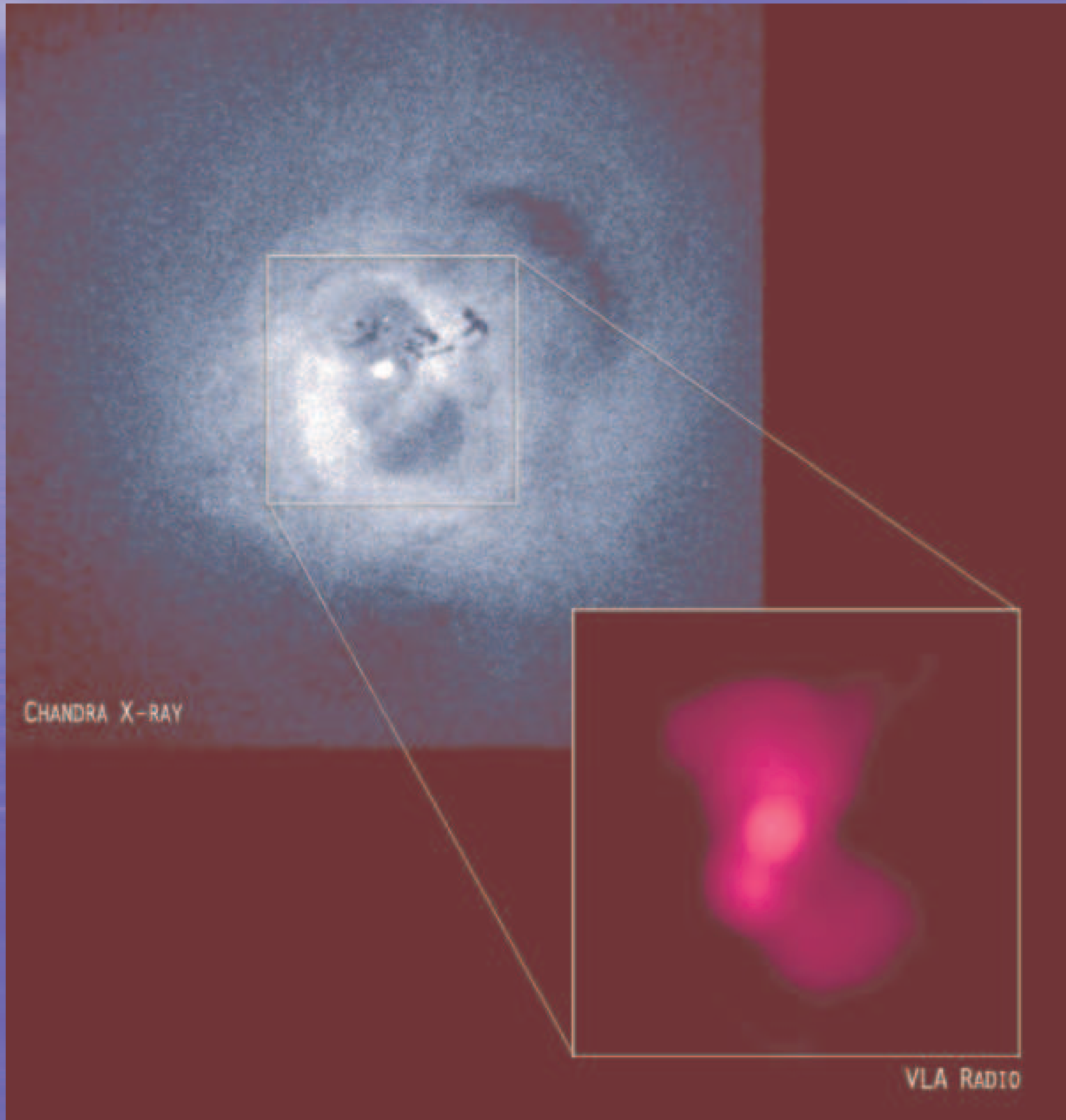
Perseus Cluster (Radio Plasma pushes the X-Ray Gas)



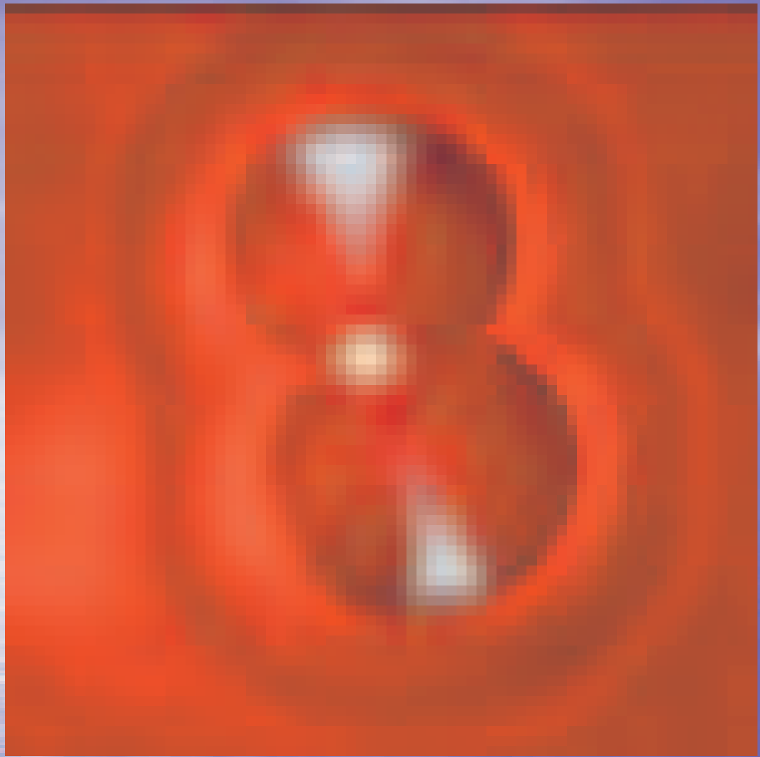
When such a galaxy and black hole are at the centre of a cluster of galaxies, the jets plough into the Intra Cluster Medium (ICM).

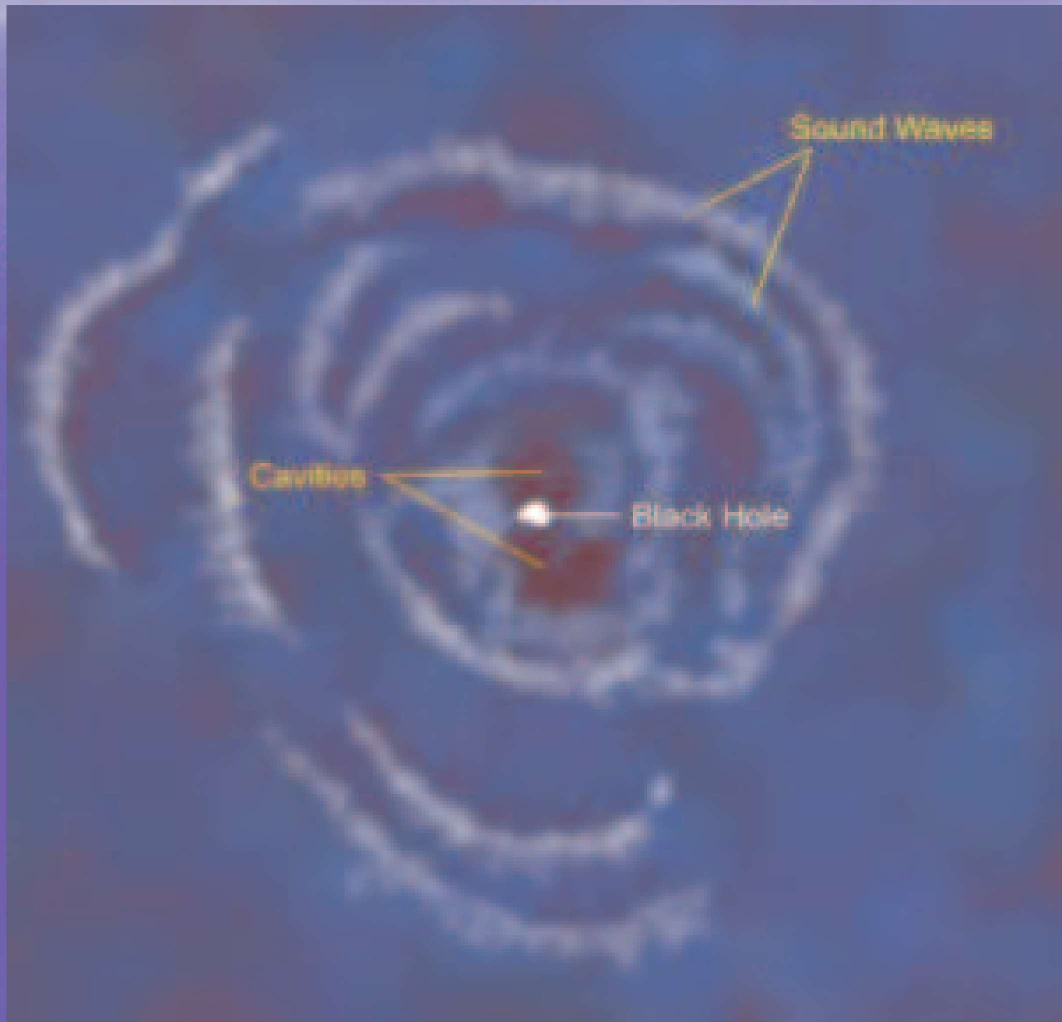
➔

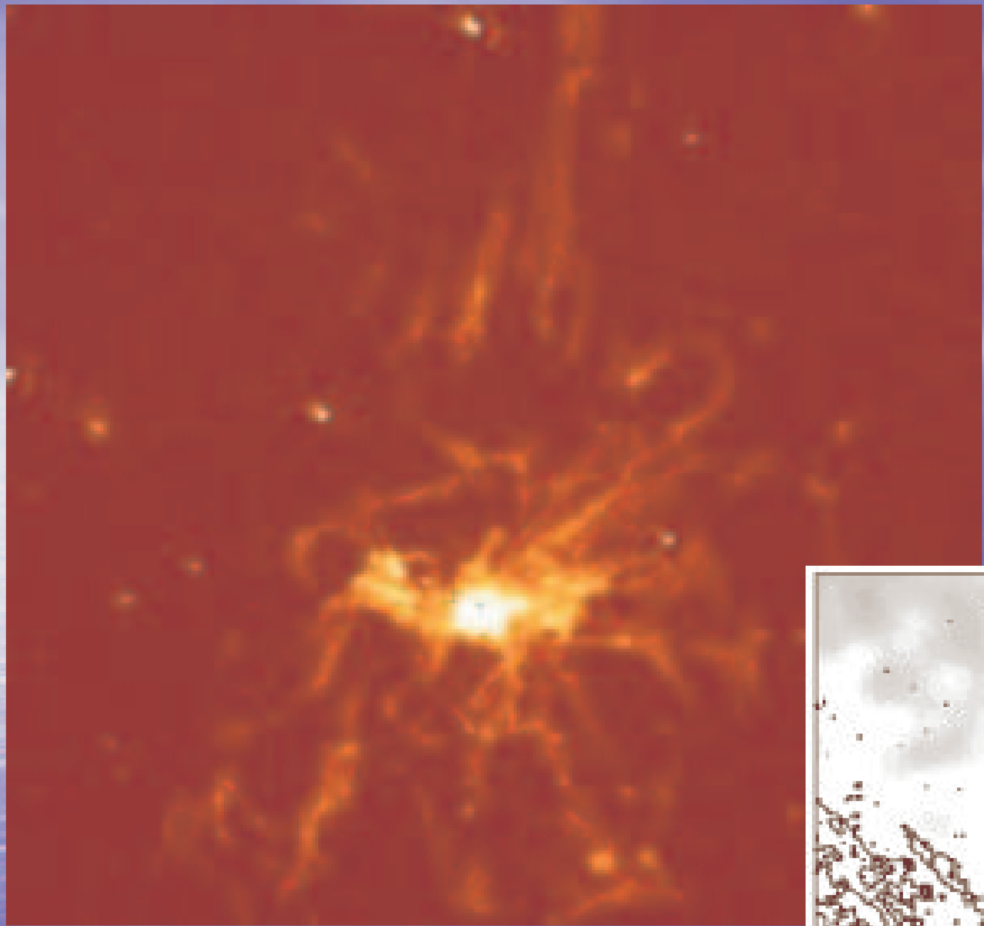
The two types of gas (plasma) do not mix very well, and the jets inflate bubbles in the ICM.



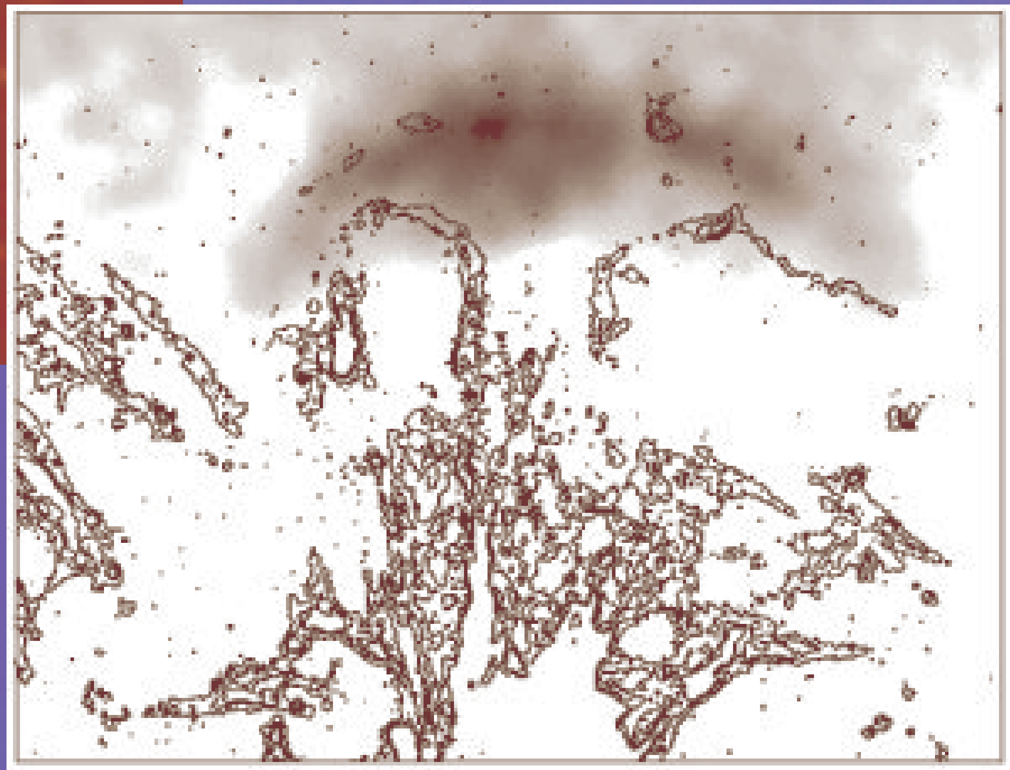
Bubble Inflation by Jets



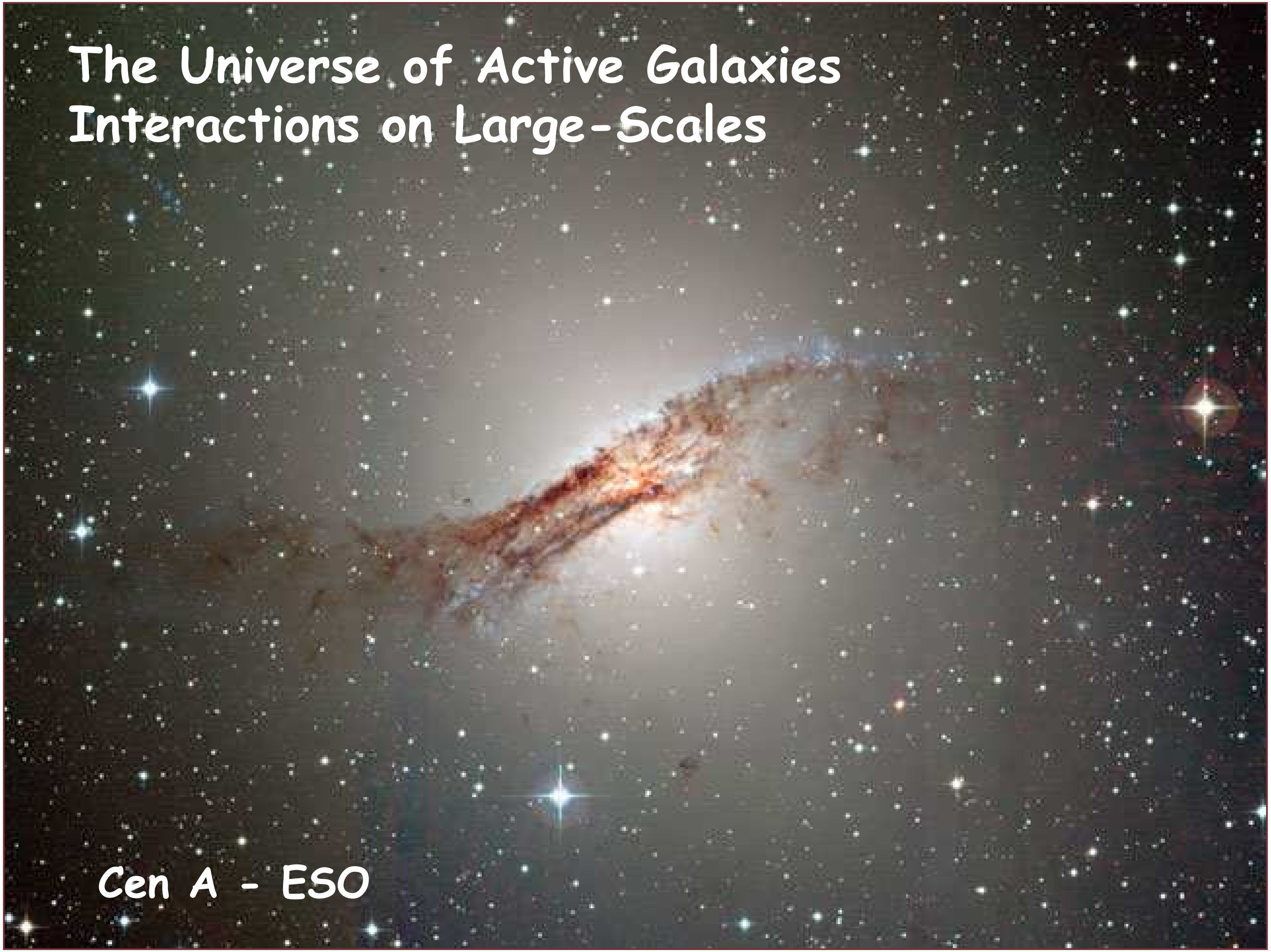




Cooling of Cluster Gas is important

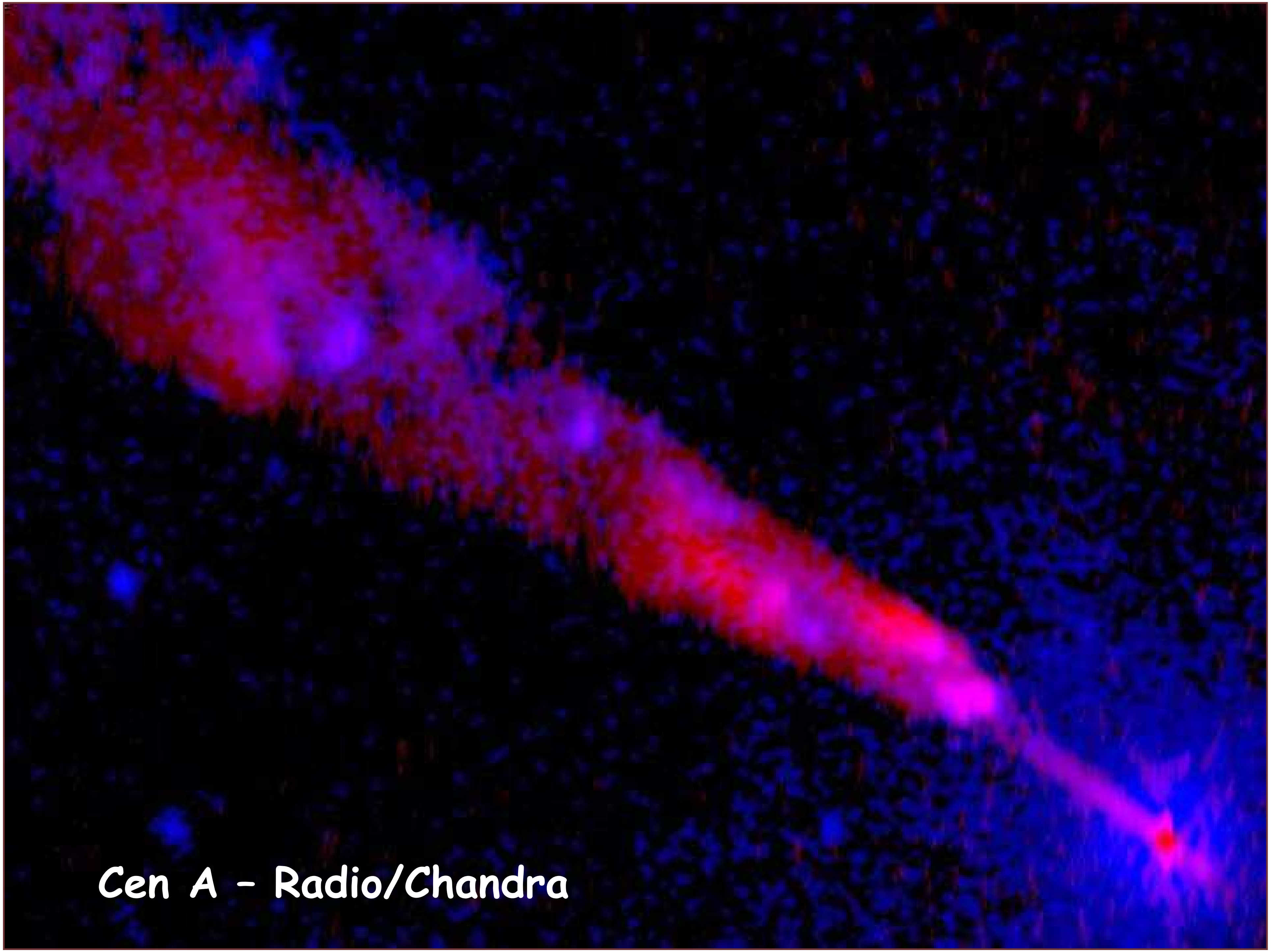


→ Radiative MHD
→ MHD & Atomic Network (NIRVANA_CP)

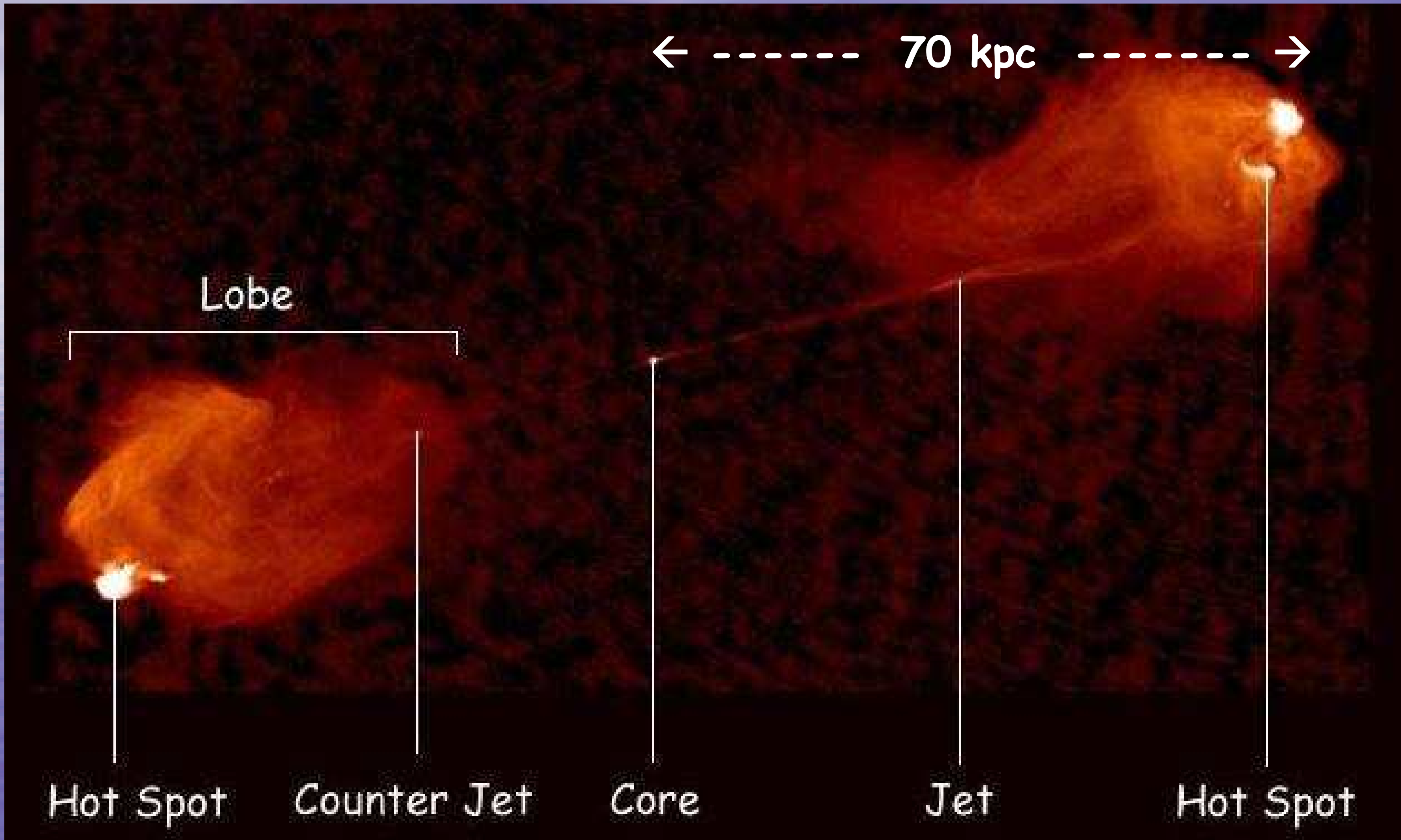

The Universe of Active Galaxies
Interactions on Large-Scales
Cen A - ESO

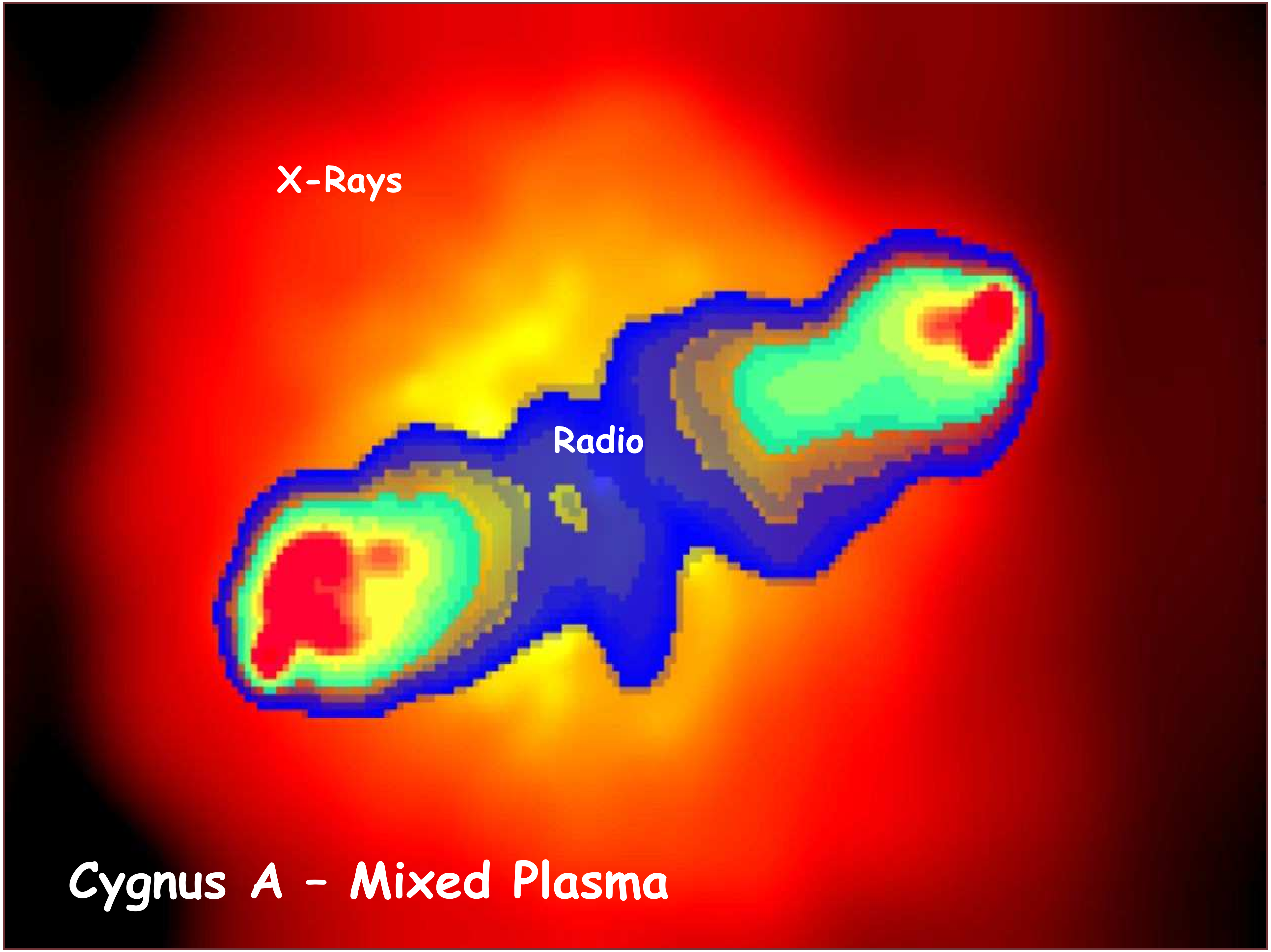
Cen A - Spitzer


Cen A - Chandra

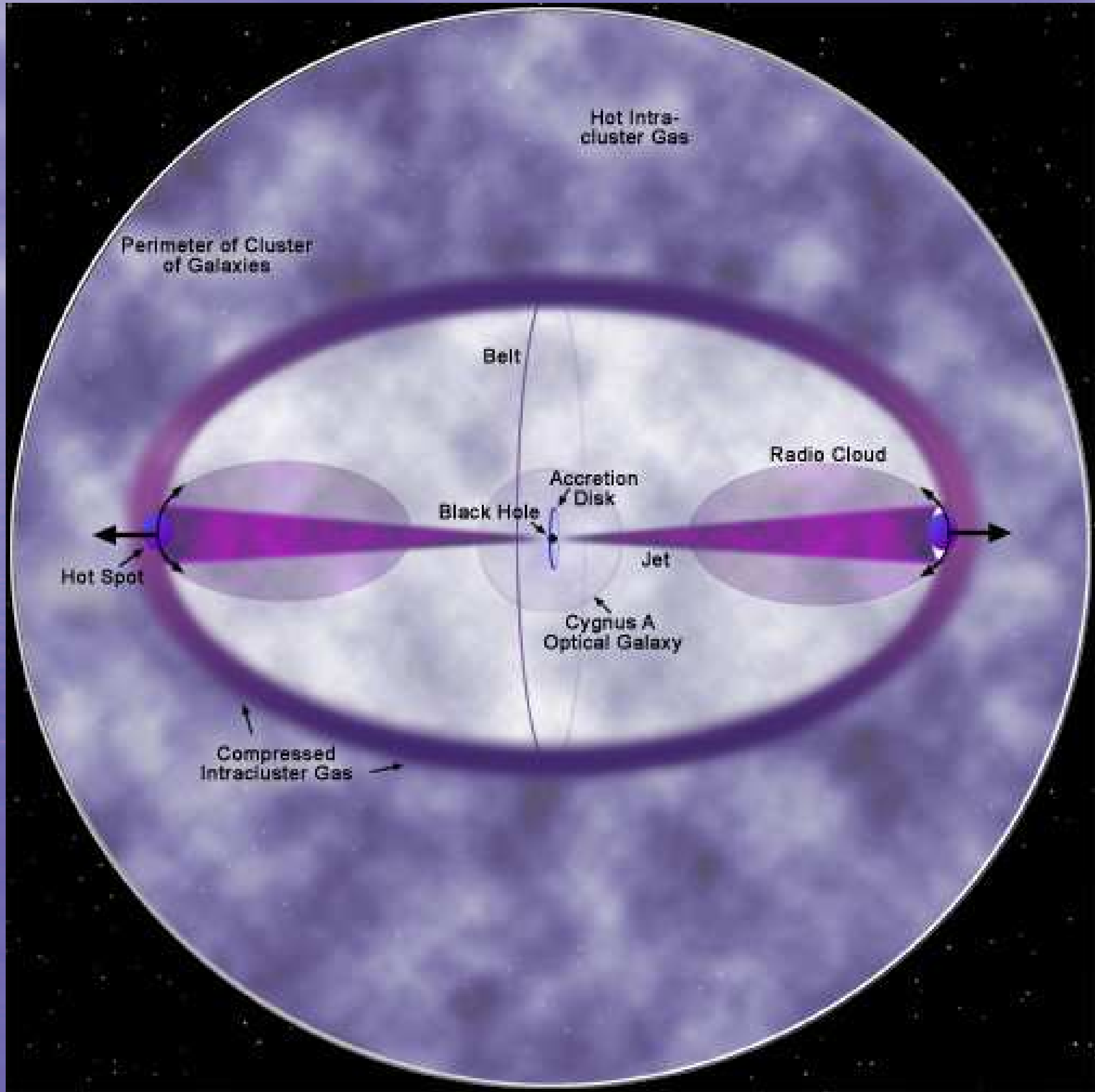

Cen A – Radio/Chandra

Cygnus A – The Archetype

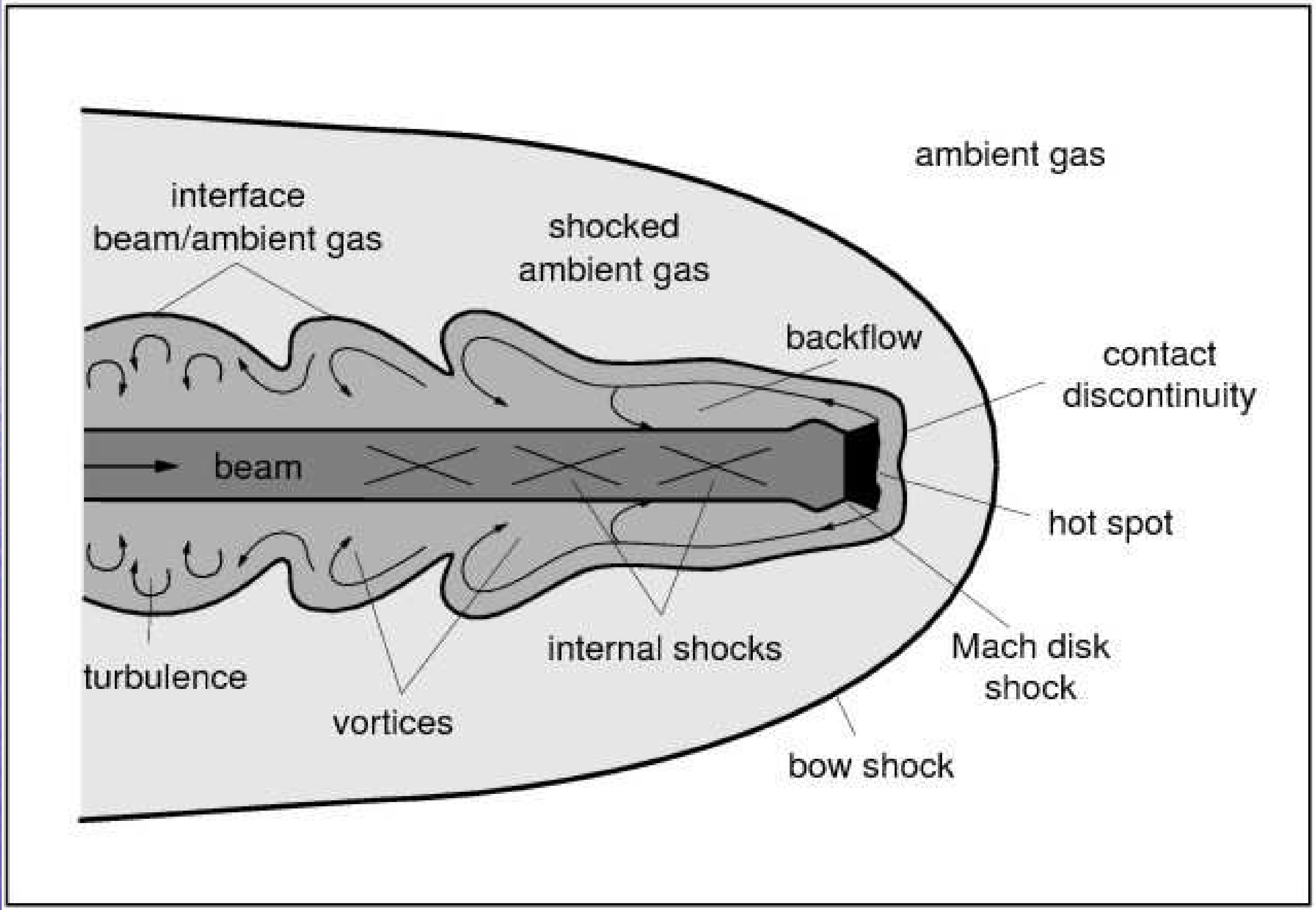



X-Rays
Radio
Cygnus A – Mixed Plasma

Cyg A - Global Struc-ture from Disks to Jets



Jet – Anatomy

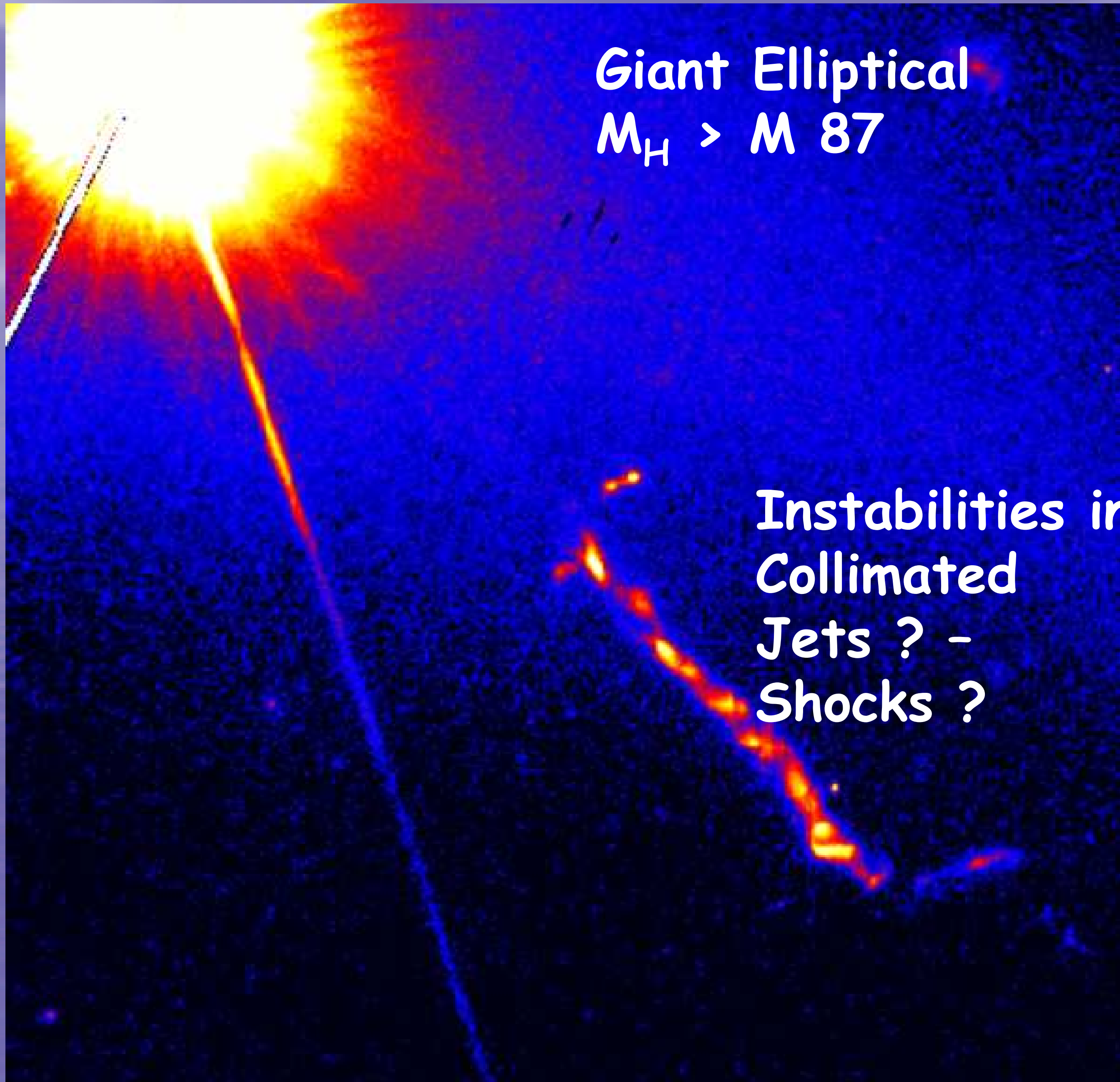


ambient gas
interface
beam/ambient gas
shocked
ambient gas
backflow
contact
discontinuity
beam
hot spot
turbulence
vortices
internal shocks
Mach disk
shock
bow shock

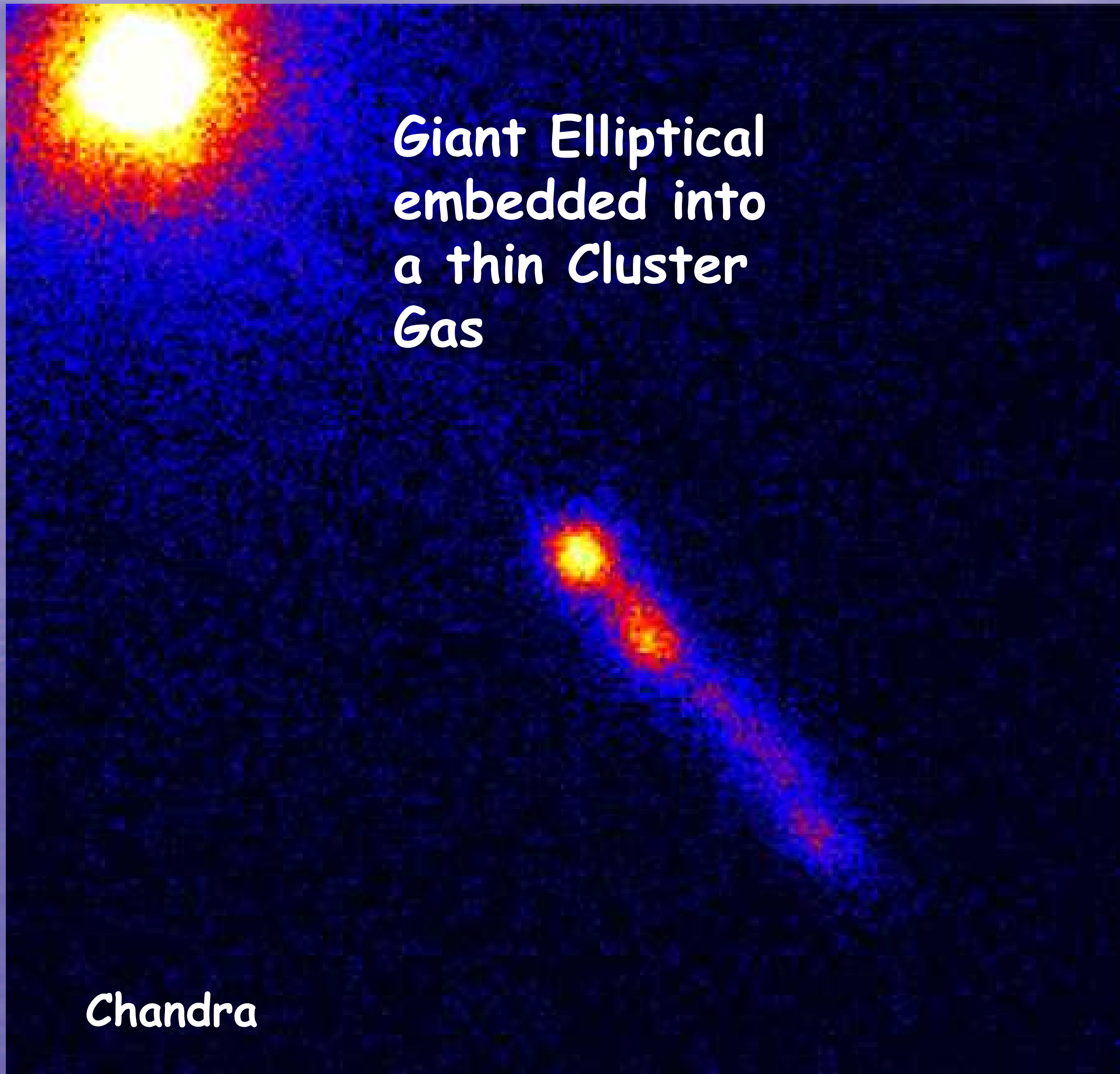
Some Numbers for Cyg A

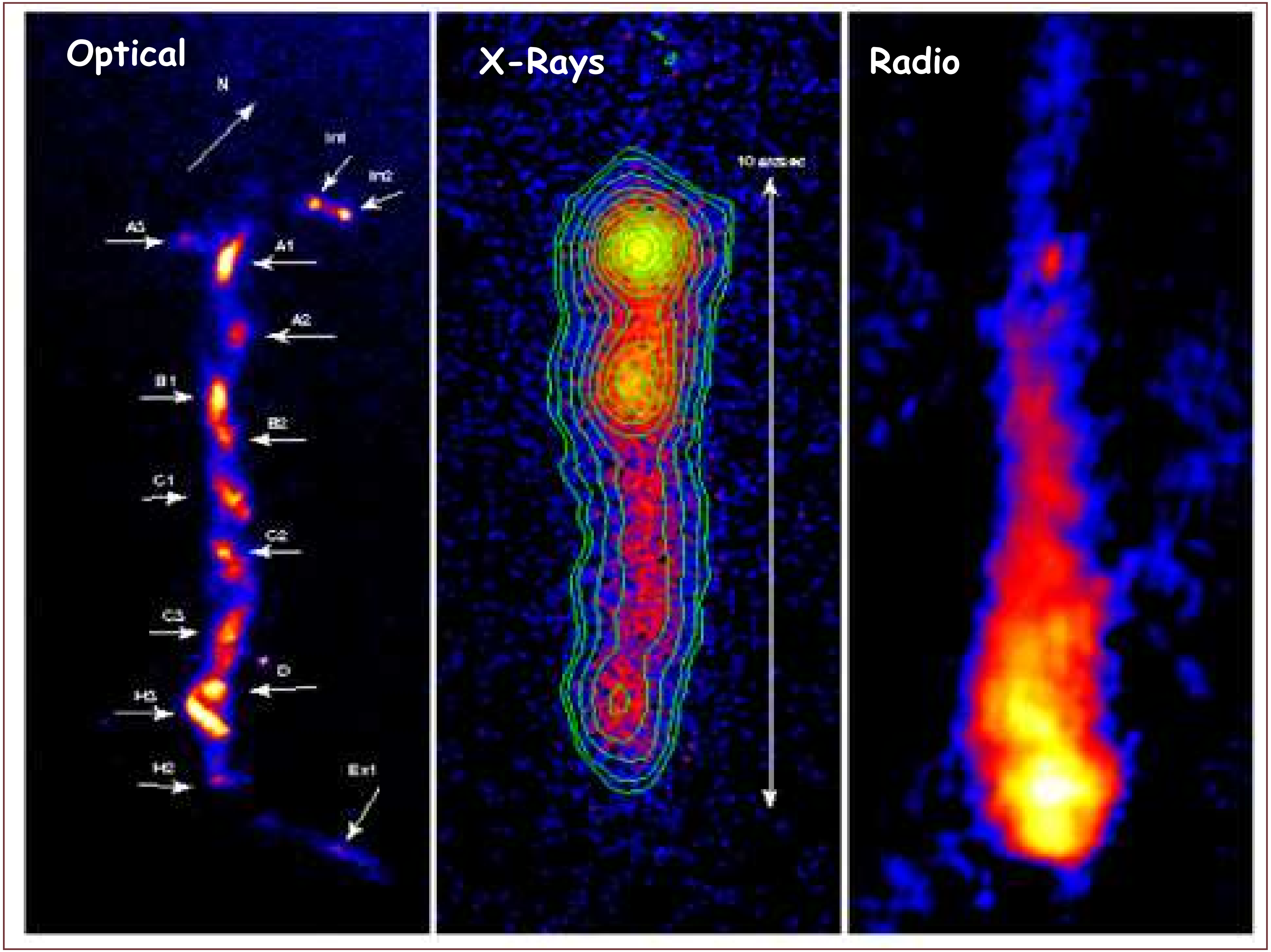
- Typical distance d = 1 kpc from center:
- Jet Power: L_{jet} = ?
- Beam radius: ?
- Number density in cluster: $n_{cl} \sim 0.05$ ccm
- Number density in beam: ?
- → density contrast $\eta = \rho_{beam}/\rho_{Medium}$ = ?
- → Magnetic field in beam ?
- → Beam velocity ?
- → Alfven velocity in beam ?

3C 273 - HST Quasar and its Jet

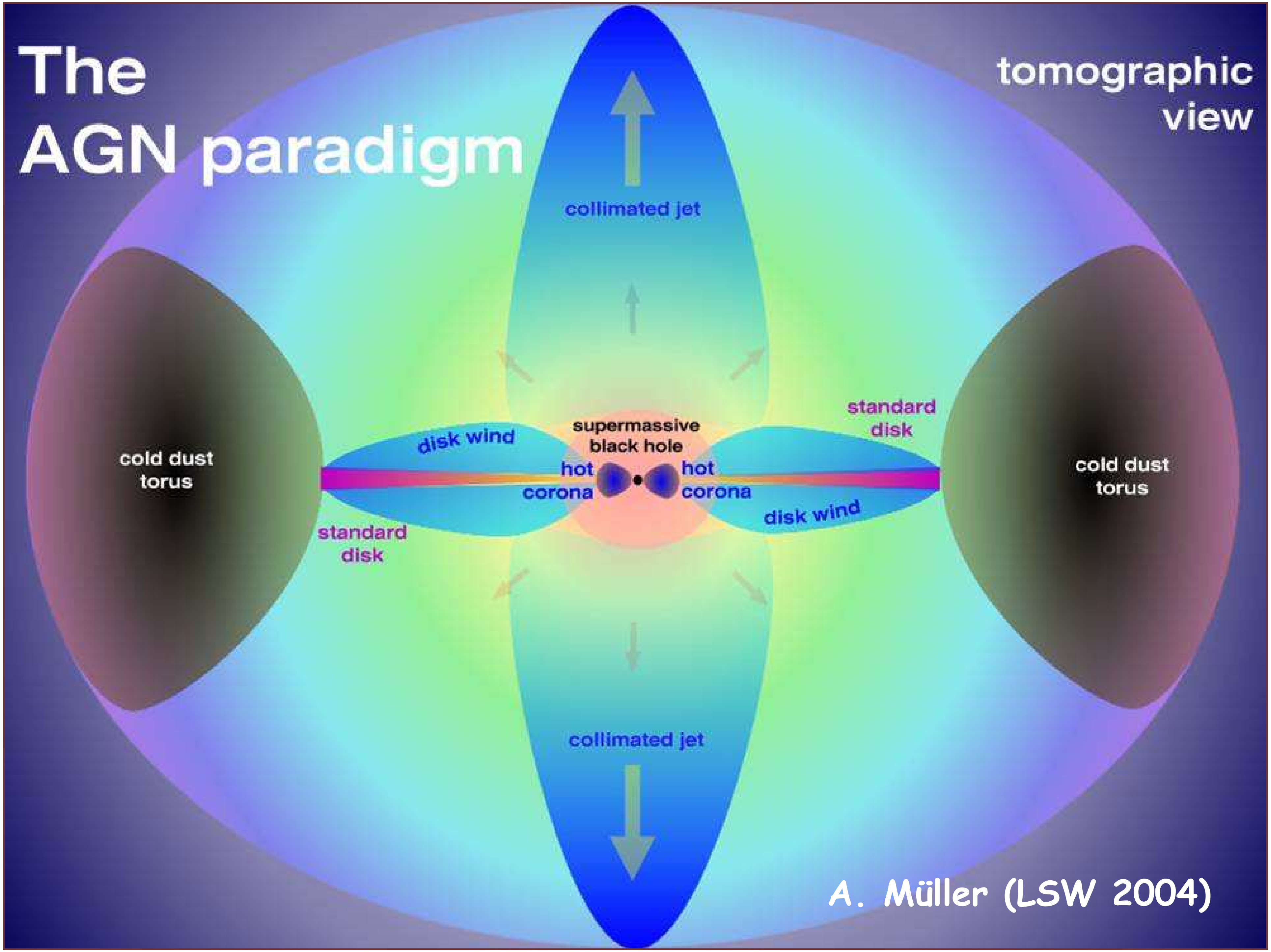


3C 273 in X-Rays





Optical
X-Rays
Radio
N
A1
B1
C1


The
AGN paradigm
tomographic
view
collimated jet
supermassive
black hole
disk wind
hot
corona
hot
corona
standard
disk
standard
disk
disk wind
cold dust
torus
cold dust
torus
collimated jet
A. Müller (LSW 2004)

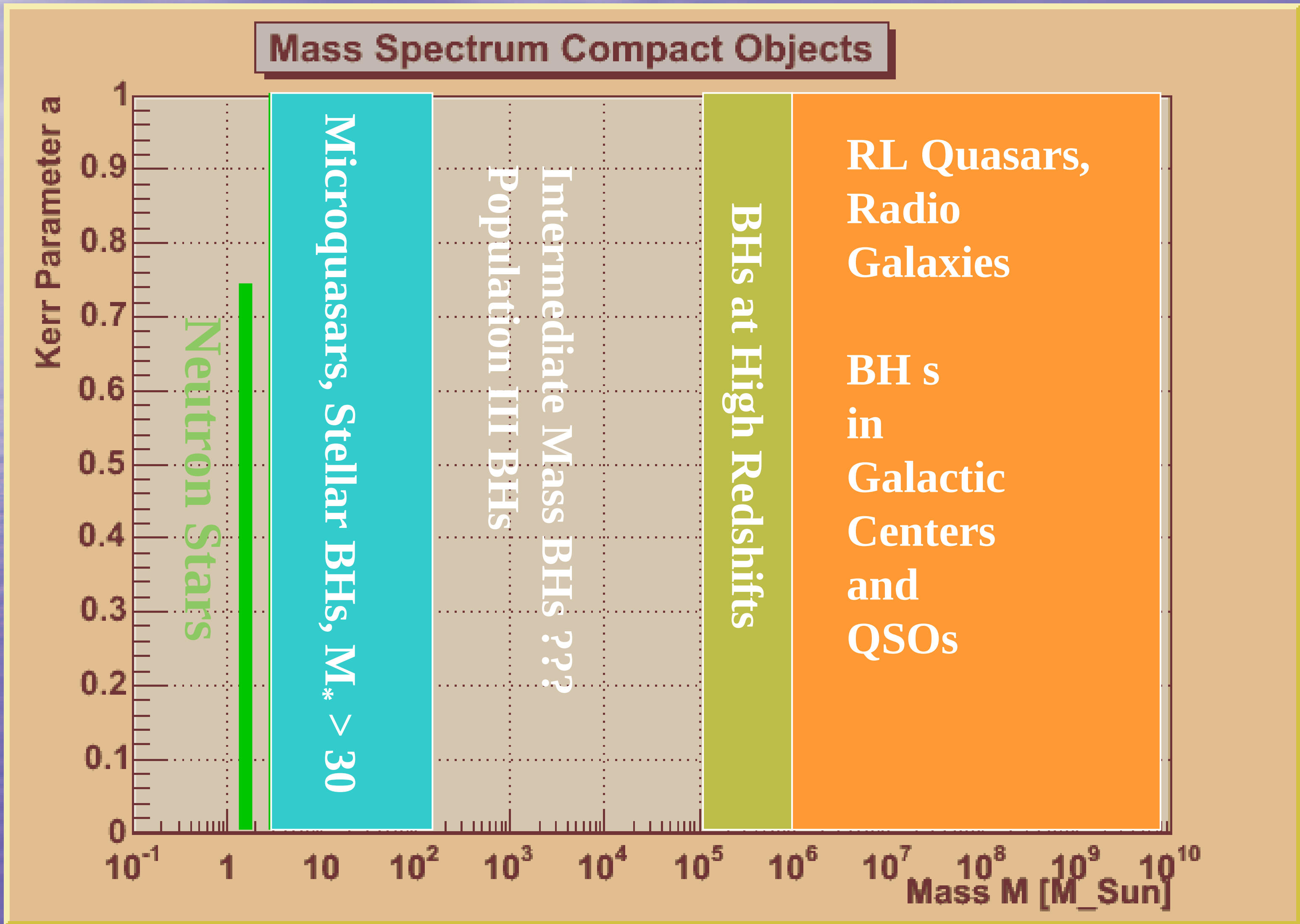
Summary 1: Space Plasmas

- **Range of Phenomena** from solar corona & Earth MS² (space weather) to hot gas in galaxy clusters (cosmology).
- These plasmas are **magnetized**:
- → Plasma **beta** parameter, is usually variable (say from disk to corona)
- → **Alfven speed vs sound speed**
- **→ $V_A > C_{sound}$ → information no longer exchanged over sound waves, but over Alfven and magnetosonic waves**
  → Hydrodynamics replaced by MHD !
- **In Quasar Jets, the plasma density varies from 10^8 to 10^{-6} ccm.**
- **→ Radiation processes are important in many space plasmas (accretion disks, cluster gas, protostellar jets [HH Flows], …).**

Basics for (Jet) MHD

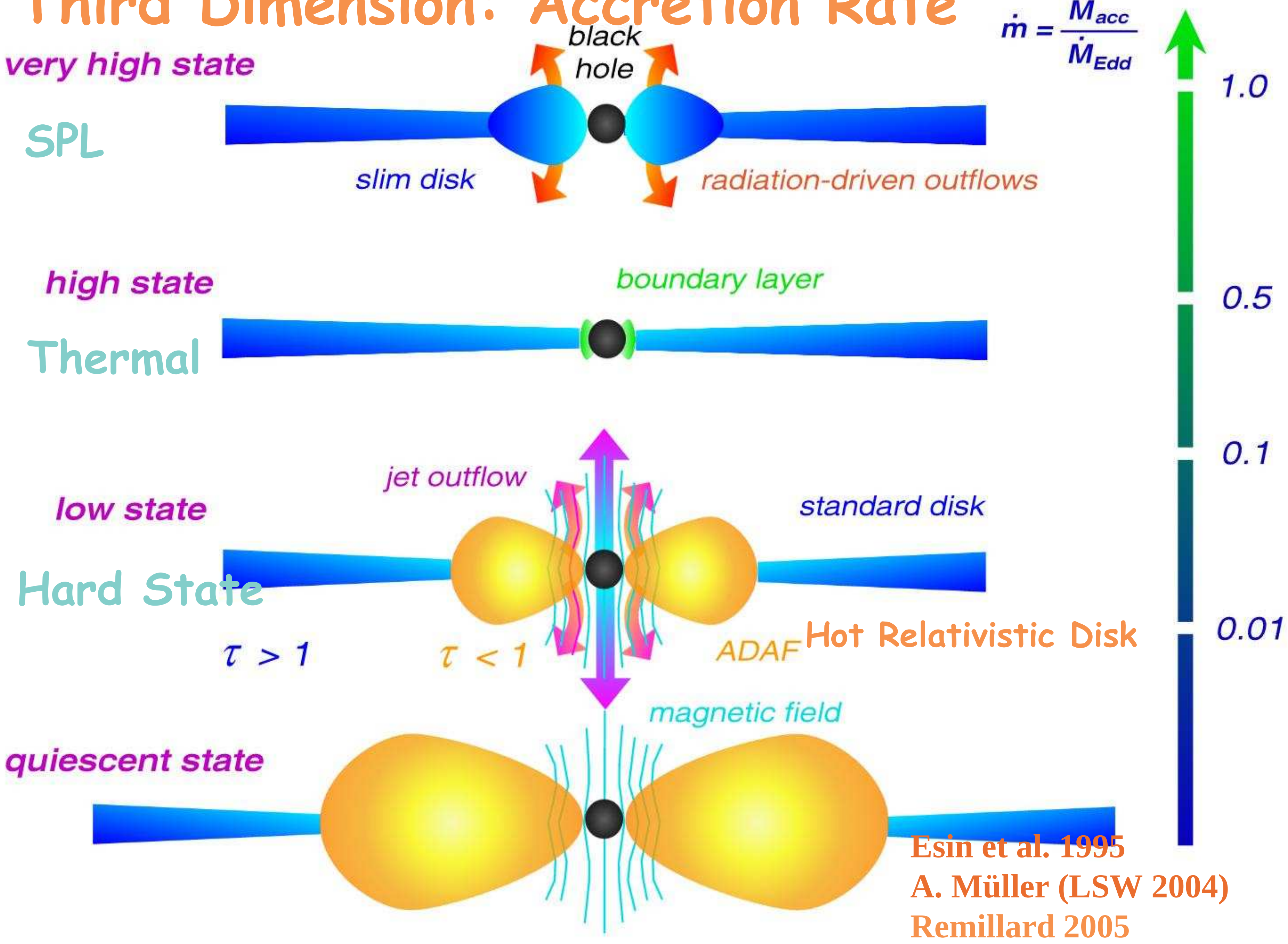
- Black Holes have three „Hairs"
- (i) Mass
- (ii) Angular Momentum
- (iii) Accretion Rate
- Axisymmetric Electrodynamics
- → Definition of magnetic flux
- → Notion of poloidal current loops
- → Grad-Shafranov Equation
- → Twisting poloidal fields by shear motion

Black Holes have „Two Hairs"



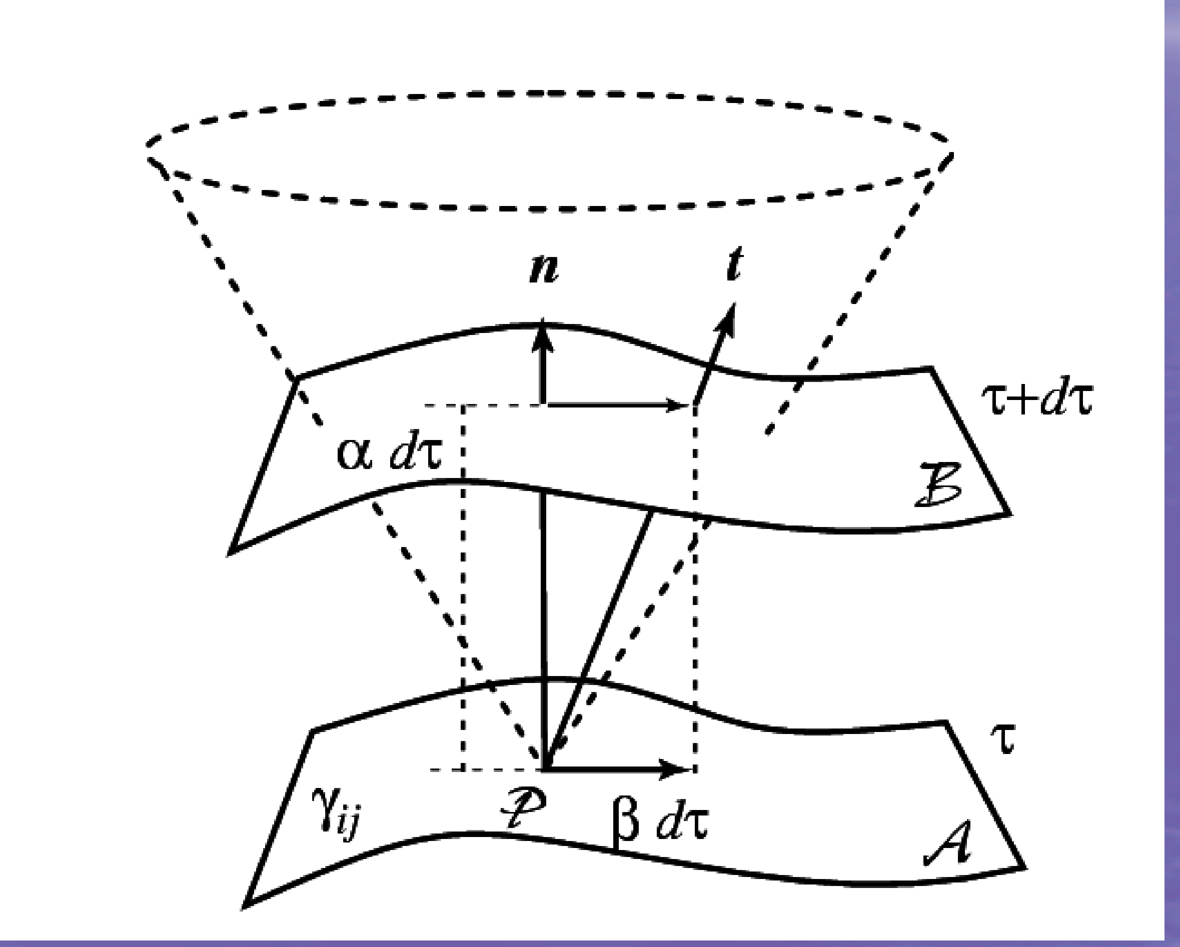
Camenzind 2005

Third Dimension: Accretion Rate

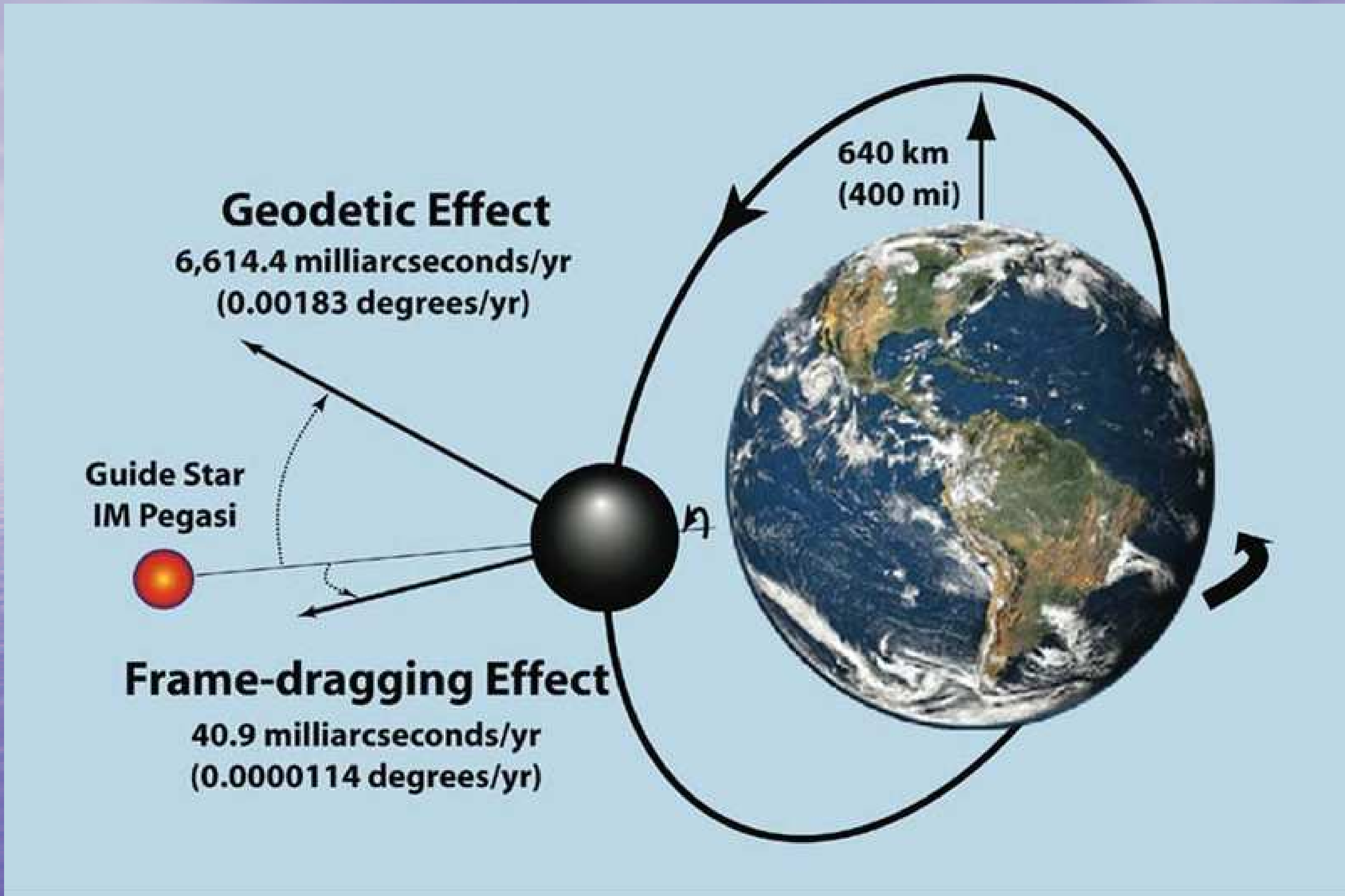


Black H's: 2 Gravitational Potentials ➔ 10 g's decomposed into α, β and γ

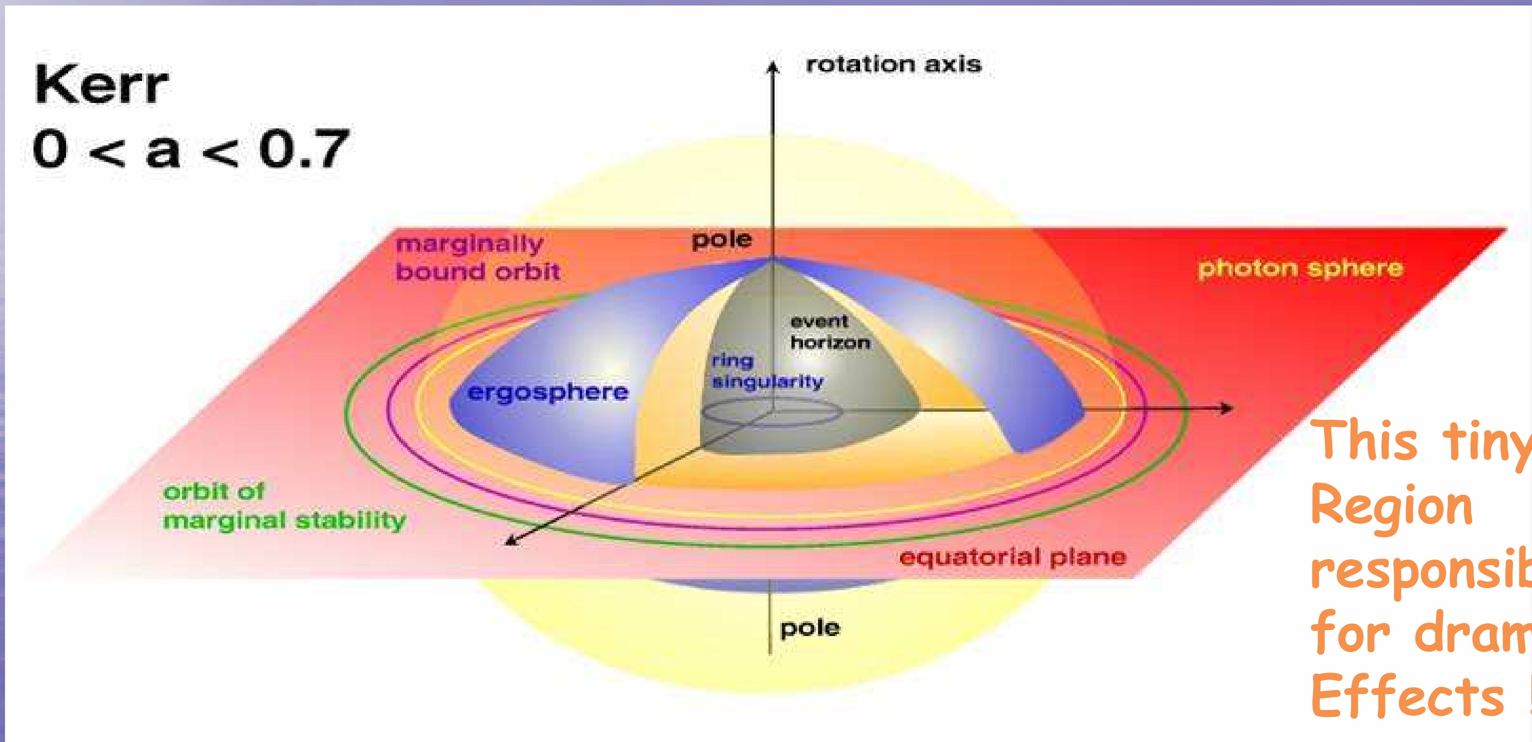
- Ordinary gravity: $g = \text{grad} \ln \alpha$ ~ Mass M_H
- Gravito-magnetic potential: β ➔ vector potential (~ED) ~ Spin : $\omega = -\beta^{\phi} \sim J_H/r^3$
- 3-metric γ of the time-slice (only geometry)



Gravity Probe-B
will confirm the
Existence of
Gravitomagnetism


640 km
(400 mi)
Geodetic Effect
6,614.4 milliarcseconds/yr
(0.00183 degrees/yr)
Guide Star
IM Pegasi
Frame-dragging Effect
40.9 milliarcseconds/yr
(0.0000114 degrees/yr)

Black Hole Accretion is Different from Newtonian

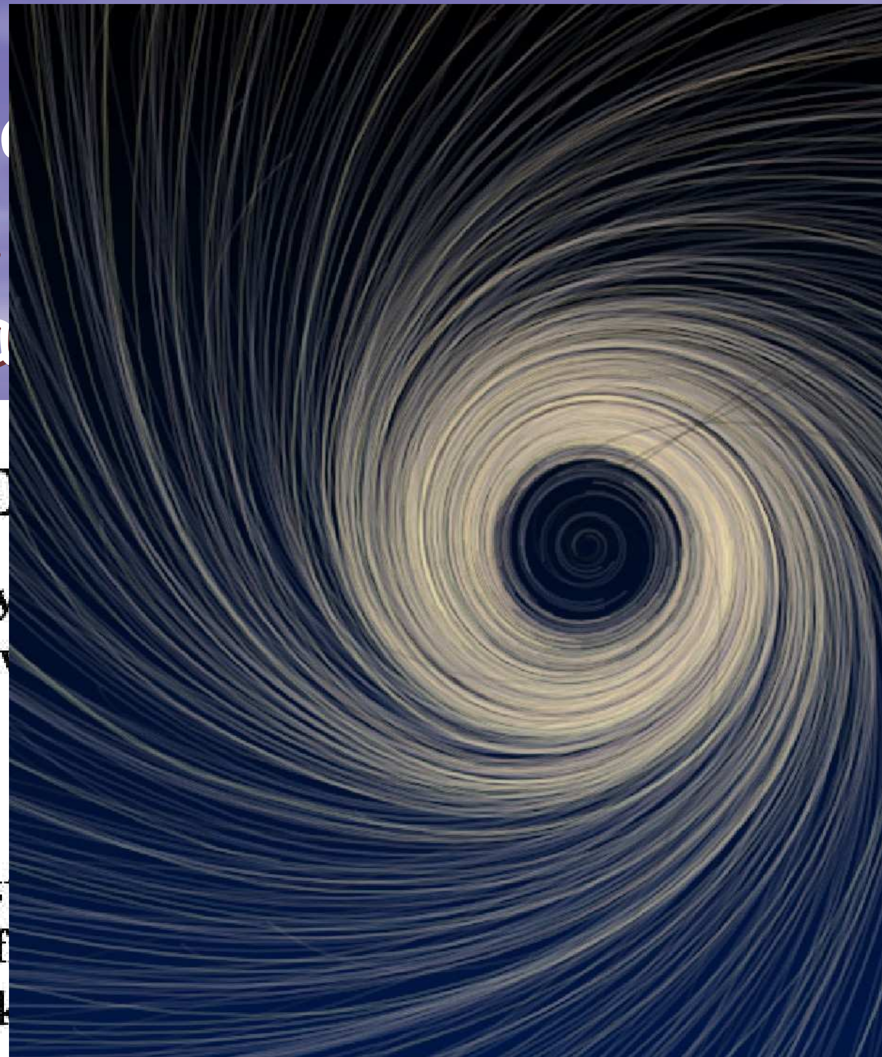


This tiny Region responsible for dramatic Effects !

In ergoregion, plasma is driven to corotation with horizon.

A. Müller LSW 2004

Each form of matter
will be driven to corot
within the ergosphere
→ Boundary Layer nea



Gravitational E

- Boundary layer: Angular frequency
observers (fixed stars) is given by (

$$\Omega = \frac{U^\phi}{U^t}$$

where U^μ is the 4–velocity of mat
poloidal motion of matter, angular f
gular momentum are related over a l

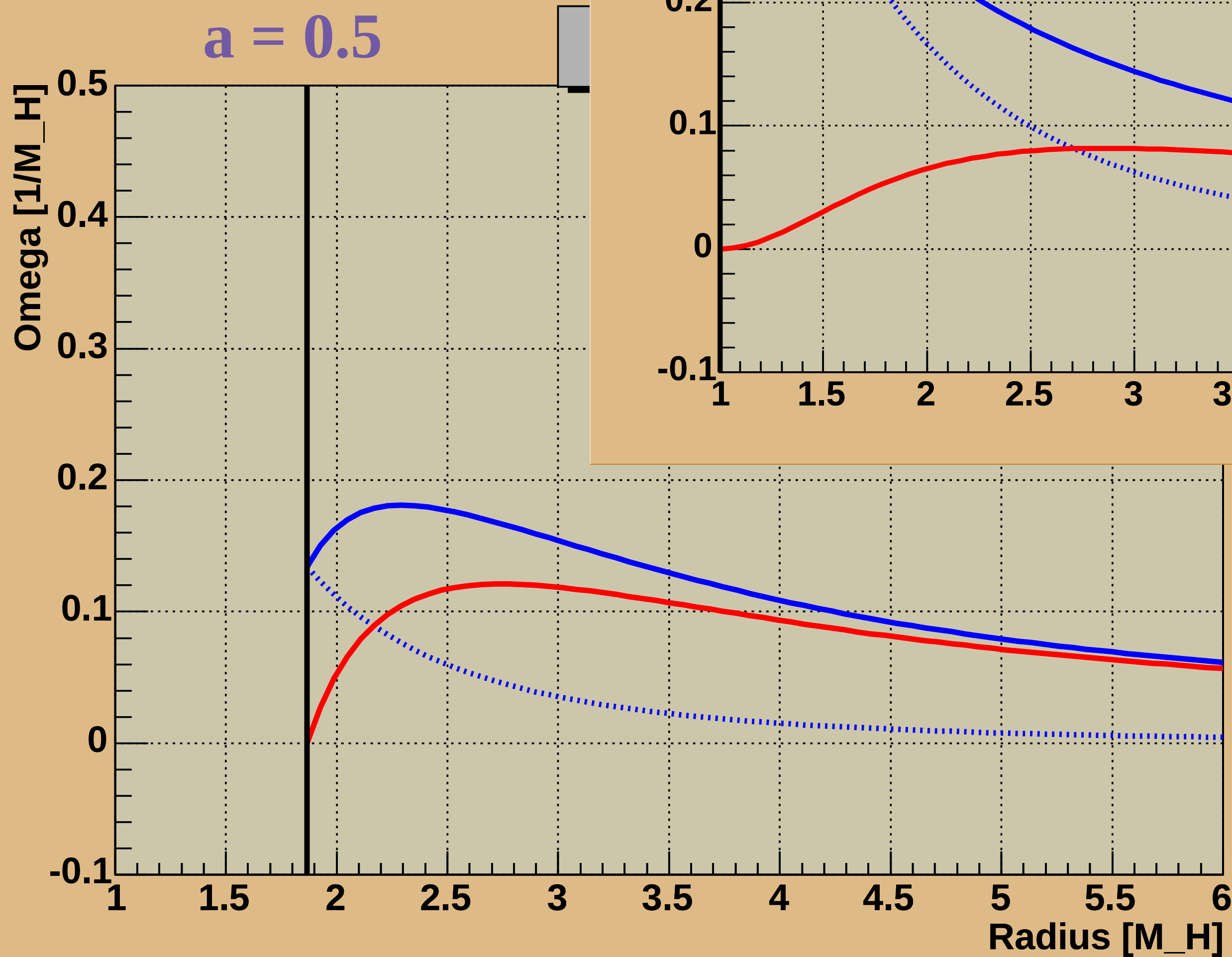
$\Omega_H = \omega(r_+)$

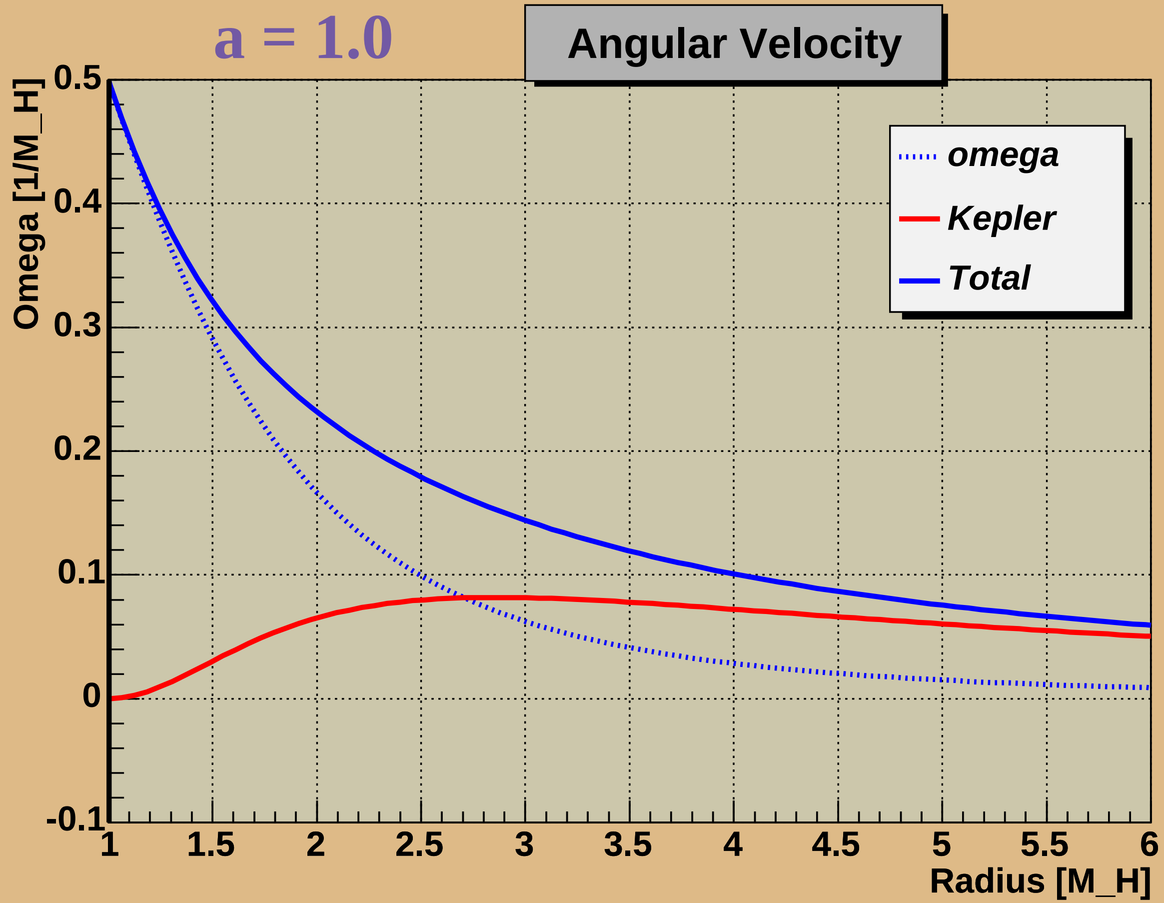
$$\Omega = \omega + \frac{\alpha^2}{R^2}\frac{\lambda}{1-\omega\lambda}. \quad (2)$$

$R \equiv \sqrt{h_{\phi\phi}}$ is the cylindrical radius.

In Schwarzschild:
→No rotation
near Horizon !

Frame Dragging near Horizon





Black Hole generates enormous shear near Horizon !

Black Holes → 2 Energy Reservoirs

- Potential energy → tapped by accretion
- Rotational energy → tapped by magnetic fields, similar to rotating neutron stars

$$L_{Rot} = E_{Rot}/t_{brake} \sim 10^{46}\ \mathrm{erg/s}\ (M_H/10^9\ M_S)\ (t_H/t_{brake})$$

$$L_{Rot} = E_{Rot}/t_{brake} \sim 10^{38}\ \mathrm{erg/s}\ (M_H/10\ M_S)\ (t_H/t_{brake})$$

$$t_{brake} = f\,(a, B, \ldots) \quad [\mathrm{BZ}\ 1977]$$

$$L_{BZ} = k\, B_H^2\, r_H^2 c\, (a/M)^2\, (\Omega_F[\Omega_H - \Omega_F]/\Omega_H^2) \sim M_H$$

Intermezzo I: Maxwell's Equations

$$\nabla \cdot \vec{E} = 4\pi\rho_e$$

$$\nabla \cdot \vec{B} = 0$$

Faraday Induction

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$$

Ampere's Law

$$\frac{\partial \vec{E}}{\partial t} = \nabla \times \vec{B} - 4\pi\vec{j}\,.$$

Lorentz Force Density

$$\vec{f}_L = \rho_e \vec{E} + \vec{j} \times \vec{B}\,.$$

Ohm's Law and Induction Equation

Ohm's Law

$$\vec{j} = \sigma \left[\vec{E} + \vec{v} \times \vec{B} \right] + \cdots ,$$

→ MHD Induction Equation

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left[\vec{v} \times \vec{B} - \frac{1}{4\pi\sigma} \nabla \times \vec{B} \right],$$

together with the constraint: div(B) = 0
for all times ! → Ampere's law is not used !
For finite conductivity → diffusion equation

INDUCTION EQUATION

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = \nabla \times (\mathbf{v} \times \mathbf{B} - \mathbf{j}/\sigma)$$

$$= \nabla \times (\mathbf{v} \times \mathbf{B}) - \eta \nabla \times (\nabla \times \mathbf{B})$$

$$= \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B},$$

where $\eta = \dfrac{1}{\mu\sigma}$ *is magnetic diffusivity*

Magnetic Reynolds Number

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

A B

(iv) **B** changes due to transport + diffusion

(v) $\frac{A}{B} = \frac{L_0 \, v_0}{\eta} = R_m$ -- * **magnetic Reynold number** *

e.g. $\eta = 1 \text{ m}^2/s$, $L_0 = 10^5$ m, $v_0 = 10^3$ m/s → $\mathbf{R_m = 10^8}$

(vi) **A** >> **B** in most of the Universe →

B frozen to plasma -- keeps its energy

Except SINGULARITIES -- j & ∇B large

Form at **NULL POINTS**, B = 0

(a) If $R_m << 1$

- The induction equation reduces to

$$\frac{\partial \mathbf{B}}{\partial t} = \eta \nabla^2 \mathbf{B}$$

- B is governed by a diffusion equation
  --> field variations on a scale L_0
  diffuse away on time * $t_d = \frac{L_0^2}{\eta}$ *
  with speed $v_d = L_0 / t_d = \frac{\eta}{L_0}$
- E.g.: sunspot (η = 1 m²/s, $L_0 = 10^6$ m), $t_d = 10^{12}$ **sec**;
  for whole Sun ($L_0 = 7\mathrm{x}10^8$ m), $t_d = 5\mathrm{x}10^{17}$ **sec**

(b) If $R_m >> 1$

The induction equation reduces to

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

and Ohm's law ➔

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

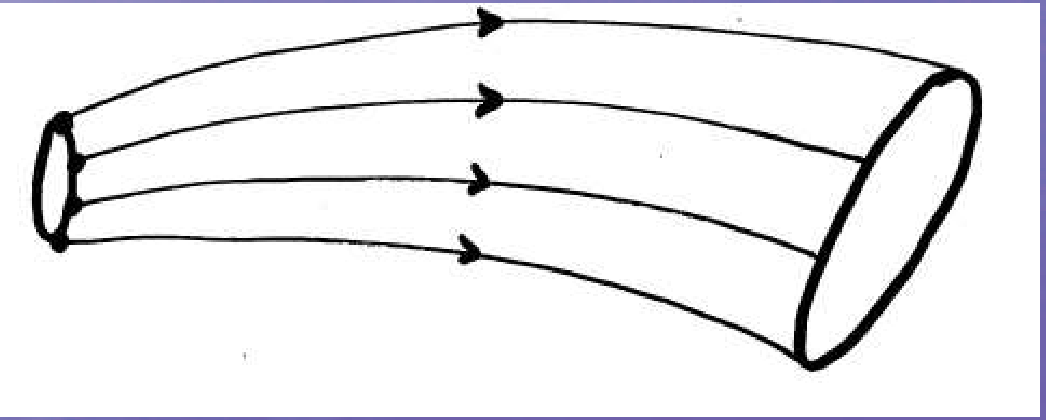
Magnetic field is " frozen to the plasma "

Concept of Magnetic Flux

- One deals often with axisymmetric magnetic structures (mangetospheres etc)
- → Poloidal and toroidal components of vector fields
  $E = E_p + E_T$, $B = B_p + B_T$
- In ED it is useful to work with the vector potential A : $B = \mathrm{rot}\, A$
- → Magnetic flux function: $\Psi = R\, A_\phi$

Magnetic Flux Tube

- Surface generated by set of field lines intersecting simple closed curve.



Strength (F) -- magnetic flux crossing a section

i.e., $$F = \int \mathbf{B.dS}$$

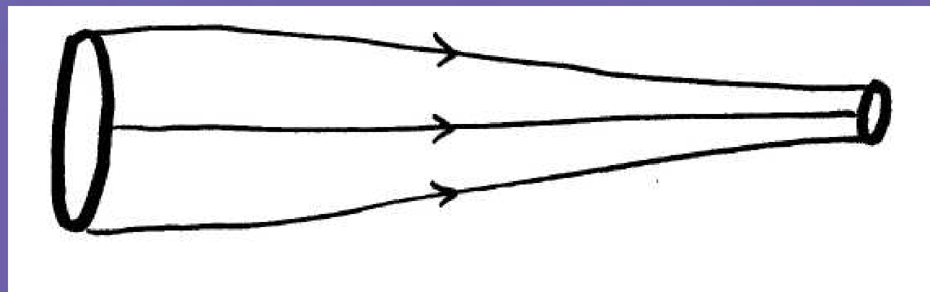
(ii) But $\nabla . \mathbf{B} = 0$

→ No flux is created/destroyed inside flux tube

So $F = \int \mathbf{B}.\mathbf{dS}$ **is constant along tube**

(iii) If cross-section is small,

$$F \approx BA$$



B lines closer → A smaller + B increases

→ Sun Spots

Axisymmetric Maxwell's Equations

(I) $$\frac{\partial A_\phi}{\partial t} = -E_\phi$$

(II) $$\frac{\partial B_\phi}{\partial t} = -\vec{e}_\phi \cdot (\nabla \times \vec{E}_p)$$

(III) $$\frac{\partial E_\phi}{\partial t} = -\mathcal{G}_2[\Psi] - 4\pi j_\phi$$

(IV) $$\frac{\partial \vec{E}_p}{\partial t} = \nabla \times \vec{B}_T - 4\pi \vec{j}_p .$$

Grad-Shafranov Operator

→ Fusion Physics

$$\mathcal{G}_2[\Psi] \equiv R\,\mathbf{Div}\left[\frac{1}{R^2}\,\nabla\Psi\right],$$

Grad-Shafranov Equation

Combining Equ (I) and (III)

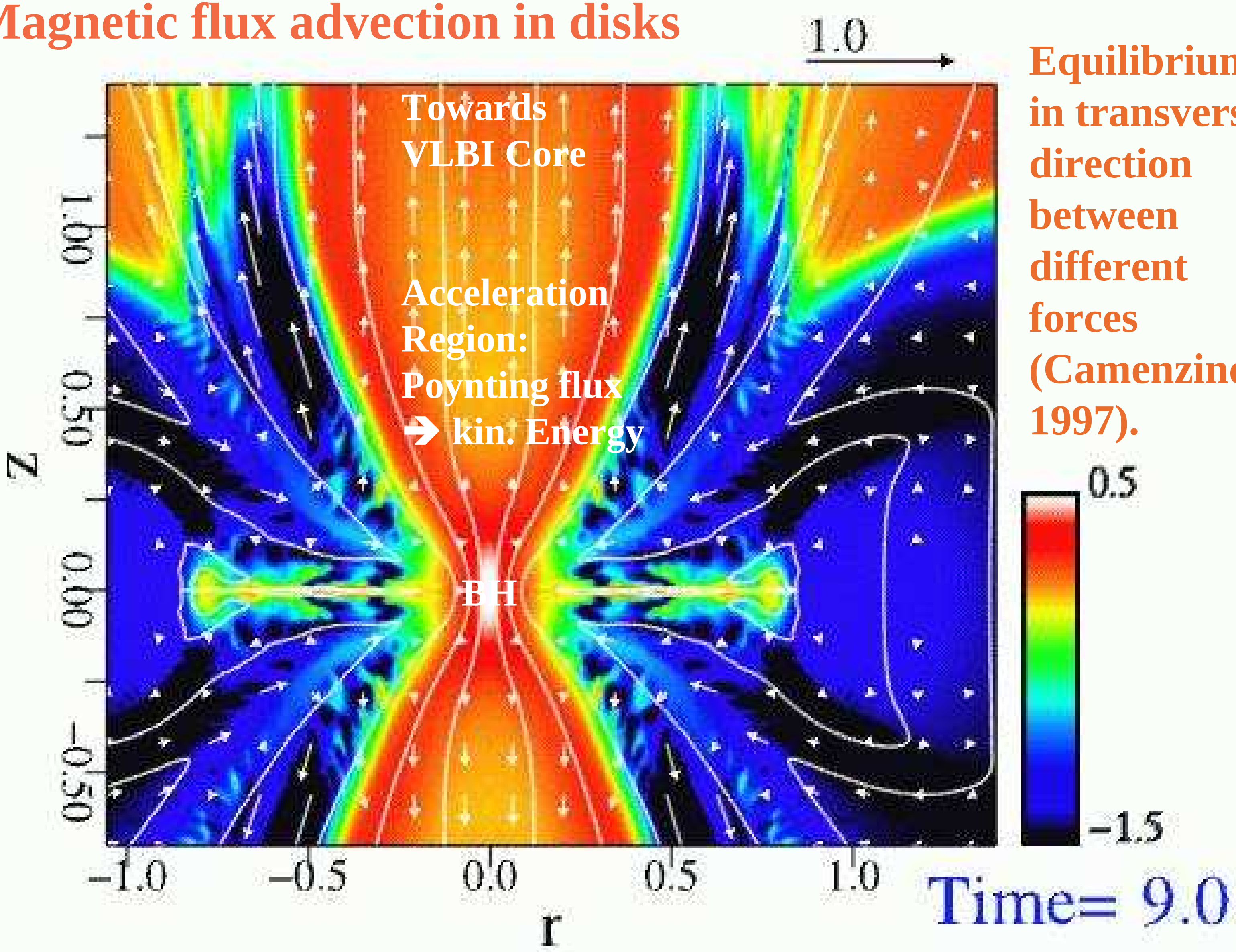
$$\frac{\partial^2 \Psi}{\partial t^2} - R\,\mathcal{G}_2[\Psi] = 4\pi R j_\phi \,.$$

Time-dependent Grad-Shafranov Equation

$$\eta \, \frac{\partial^2 \Psi}{c^2 \partial t^2} + \frac{\partial \Psi}{\partial t} + (\vec{v}_p \cdot \nabla)\Psi - \eta\, R\,\mathcal{G}_2[\Psi] = 0 \,.$$

→Magnetic flux changes by advection (disks, Jets) and diffusion against plasma flows.
Without diffusion → $v_p \parallel B_p$ → Jet flows !

Magnetic flux advection in disks



Equilibrium in transverse direction between different forces (Camenzind 1997).

Uchida & Shibata 1996

Axisymmetric Maxwell's Equations

(I) $$\frac{\partial A_\phi}{\partial t} = -E_\phi$$

(II) $$\frac{\partial B_\phi}{\partial t} = -\vec{e}_\phi \cdot (\nabla \times \vec{E}_p)$$

(III) $$\frac{\partial E_\phi}{\partial t} = -\mathcal{G}_2[\Psi] - 4\pi j_\phi$$

(IV) $$\frac{\partial \vec{E}_p}{\partial t} = \nabla \times \vec{B}_T - 4\pi \vec{j}_p \,.$$

Grad-Shafranov Operator
→ Fusion Physics

$$\mathcal{G}_2[\Psi] \equiv R\,\mathbf{Div}\left[\frac{1}{R^2}\,\nabla\Psi\right],$$

Shearing of Magnetic Fields

- From Equ (II) + Ohm's law
  ➔ Equ for B_ϕ ➔ Equ for RB_ϕ
- Differential rotation will twist magnetic fields (e.g. in disks) ➔ toroidal fields
- Current function: $T(t, R, z) = RB_\phi(t, R, z)$
- Source of T is the shear in differential rotation
  ➔ Magnetorotational Instability

Source Term !

$$\frac{\partial T}{\partial t} + (\vec{v}_p \cdot \nabla)T - R^2\,\nabla \cdot \left(\frac{T}{R^2}\vec{v}_p\right) - R^2\,\nabla \cdot \left(\frac{\eta}{R^2}\nabla T\right) = R^2\,\vec{B}_p \cdot \nabla\Omega\,.$$

Extension: Black Hole Electrodynamics

$\omega \sim J_H/r^3$

Frame Dragging due to Spin of BH

$$\frac{\partial A^{\hat{\phi}}}{\partial t} = -\alpha E^{\hat{\phi}} \qquad \boldsymbol{B}_p = \frac{1}{\tilde{\omega}} \nabla \Psi \times \boldsymbol{e}_\phi \,.$$

$$\frac{\partial B^{\hat{\phi}}}{\partial t} = \tilde{\omega} \boldsymbol{B}_p \cdot \nabla \omega - \boldsymbol{e}_\phi \cdot (\nabla \times \alpha \boldsymbol{E}_p)$$

$$\frac{\partial E^{\hat{\phi}}}{\partial t} = \tilde{\omega} \boldsymbol{E}_p \cdot \nabla \omega - \mathcal{G}_2[A^{\hat{\phi}}] - 4\pi\alpha j^{\hat{\phi}}$$

$$\frac{\partial \boldsymbol{E}_p}{\partial t} = \nabla \times (\alpha B^{\hat{\phi}}) - 4\pi\alpha \boldsymbol{j}_p \,.$$

$$\mathcal{G}_2[\Psi] \equiv \tilde{\omega}\, \mathbf{Div} \left[\frac{\alpha}{\tilde{\omega}^2} \nabla \Psi \right] ,$$

Grad-Shafranov Operator

Camenzind 1997

Rel. Grad-Shafranov Equation

- Combine Confinement Equs with Ohm's law

$$\boldsymbol{j_p} = \sigma\gamma \left[\boldsymbol{E_p} + \boldsymbol{v_T} \times \boldsymbol{B_p} + \boldsymbol{v_p} \times \boldsymbol{B_T} \right]$$

$$j^{\hat{\phi}} = \sigma\gamma \left[E^{\hat{\phi}} + \boldsymbol{e_\phi} \cdot (\boldsymbol{v_p} \times \boldsymbol{B_p}) \right] ,$$

$$\frac{\partial^2 \Psi}{\partial t^2} + 4\pi\gamma\sigma\,\alpha \frac{\partial \Psi}{\partial t} - \alpha\tilde{\omega}\,\mathcal{G}_2[\Psi] = -\tilde{\omega}^2 \alpha \boldsymbol{E_p} \cdot \nabla\omega + 4\pi\alpha^2\gamma\sigma\tilde{\omega}\boldsymbol{e_\phi} \cdot (\boldsymbol{v_p} \times \boldsymbol{B_p}) .$$

Source for magnetic flux near Horizon !

Khanna & Camenzind 1997

Force-Free Grad-Shafranov Equ

- In force-free limit, $\mathbf{f}_L = 0$, current densities are determined by fields (BZ 1977, …, Okamoto 1992, Beskin 2000, Fendt & Memola 2001)

$$\nabla \cdot \left[\frac{\alpha D}{\tilde{\omega}^2} \nabla \Psi \right] + \frac{\Omega_F - \omega}{\alpha} \frac{d\Omega_F}{d\Psi} |\nabla \Psi|^2 + \frac{16\pi^2 I}{\alpha \tilde{\omega}^2} \frac{dI}{d\Psi} = 0 \, .$$

$$D = 1 - \frac{(\Omega_F - \omega)^2 \tilde{\omega}^2}{c^2 \alpha^2}$$

Light Cylinder Function
$D = 0$ → Outer and Inner Light Cylinder around BHs

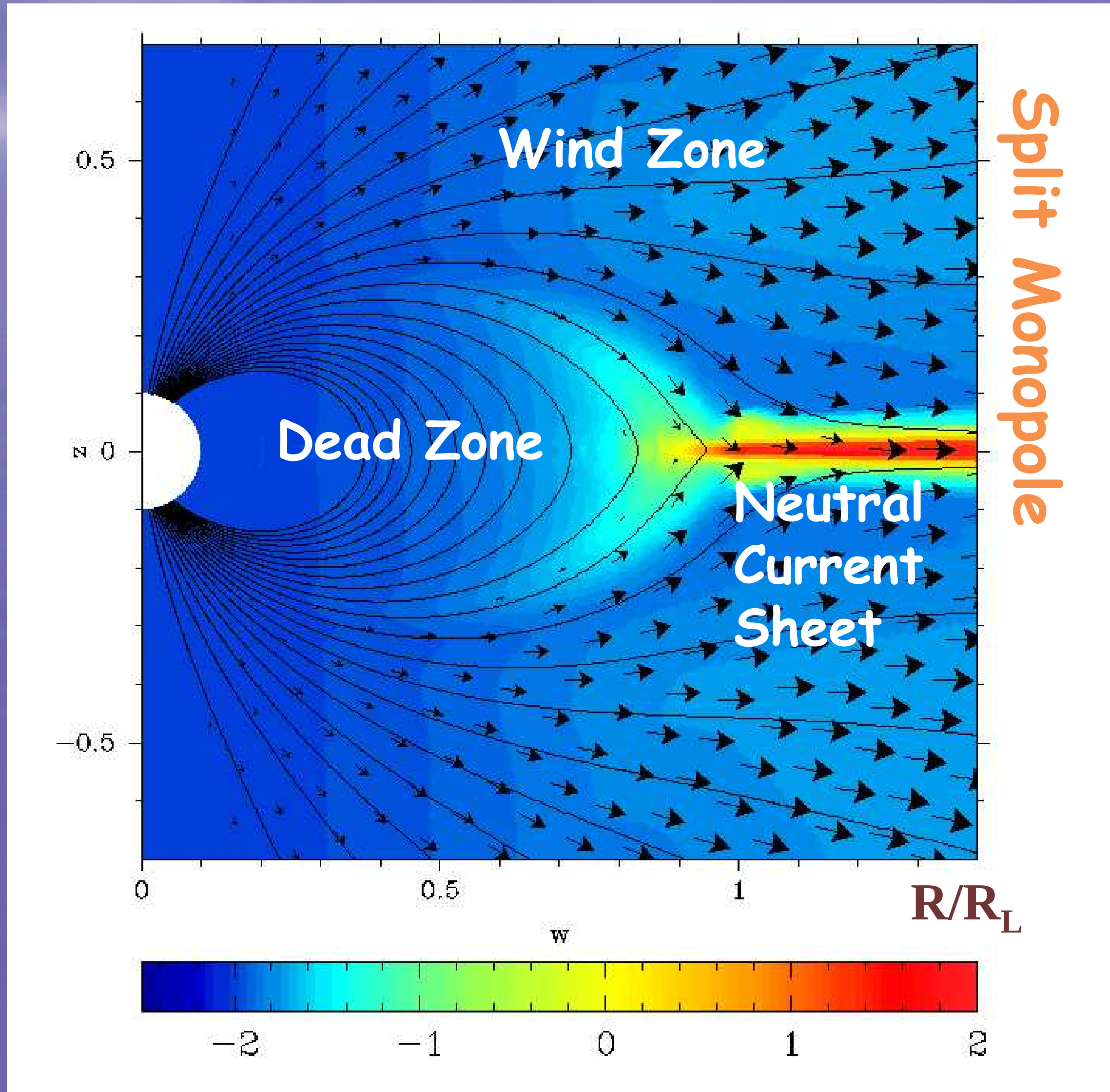
Pulsar MagSphere ($\alpha = 1$, $\omega = 0$)

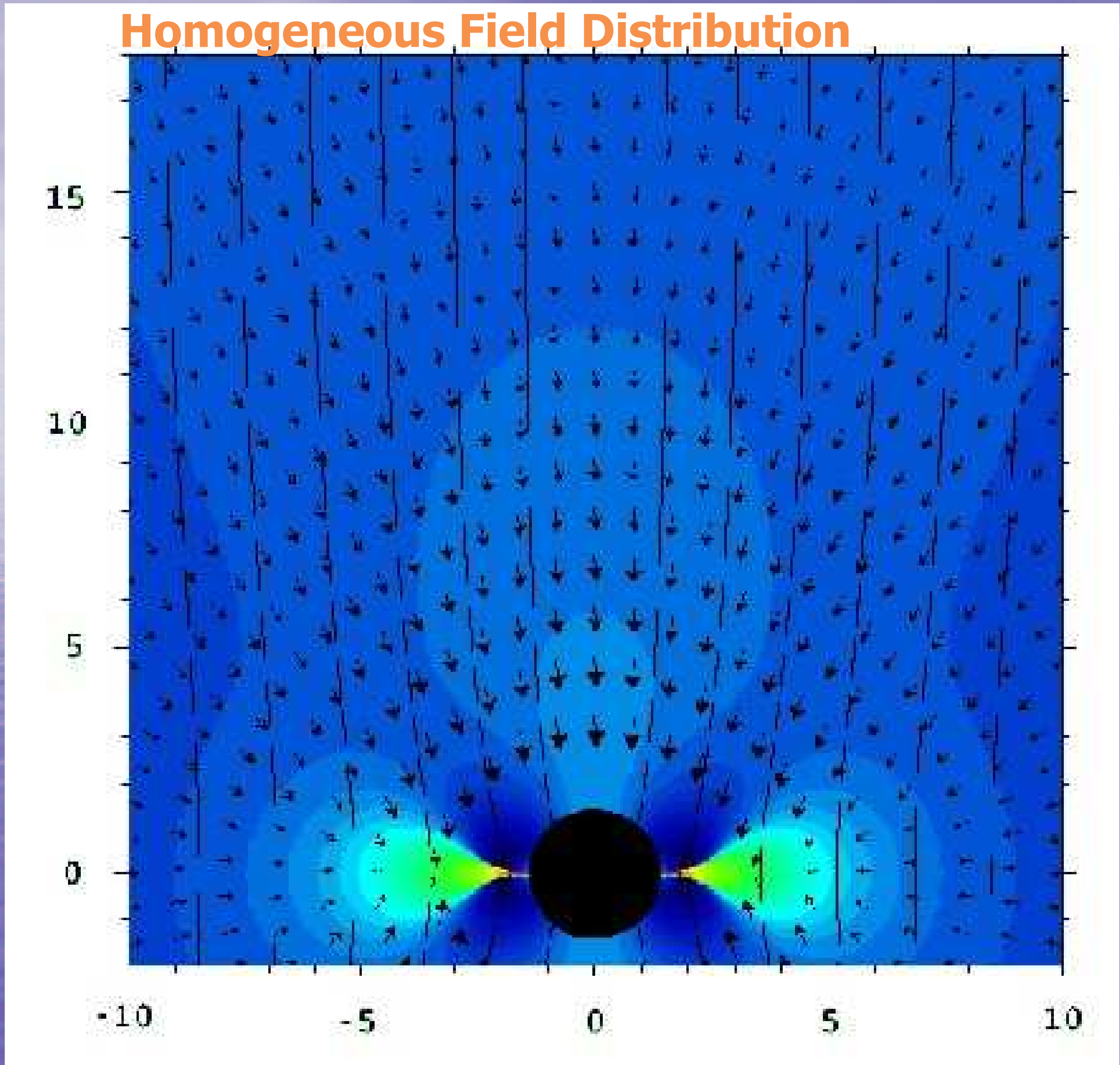
- Problem has been formulated by Michel (1973), Scharleman & Wagoner (1973)
- Contopoulos et al. (FF, 1999)

➔ Time-depend S. Komissarov (2005)

➔ Neutral current sheet beyond LC, $R_L = c/\Omega_F$

➔ Smooth transition LC





Black Hole Magneto-sphere is different

→ External MagSphere

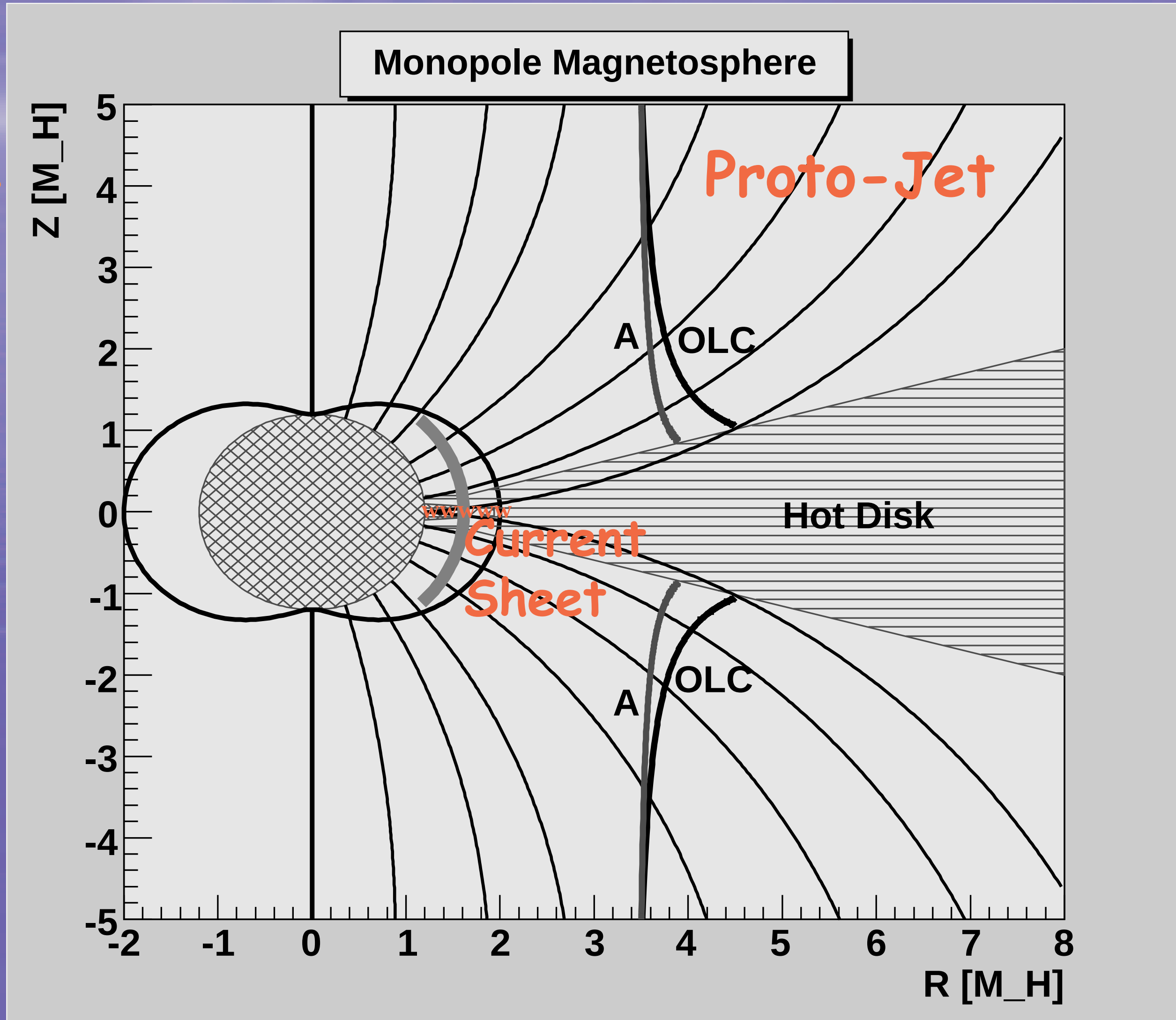
-

Initial Configura-tion, e.g. Wald solution

Black Hole Split-Monopole MS²

OLC: Outer Light Surface, compact for Black Holes
A: Alfven Surface

Plasma injection from near ms orbit; Plasma accretion causal: slow ms, Alfven and fast ms points



Twisting of Magnetic Fields

- Except for induction terms, evolution of toroidal magnetic field ~ Newtonian MHD
- ➔ Source: Differential plasma rotation
- ➔ Schwarzschild: no shear !
- ➔ Extreme Kerr: biggest effect !

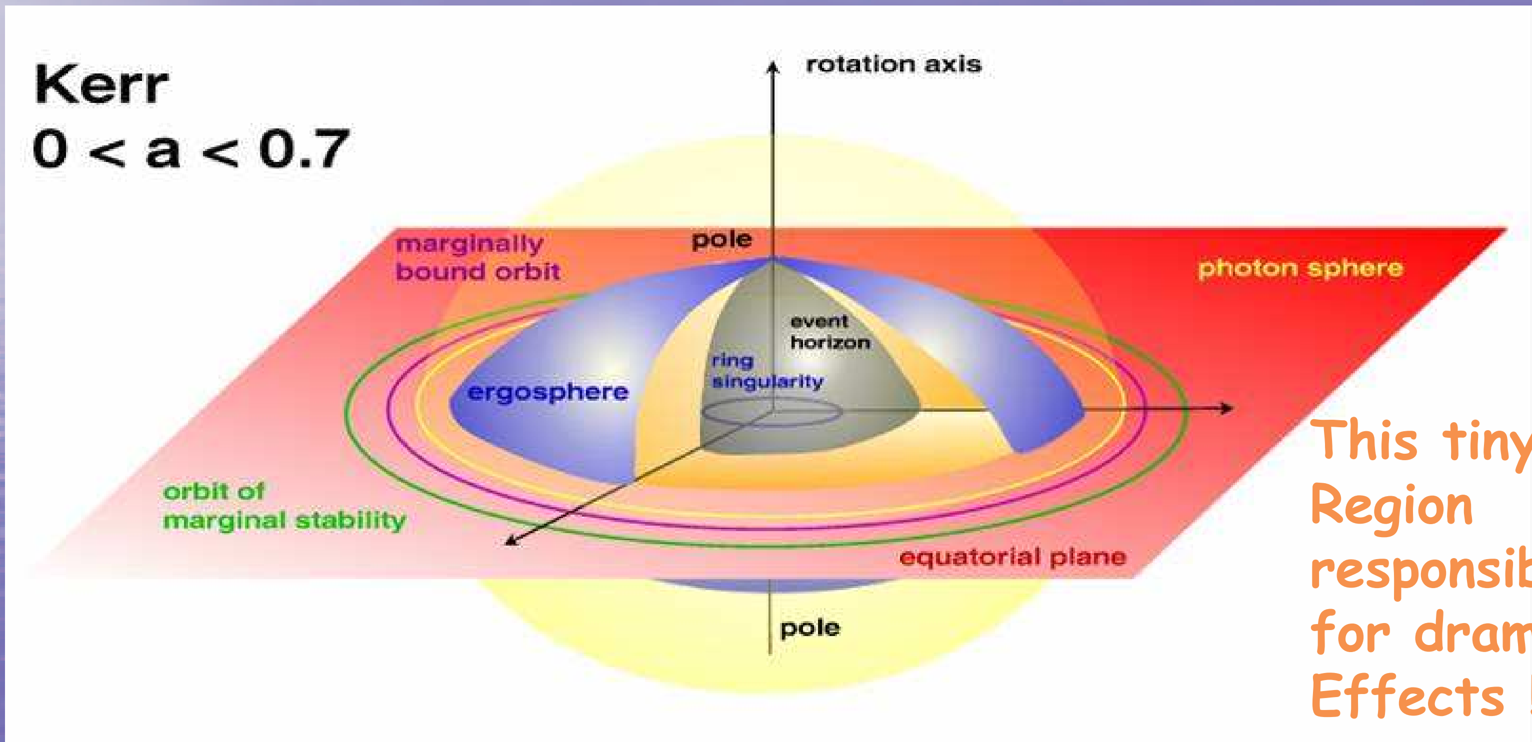
$$\frac{\partial T}{\partial t} + \alpha(\boldsymbol{v}_p \cdot \nabla)T - \alpha\tilde{\omega}^2 \nabla \cdot \left(\frac{T}{\tilde{\omega}^2}\boldsymbol{v}_p\right) - \alpha\tilde{\omega}^2 \nabla \cdot \left(\frac{\eta}{\gamma\tilde{\omega}^2}\nabla T\right)$$

$T \sim RB_\phi$

$$= \alpha\tilde{\omega}^2 \boldsymbol{B}_p \cdot \nabla\Omega + \alpha\tilde{\omega}\, \boldsymbol{e}_\phi \cdot \nabla \times \left(\frac{\eta}{\gamma}\frac{\partial \boldsymbol{E}_p}{\partial t}\right) .$$

Operates outside horizon

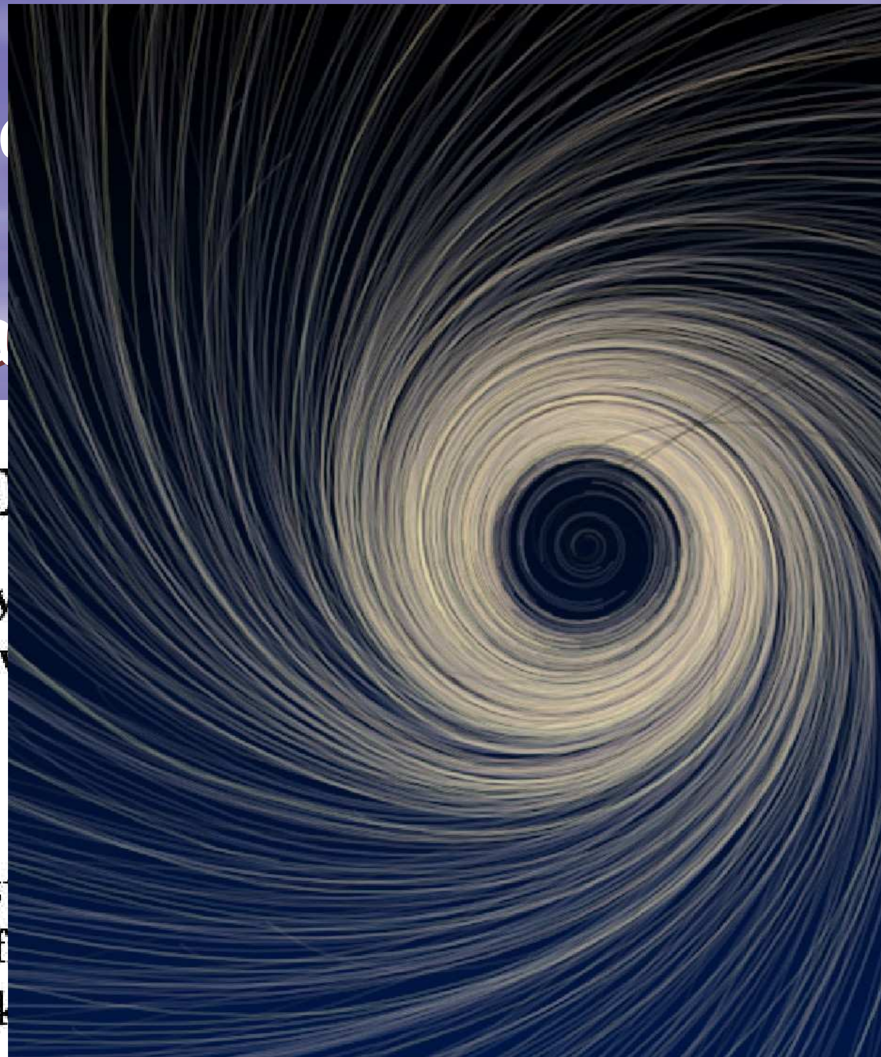
Black Hole Accretion is Different from Newtonian



This tiny Region responsible for dramatic Effects !

In ergoregion, plasma is driven to corotation with horizon.

Each form of matter
will be driven to corot
within the ergosphere
→ Boundary Layer nea



Gravitational E

- Boundary layer: Angular frequency
observers (fixed stars) is given by (

$$\Omega = \frac{U^\phi}{U^t}$$

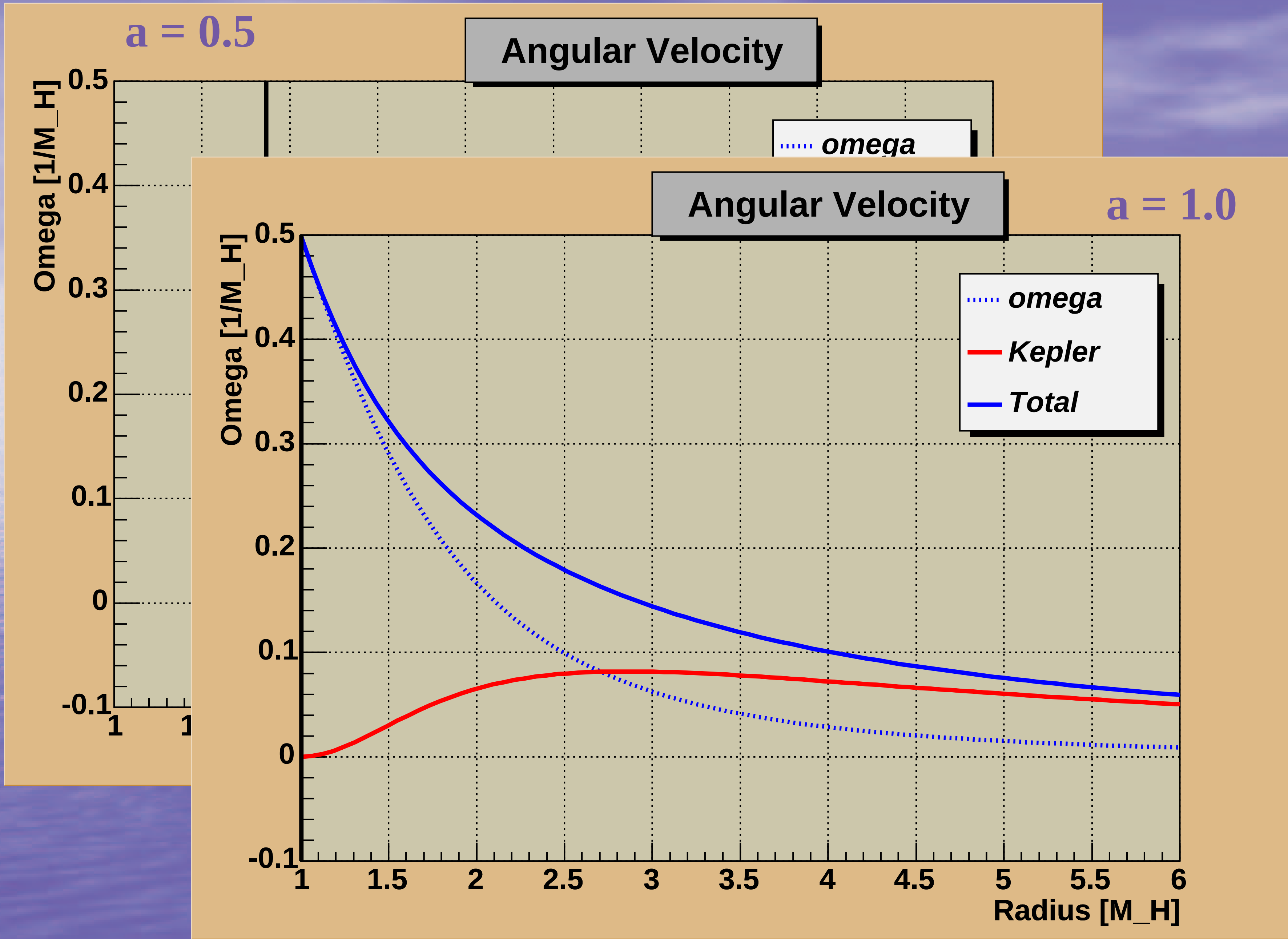
where U^μ is the 4–velocity of mat
poloidal motion of matter, angular f
gular momentum are related over a l

$\Omega_H = \omega(r_H)$

$$\Omega = \omega + \frac{\alpha^2}{R^2}\frac{\lambda}{1-\omega\lambda}. \quad (2)$$

$R \equiv \sqrt{h_{\phi\phi}}$ is the cylindrical radius.

In Schwarzschild:
→No rotation
near Horizon !


a = 0.5
Angular Velocity
Omega [1/M_H]
omega
a = 1.0
Angular Velocity
Omega [1/M_H]
omega
Kepler
Total
0.5
0.4
0.3
0.2
0.1
0
-0.1
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
Radius [M_H]

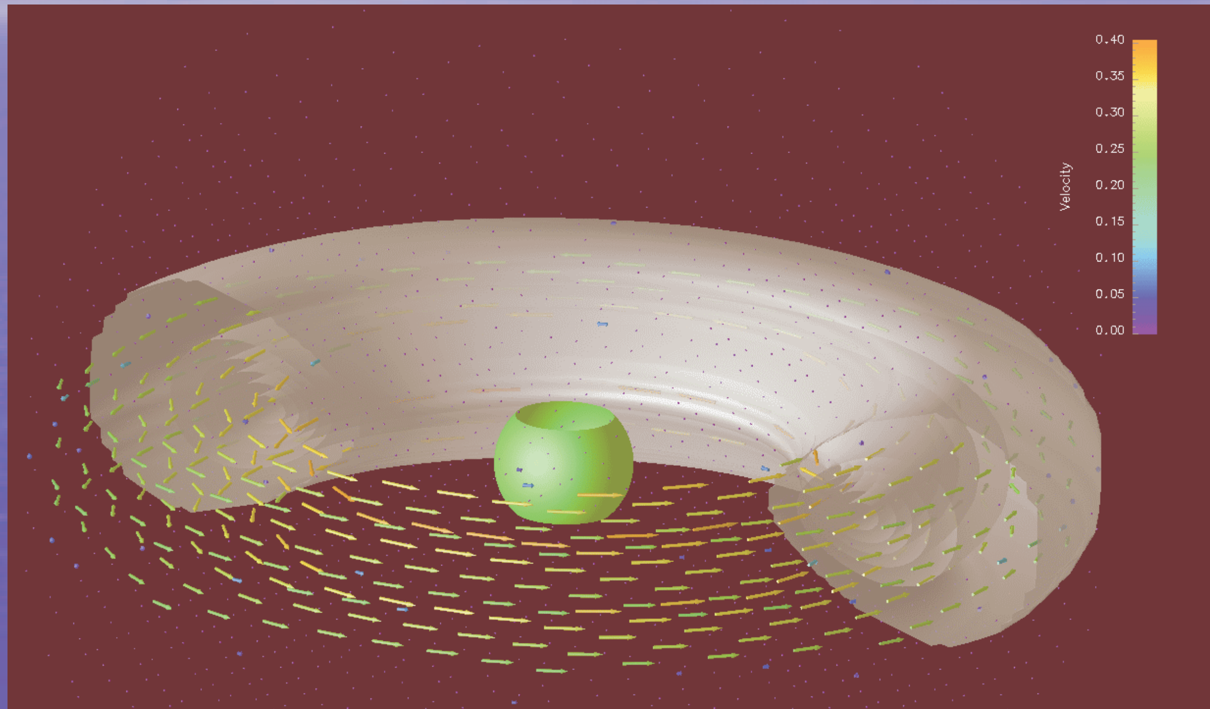
GRMHD Accretion from a Torus + B
→ Non-Radiative Accretion Flows

Koide et al. (2000);

De Villiers, Hawley & Krolik 2003 - 2005 (3D non-conservative GRMHD in BL);

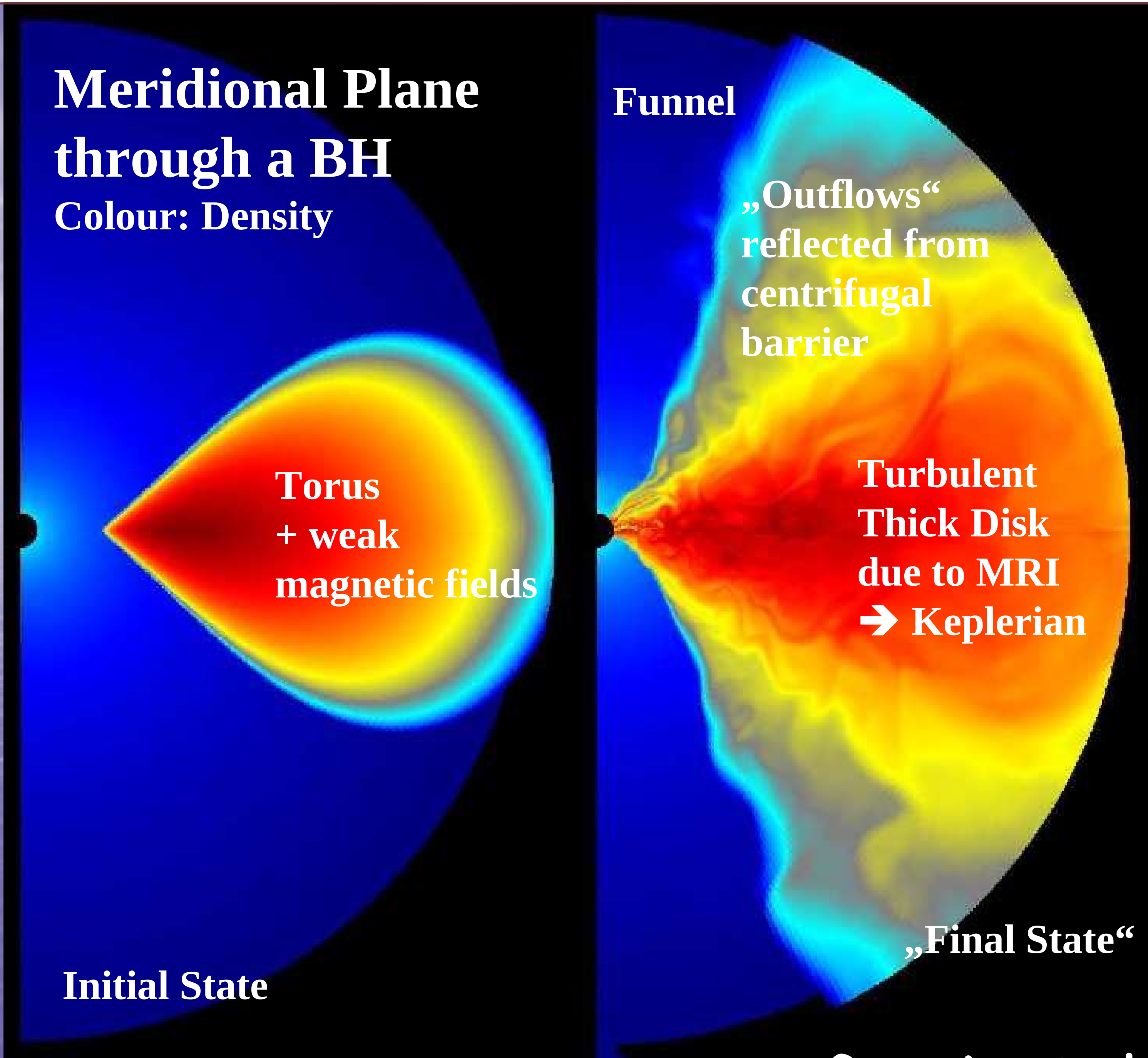
Gammie et al. 2003, 2004 (2D conservative GRMHD in BL coordinates);

Anton et al. (2005), based on GENESIS (conservative GRMHD)



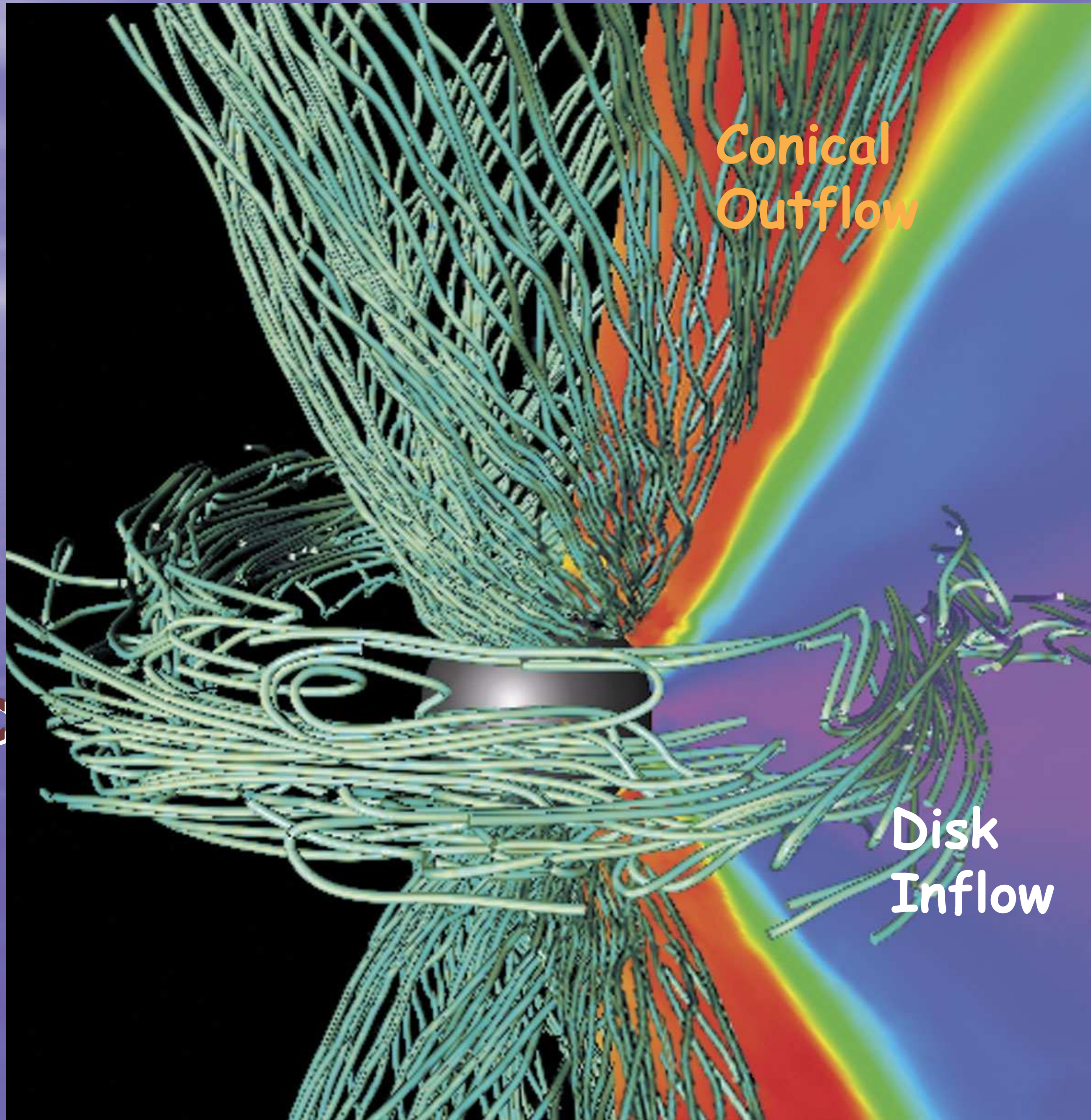
Initial condition (exact mech. equilibrium + weak magnetic fields)

Standard Paradigm of GRMHD



Gammie et al. 2004

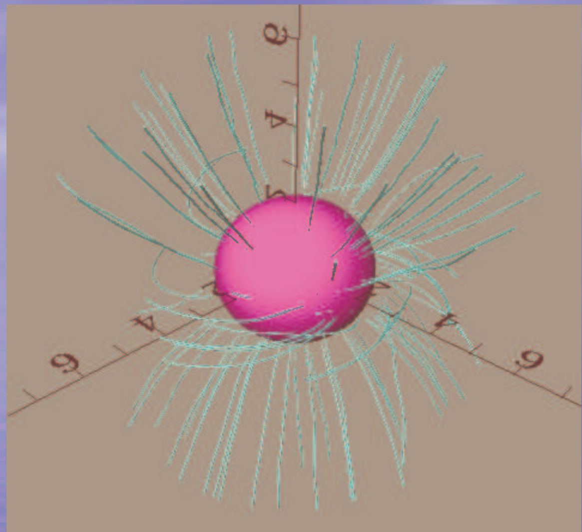
Outflows in Quasars & Micro-Quasars ? ➔ „Stochastic Funnel-Flow"



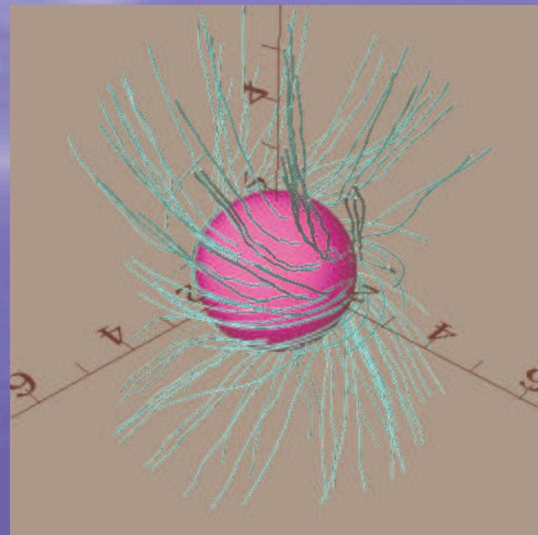
Krolik 2005

Field line twisting by rotating Black Holes

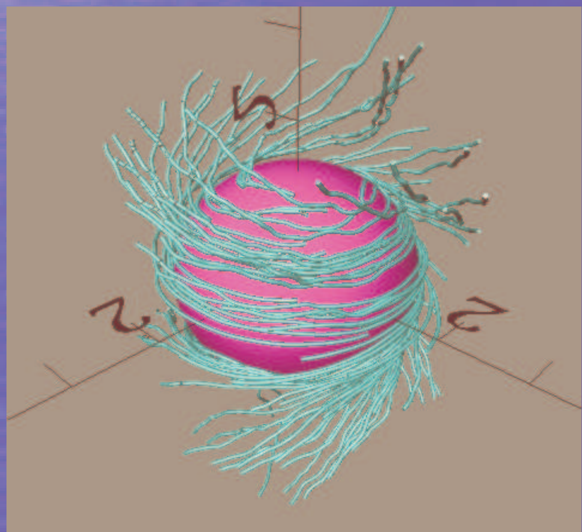
a = 0



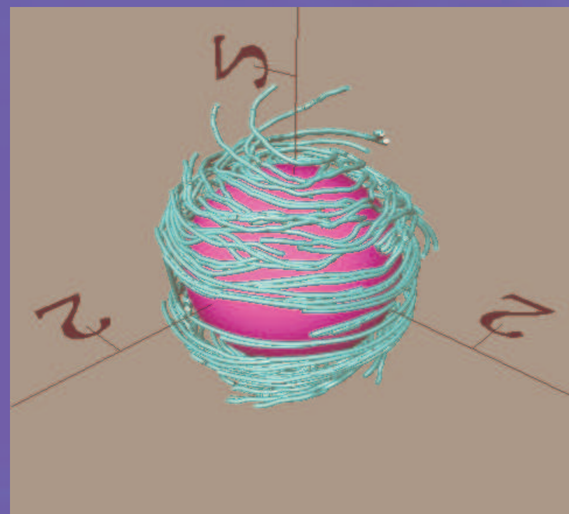
a = 0.5



a = 0.9



a = .998



GRMHD Simulations (Hawley et al. 2005)

Black Holes → 2 Energy Reservoirs

- Potential energy → tapped by accretion
- Rotational energy → tapped by magnetic fields, similar to rotating neutron stars

$$L_{Rot} = E_{Rot}/t_{brake}$$
$$\sim 10^{46}\ \mathrm{erg/s}\ (M_H/10^9\ M_S)\ (t_H/t_{brake})$$

$$L_{Rot} = E_{Rot}/t_{brake}$$
$$\sim 10^{38}\ \mathrm{erg/s}\ (M_H/10\ M_S)\ (t_H/t_{brake})$$

$$t_{brake} = f\,(a, B, \ldots) \quad [\mathrm{BZ}\ 1977]$$

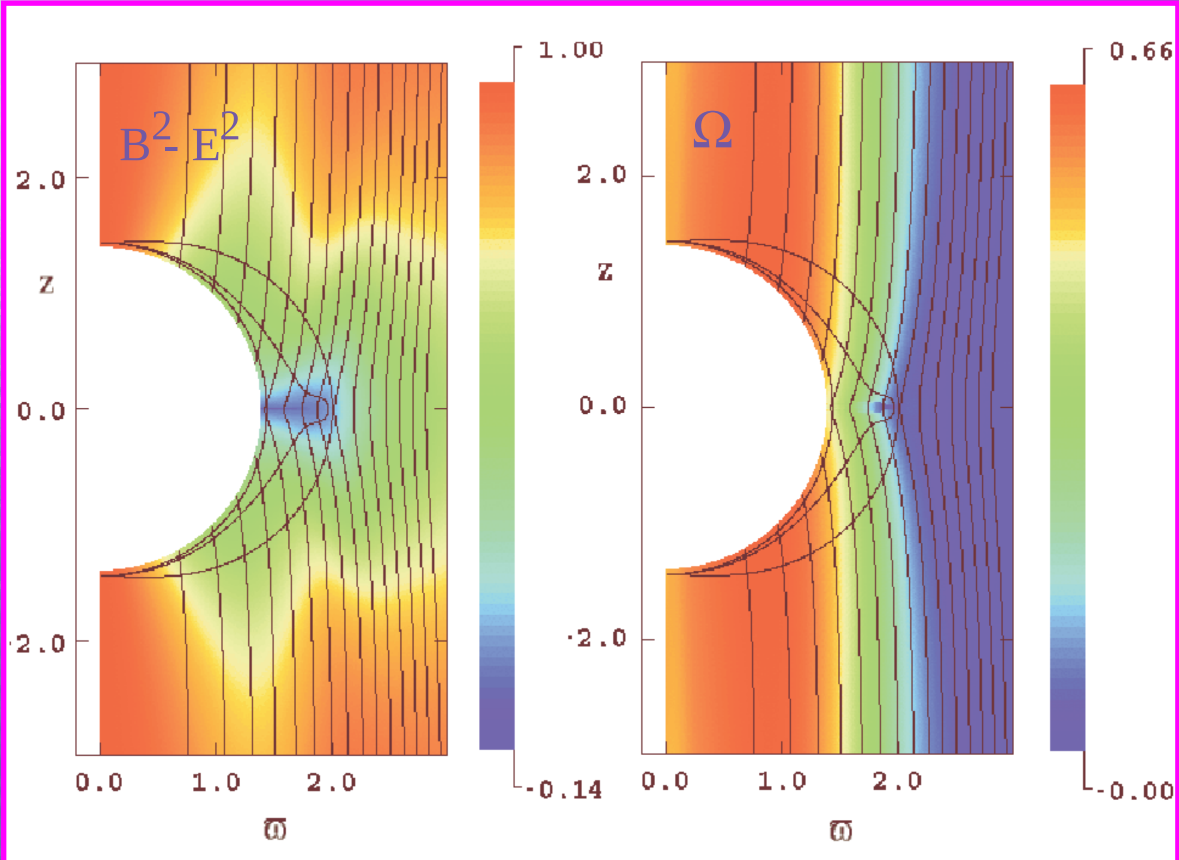
$$L_{BZ} = k\, B_H^2\, r_H^2 c\, (a/M)^2\, (\Omega_F[\Omega_H - \Omega_F]/\Omega_H^2) \sim M_H$$

FF Blandford-Znajek Process

Example: Wald solution (non-rotating) ➔ Testbed for Blandford-Znajek

➔ **Field lines connected to ergosphere are driven to rotation**

➔ Poynting flux ➔ Energy extraction from ergosphere !



Kerr black hole in uniform magnetic field at infinity; Thin plasma version BH magnetosphere.

a = 0.9

➔ **$\Omega_F \sim \Omega_H/2$!**

S. Komissarov 2004

Ingredients Formation Relativistic Jets

- **Strong magnetosphere** with rapid rotation (disk around BHs) → Strong field limit.
- **Plasma injection** (disk plasma, pairs from MS²). High initial Poynting flux (BZ) is converted into kinetic energy by collimating flux-tubes (Camenzind 1996)
  → Michel's parameter: $\Gamma \sim \sigma \sim 10$:
  σ = Poynting flux/Total energy flux | Inj.
- **Flux-tubes are collimated** by current system: hoop stress (B_ϕ) > 100 M_H.
- **For high spin**, a > 0.9, magnetic flux-tubes connected to horizon → Poynting-flux (BZ).

Summary: Plasma & Maxwell

- **Space plasmas** in general extremely thin
- → 10^6 ccm – 10^{-5} ccm (kpc Jets)
- → **collisionless plasmas**: shock heating mediated by waves and not collisions
- → continuum approximation valid beyond **Debye length** << Dimension of Plasma
- → with magnetic fields → **Alfven and magnetosonic waves**, not sound waves !
- → Alfven speed can be near speed of light, even in solar system ! → Relativistic MHD
- Example: Axisymmetric Field Structures (Flux tubes, magnetospheres).
- → Can easily be extended to Relativistic MHD