Cosmic Ray Acceleration

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SN1006: A supernova remnant 7,000 light years from Earth
Mainly protons

Cosmic ray spectrum arriving at earth
(Nagano & Watson 2000)
Galactic Supernova remnants

(II) Probably galactic

(III) Extra-galactic ?

~$E^{-2.6}$

~$E^{-3}$

log(FLUX $\times E^3$ in eV$^2$ m$^{-2}$ s$^{-1}$ Sr$^{-1}$)

log(ENERGY in eV)
Historical shell supernova remnants

Tycho 1572AD
Kepler 1604AD
SN1006
Cas A 1680AD

Chandra observations

NASA/CXC/Rutgers/
J.Hughes et al.

NASA/CXC/Rutgers/
J.Warren & J.Hughes et al.

NASA/CXC/NCSU/
S.Reynolds et al.

NASA/CXC/MIT/UMass Amherst/
M.D.Stage et al.
Cassiopeia A

Radio (VLA)

Infrared (Spitzer)

Optical (Hubble)

X-ray (Chandra)

Mixture of line radiation & synchrotron continuum

Synchrotron in magnetic field $\sim 0.1\text{-}1\text{mG}$

Radio ($h\nu \sim 10^{-5}\text{eV}$): electron energy $\sim 1\text{ GeV}$

X-ray ($h\nu \sim 10^{3}\text{eV}$): electron energy $\sim 10\text{ TeV}$
HESS: $\gamma$-rays directly produced by TeV particles

SNR RX J1713.7-3946

Aharonian et al
Arrival directions of highest energy cosmic rays

Auger (2007)

Fig. 2. Aitoff projection of the celestial sphere in galactic coordinates with circles of radius 3.1° centered at the arrival directions of the 27 cosmic rays with highest energy detected by the Pierre Auger Observatory. The positions of the 472 AGN (318 in the field of view of the Observatory) with redshift $z \leq 0.018$ ($D < 75$ Mpc) from the 12th edition of the catalog of quasars and active nuclei (12) are indicated by red asterisks. The solid line represents the border of the field of view (zenith angles smaller than 60°). Darker color indicates larger relative exposure. Each colored band has equal integrated exposure. The dashed line is the supagalactic plane. Centaurus A, one of our closest AGN, is marked in white.
Active galaxies

Cygnus A

VLA

Centaurus A

X-ray (Chandra)
Cosmic ray acceleration by shocks
Cosmic ray acceleration

Due to scattering, CR recrosses shock many times
Gains energy at each crossing
Shock acceleration energy spectrum

Fractional CR loss per shock crossing
\[ \frac{\Delta n}{n} = \frac{u_s}{c} \]

Fractional energy gain per shock crossing
\[ \frac{\Delta \epsilon}{\epsilon} = \frac{u_s}{c} \]

Differential energy spectrum
\[ N(\epsilon) d\epsilon \propto \epsilon^{-2} d\epsilon \]
Maximum CR energy
CR upstream of shock

Exponential density

\[ n_{cr} = n_0 e^{ux/D} \]

Scaleheight

\[ L = \frac{D}{u} \]
CR acceleration time

Average time per shock crossing: \[ \Delta t = \frac{4L}{u_{\text{shock}}} \]

Energy gain per crossing: \[ \Delta \epsilon / \epsilon = \frac{u_{\text{shock}}}{c} \]

\[ \frac{\Delta \epsilon}{\Delta t} = \frac{\epsilon}{4} \frac{u_{\text{shock}}^2}{D_{\text{upstream}}} \]

Time needed for acceleration (Lagage & Cesarsky) \[ \tau = \frac{4D_{\text{upstream}}}{u_{\text{shock}}^2} + \frac{4D_{\text{downstream}}}{(u_{\text{shock}} / 4)^2} \]
Maximum CR energy

(Lagage & Cesarsky)

Acceleration time: \( \tau = \frac{8D}{u_{\text{shock}}^2} \)

Smallest diffusion coefficient: \( D = \frac{r_g c}{3} \)

Maximum CR energy: \( \frac{3}{8} e u_{\text{shock}} B R \)

Typically for young SNR

Max CR energy
\[
\left( \frac{E}{eV} \right) = 3 \times 10^{13} \left( \frac{u_{\text{shock}}}{5000 \text{ km s}^{-1}} \right)^2 \left( \frac{B}{3 \mu \text{G}} \right) \left( \frac{t}{300 \text{ yr}} \right)
\]

Appears that:
- Acceleration too slow to get to PeV
- CR precursor too large
- Magnetic field too small
Hillas diagram

(condition on $uRB$)

Figure 1  Size and magnetic field strength of possible sites of particle acceleration. Objects below the diagonal line cannot accelerate protons to $10^{20}$ eV.
Perpendicular shocks

(Jokipii 1982, 1987)
CR trajectory at perpendicular shock (no scattering)

\[ E = -u_{\text{shock}} \times B \]

\[ v_{\text{drift}} = \frac{E \times B}{B^2} \]

CR gain energy by drifting in E field
CR acceleration at perpendicular shock: with scattering

Diffusive shock theory applies
Provided gyrocentre diffuses over distances
greater than Larmor radius during shock transit
Same power law (see later)

No scattering

Weak scattering

Strong scattering

Not to scale
CR acceleration at perpendicular shock

\[ E = -u_{\text{shock}} \times B \]

Transit between pole & equator: energy gain ~ \( eER = eu_{\text{shock}}BR \)

Hillas parameter as with parallel shock: similar max CR energy
The case of SN1006

At perpendicular shocks

- Acceleration is faster – potentially higher CR energy
- CR energy limited to $euBR$ (Hillas) by space rather than time
- CR scattering frequency has to be in right range

Room for discussion!

(Polar x-ray synchrotron emission? (Rothenflug et al 2004))
Magnetic field

1) Need larger field

2) Structured on scale of CR Larmor radius

\[
\left( \frac{E}{eV} \right) = 3 \times 10^{13} \left( \frac{u_{\text{shock}}}{5000 \text{ km s}^{-1}} \right)^2 \left( \frac{B}{3 \mu \text{G}} \right) \left( \frac{t}{300 \text{ yr}} \right)
\]
Electric currents carried by CR and thermal plasma

Density of $10^{15}$eV CR: $\sim 10^{-12}$ cm$^{-3}$
Current density: $j_{cr} \sim 10^{-18}$ Amp m$^{-2}$

CR current must be balanced by current carried by thermal plasma

$$j_{\text{thermal}} = -j_{cr}$$

$j_{\text{thermal}} \times B$ force acts on plasma to balance $j_{cr} \times B$ force on CR
Streaming instabilities amplify magnetic field
Lucek & Bell (2000)

CR treated as particles

Thermal plasma as MHD
1) CR currents set plasma in motion

\[ \rho \frac{du}{dt} = -j_{CR} \times B \]

2) Motion stretches field lines

\[ \frac{\partial B}{\partial t} = \nabla \times (u \times B) \]

Need CR current to drive turbulence on scale of CR Larmor radius \( r_g \) in time \( t \)

\[ \frac{1}{2} \frac{j_{CR} B t^2}{\rho} > \frac{p_{CR}}{eB} \]

CR current carries energy flux \( Q_{CR} \)

\[ \frac{Q_{CR}}{\rho u_{\text{shock}}^3} > 6 \left( \frac{E_{CR}}{\text{PeV}} \right)^2 \left( \frac{B}{3 \mu G} \right)^{-2} \left( \frac{u_{\text{shock}}}{5000 \text{ km s}^{-1}} \right)^{-3} \left( \frac{t}{300 \text{ yr}} \right)^{-2} \]
CR-driven instability amplifies magnetic field

$$\rho \frac{du}{dt} = -j_{CR} \times B$$

$$\frac{\partial B}{\partial t} = \nabla \times (u \times B)$$
Streaming CR excite instabilities

CR streaming ahead of shock
Excite instabilities
Amplify magnetic field

Shock
upstream
downstream
Simplest form: expanding loops of $B$

$j \times B$ expands loops

$\rightarrow$ stretches field lines

$\rightarrow$ more $B$

$\rightarrow$ more $j \times B$
Simplest form: expanding loops of $B$

$j \times B$ expands loops

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$\rightarrow$ more $B$

$\rightarrow$ more $j \times B$

CR current
Streaming instabilities amplify magnetic field

Lucek & Bell (2000)

B field lines, $t = 0$

CR treated as particles

Thermal plasma as MHD
Dispersion relation

\[ \gamma_{\text{max}} = j_{CR} \left( \frac{\mu_0}{2\rho} \right)^{1/2} \]

Red line is growth rate

Max growth rate

Wavelength longer than Larmor radius
CR follow field lines.
jxB drives weak instability

\[ \rho \frac{du}{dt} = -j_{CR} \times B \]

Magnetic tension inhibits instability

\[ \frac{\partial B}{\partial t} = \nabla \times (u \times B) \]

\[ \gamma = \left( \frac{kB_0 j_{CR}}{\rho} \right)^{1/2} \]
Non-linear growth – expanding loops

Slices through $|B|$ - time sequence (fixed CR current)

Field lines: wandering spirals

Cavities and walls in $|B|$ & $\rho$
Non-linear growth

\[ \gamma_{\text{max}} = 0.8 \]

Strong fields for \( t > 10^{\gamma_{\text{max}}^{-1}} \)

\[ \gamma_{\text{max}} = j_{\text{CR}} \left( \frac{\mu_0}{2\rho} \right)^{1/2} \quad \implies \quad j_{\text{CR}} > 10 \left( \frac{2\rho}{\mu_0} \right)^{1/2} t \]

\[ \implies E_{\text{max}} = 4 \times 10^{15} \left( \frac{Q_{\text{CR}}}{\rho u_{\text{shock}}^3} \right) \left( \frac{u_{\text{shock}}}{5000 \text{ km s}^{-1}} \right)^3 \left( \frac{t}{300 \text{ yr}} \right) \left( \frac{n_e}{\text{cm}^{-3}} \right)^{1/2} \]
Limits on magnetic field
Limits on magnetic field growth

1) Magnetic tension
\[ \frac{B^2}{\mu_0} \approx \left( \frac{u_{\text{shock}}}{c} \right) \left( \frac{Q_{\text{CR}}}{\rho u_{\text{shock}}^3} \right) \rho u_{\text{shock}}^2 \]

2) Low instability growth rate, but normally compensated by drop in CR escape energy
\[ \text{CR current} \propto \frac{\rho u_{\text{shock}}^3}{\text{CR escape energy}} \]

3) Confines CR to shock, inhibits CR current, reduces instability growth
\[ \frac{B^2}{\mu_0} \approx \frac{1}{20} \left( \frac{Q_{\text{CR}}}{\rho u_{\text{shock}}^3} \right)^2 \rho u_{\text{shock}}^2 \]
**Dispersion relation**

Max growth rate

\[ \gamma_{\text{max}} = j_{CR} \left( \frac{\mu_0}{2 \rho} \right)^{1/2} \]

Red line is growth rate

Wavelength longer than Larmor radius

CR follow field lines.

jxB drives weak instability

**Magnetic tension inhibits instability**

\[ \rho \frac{d\mathbf{u}}{dt} = -j_{CR} \times \mathbf{B} \]

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) \]

\[ \gamma = \left( \frac{kB_0 j_{CR}}{\rho} \right)^{1/2} \]

k in units of \( r_g^{-1} \)

\( \omega \) in units of \( v_S^2/cr_g \)
Magnetic field limited by magnetic tension

\[ B^2/(8\pi\rho) \] (cgs)

Vink (2008)

Data for RCW86, SN1006, Tycho, Kepler, Cas A, SN1993J

Fit to obs (Vink):

\[ B \approx 700 \left( \frac{u}{10^4 \text{km s}^{-1}} \right)^{3/2} \left( \frac{n_e}{\text{cm}^{-3}} \right)^{1/2} \mu \text{G} \]

Theory:

\[ B \approx 400 \left( \frac{u}{10^4 \text{km s}^{-1}} \right)^{3/2} \left( \frac{n_e}{\text{cm}^{-3}} \right)^{1/2} \left( \frac{\eta}{0.1} \right)^{1/2} \mu \text{G} \]
Can we observe the turbulence?
Historical shell supernova remnants
(Vink & Laming, 2003; Völk, Berezhko, Ksenofontov, 2005)

Chandra observations

NASA/CXC/Rutgers/ J.Hughes et al.
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Characteristic distances and times

Slices through $|B|$ - time sequence (fixed CR current)

Larmor radius of CR at $10^{15}$eV in 100$\mu$G field = $3 \times 10^{14}$m = 0.5 arc sec at 3kpc

Shock at 5000 km s$^{-1}$ traverses Larmor radius in 2 years
Two SN remnants with varying shock structure

Shock sweeps out pre-shock medium

RX J1713.7-3946 (SN of 393AD)

Cas A (1680AD)

Uchiyama et al 2007

NASA/CXC/MIT/UMass Amherst/Stage et al.
The situation as I see it...

We probably understand the basic microphysics of shock acceleration
• 1st order Fermi produces $\sim E^{-2}$
• CR are scattered by self-excited turbulence

But the global picture is incomplete...
• Lack of clinching observational evidence that SNR accelerate protons to PeV
• Acceleration to a few PeV stretches theory to limit
• Are protons accelerated to 10-100PeV and how?
• How do CR escape into the interstellar medium?
• Extra-Galactic accelerators