Problem

Given that

- $I_\nu = (1 - e^{-\tau})\, B_\nu(T)$
- brightness temperature $T_b(\nu) \equiv 2\, k_B\, \nu^2 / I_\nu c^2$

Show that in the Rayleigh-Jeans regime:

$e^{-\tau} = (T - T_b)/T.$

At 10 GHz, the ARCADE experiment found:

$T_b = 2.721 \pm 0.01$ K (Fixsen et al. 2004).

Find the 2σ lower limit on the optical depth given $T_{\text{CMB}} = 2.725 \pm 0.002$ K (COBE).

Problem

- Show that if the sound speed is relativistic, $c_s = c/\sqrt{3}$, the Friedman equation implies that in a flat universe
  - $\lambda_J = (\sqrt{8\pi/3})\, c/H$
- What does this imply about fluctuations in the early universe?

Problem

- Estimate the thickness of the last scattering shell (Silk scale):
- Mean time between collisions
  - $t_c \sim (c n_e \sigma_T)^{-1}$
  - Assume Silk scale is distance travelled in Hubble time 1/H at z_{ls} by a photon (random walk)
- To get $H(t_{ls})$, recall $\Omega_m(t) \propto \rho_m(t)/H^2(t)$
  - What is $\Omega_m(t_{ls})$?
- Data:
  - $\Omega_m = 0.135\ h^{-2}$; $\Omega_b = 0.0224\ h^{-2}$; $\sigma_T = 6.65\times10^{-25}$ cm^{-2}

Problem

- The *COBE* fluctuations corresponded to T = 18 μK. Show that this leads to a value for the density fluctuation on horizon entry of 2×10^{-5}.